Control Technologies for Ocean Engineering

Control technologies as the second axis

Kyushu University
Prof. H.Kajiwara
(2011.11.18,at PNU)

Outline

1 **LQI Control**

Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 **LPV Control**

Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control

Applications

3 Underwater Vehicle

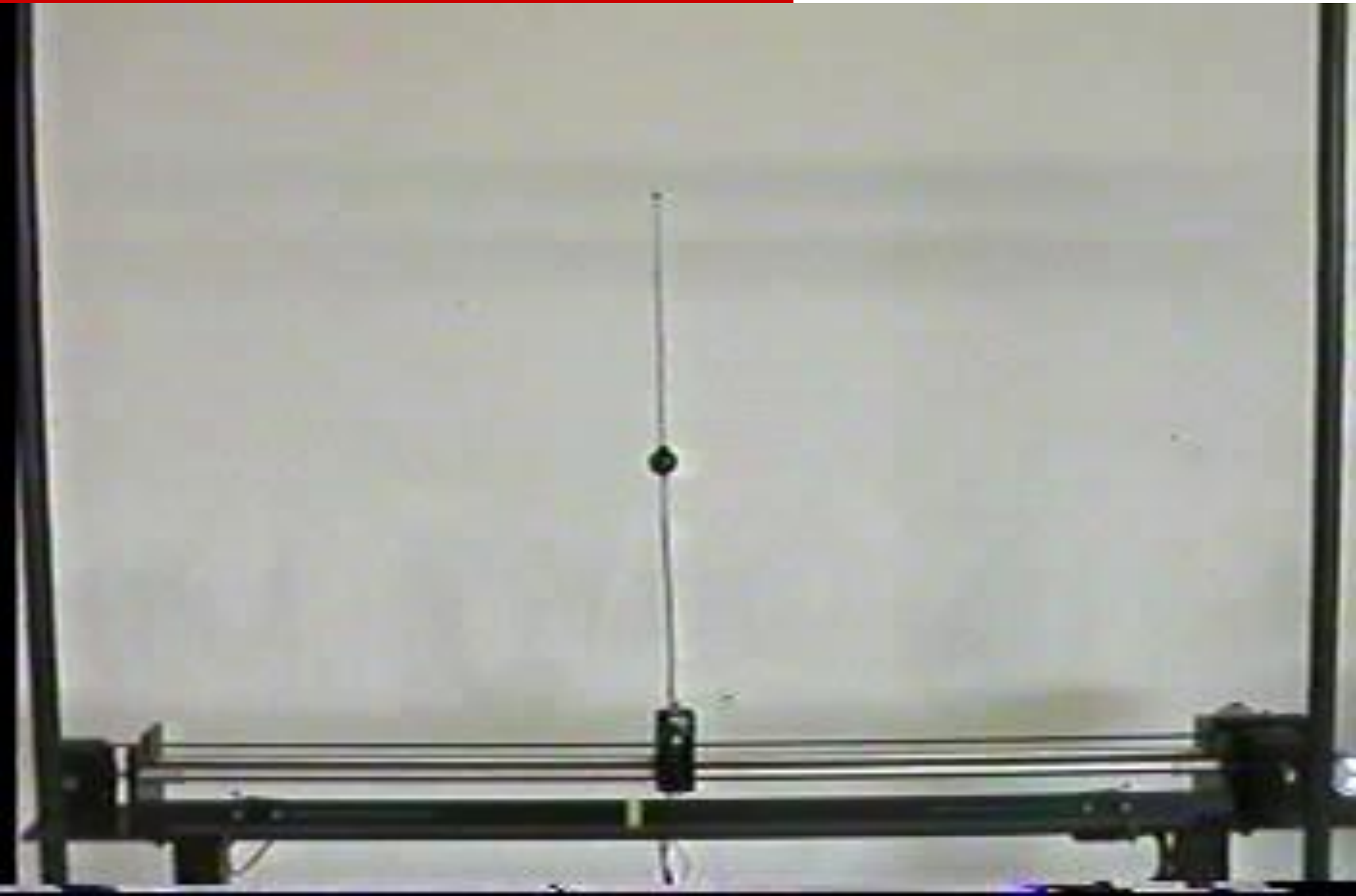
4 Flexible Riser

5 Azimuth thrusters

6 Nomoto’s Model

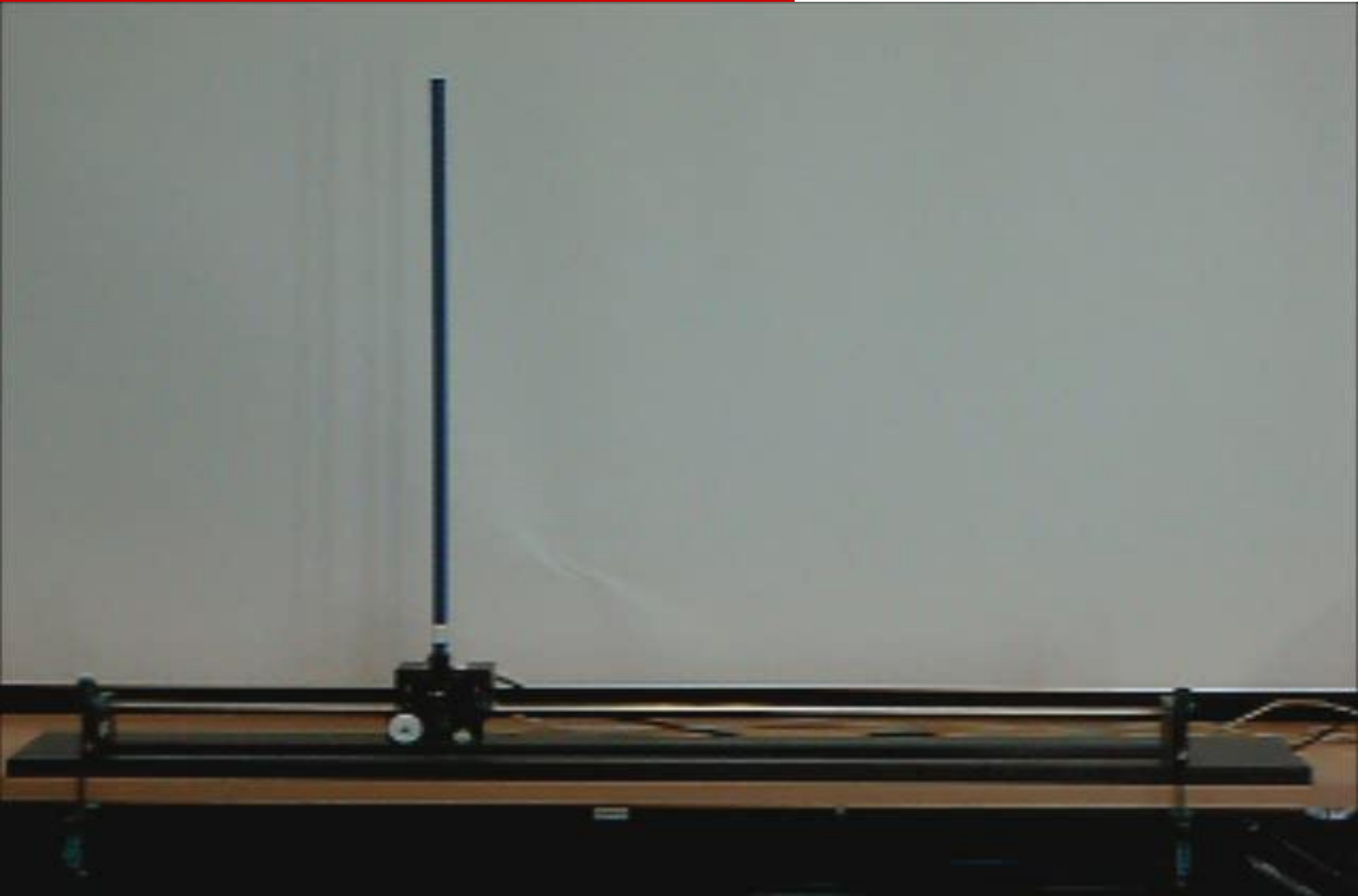
7 Wind Turbine

Double Inverted Pendulum (1979) [3]

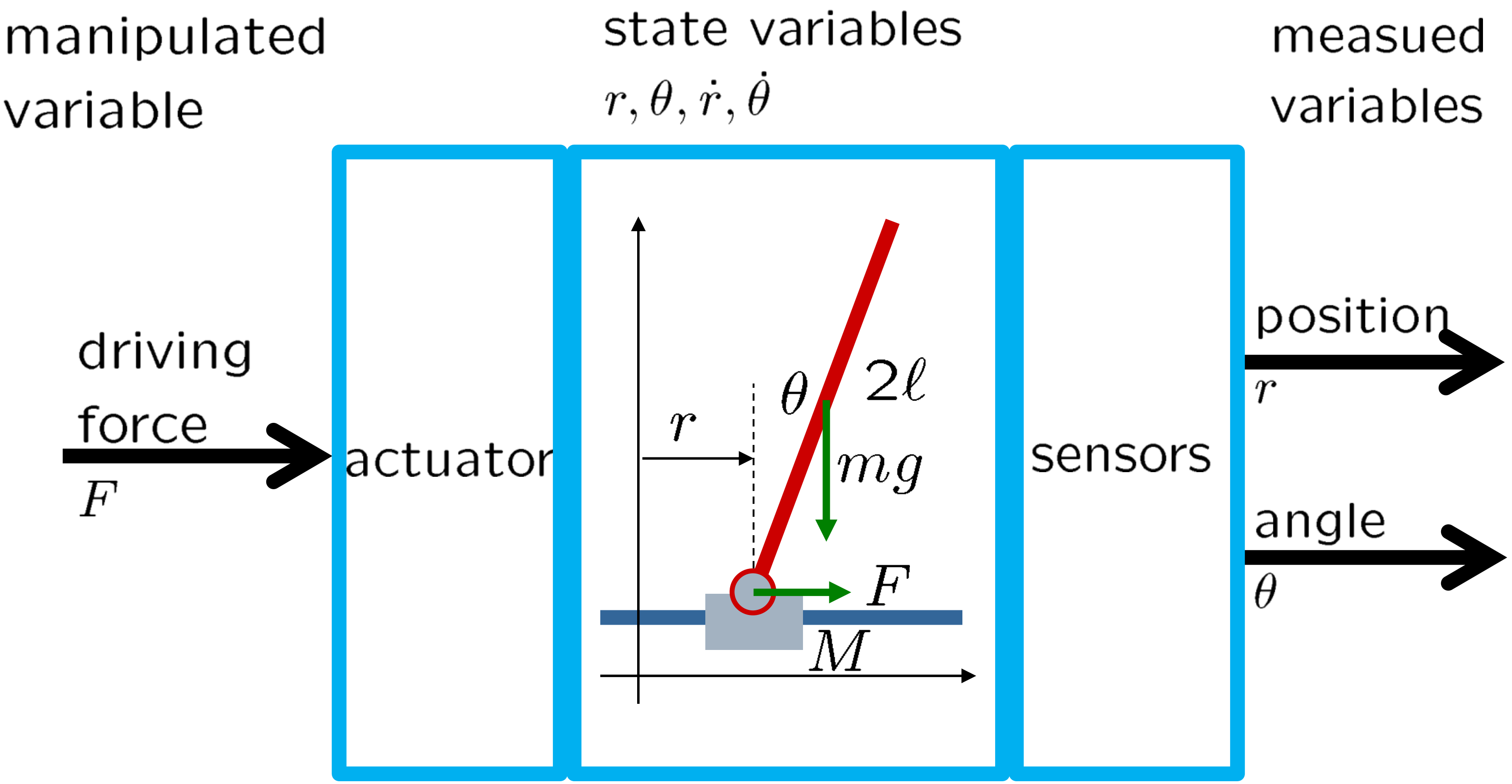


Inverted Pendulum (IP)

[4]



Inverted Pendulum

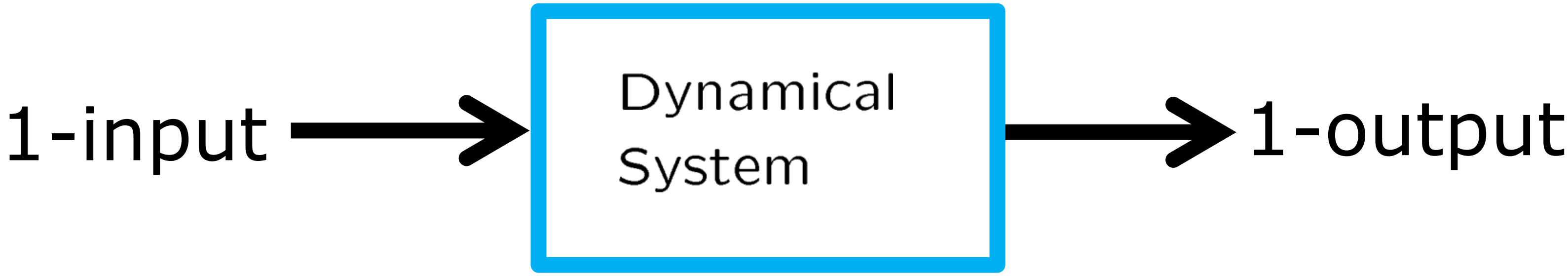


$$\underbrace{\begin{bmatrix} M+m & m\ell\cos\theta \\ m\ell\cos\theta & \frac{4}{3}m\ell^2 \end{bmatrix}}_{M(\xi_1)} \underbrace{\begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix}}_{\dot{\xi}_2} + \underbrace{\begin{bmatrix} -m\ell\dot{\theta}^2\sin\theta \\ 0 \end{bmatrix}}_{C(\xi_1,\xi_2)} + \underbrace{\begin{bmatrix} 0 \\ -m\ell g\sin\theta \end{bmatrix}}_{G(\xi_1)} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{e_1} \underbrace{F}_{\zeta}$$

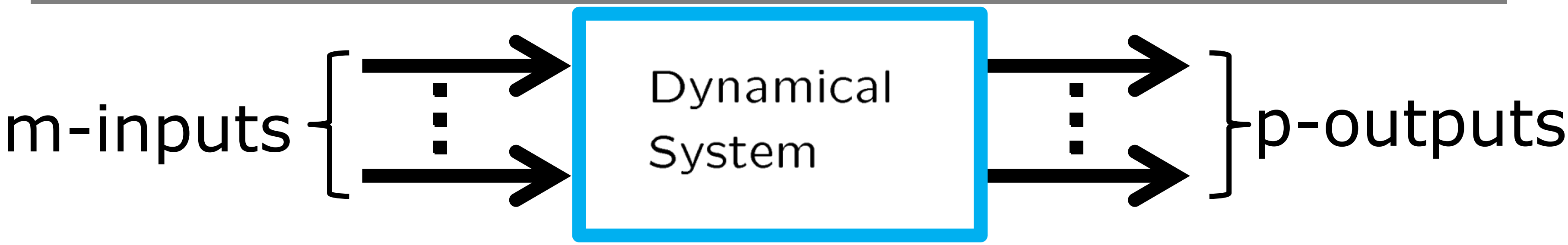
Under-actuated System

Under-actuated System

SISO (Single-Input Single-Output) System



MIMO (Multi-Input Multi-Output) System

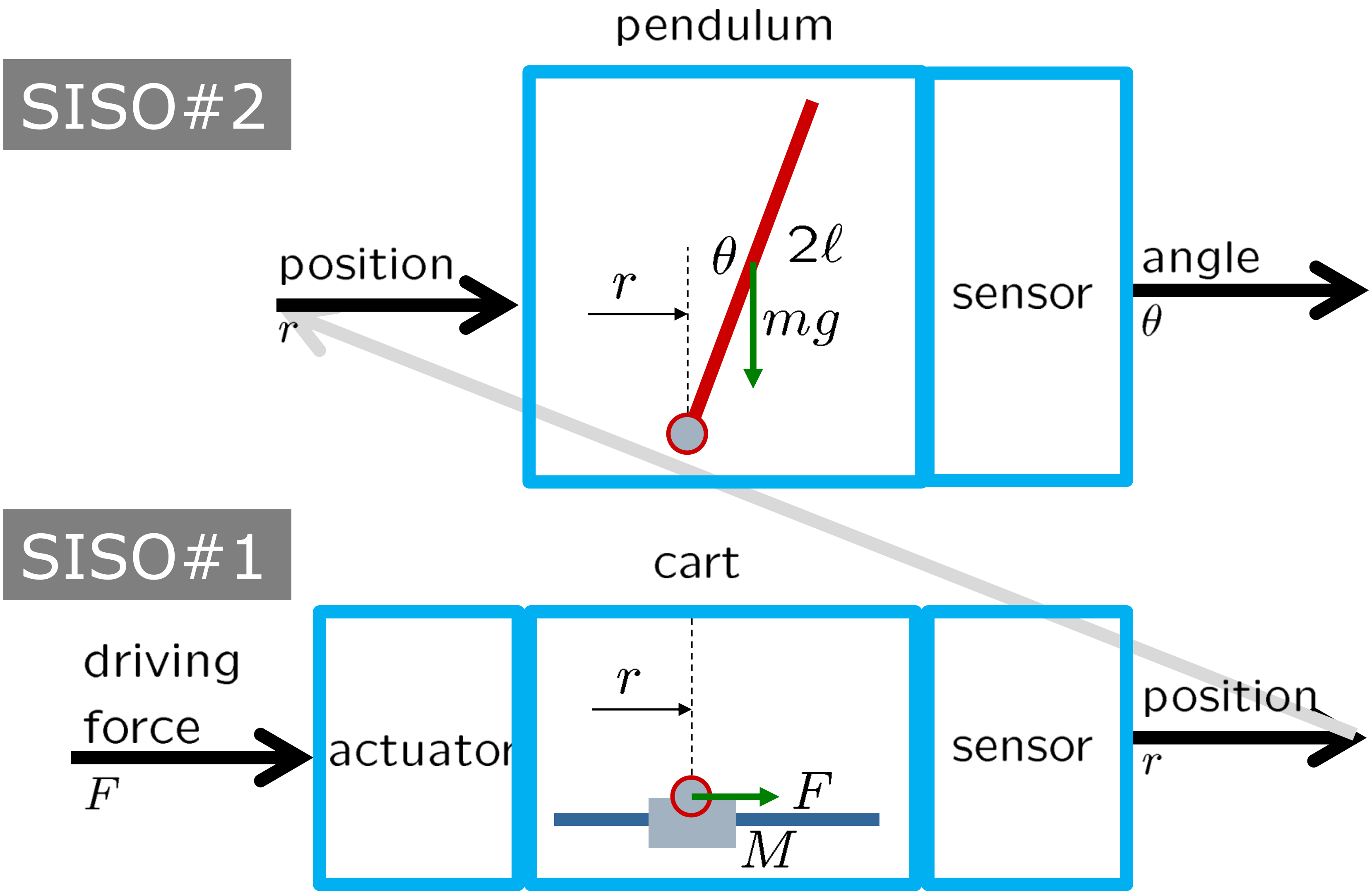


m<p ⇒ Under-actuated System

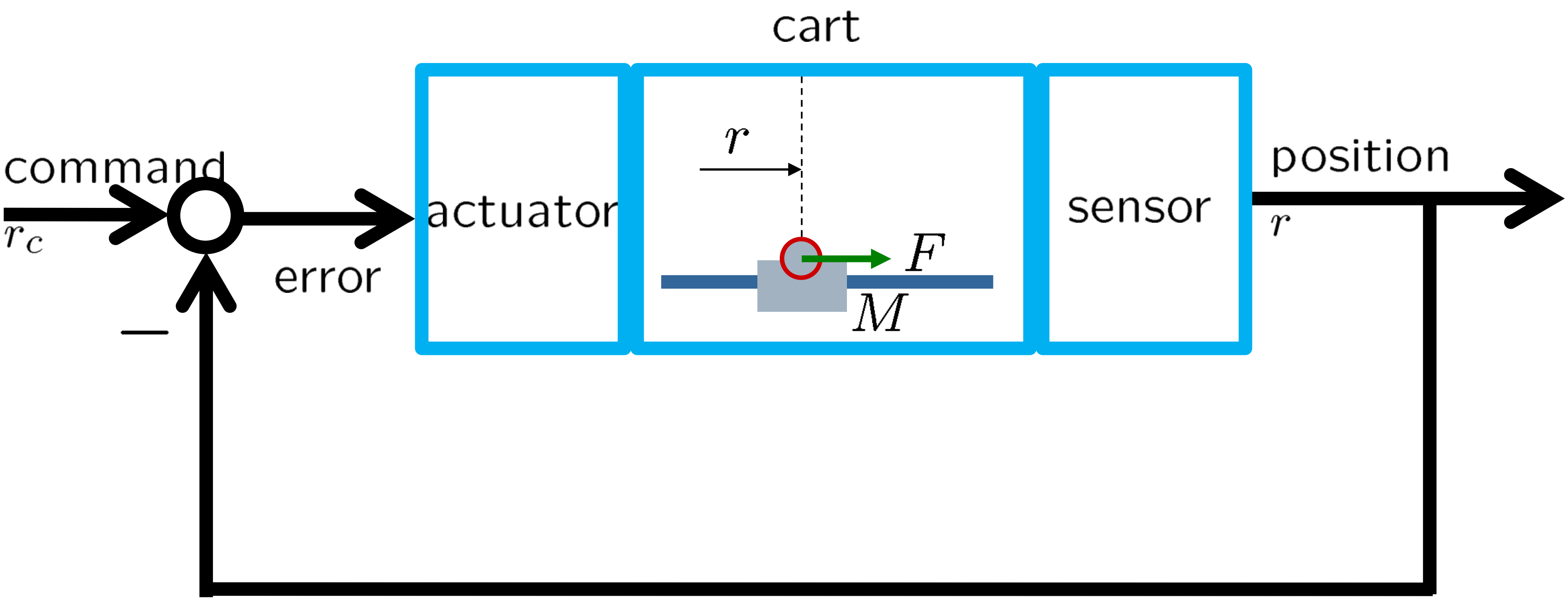
controlled variables as many as manipulated m-variables based on measured p-variables

If you chase after two rabbits, you won't catch either.

Andersen's Approach

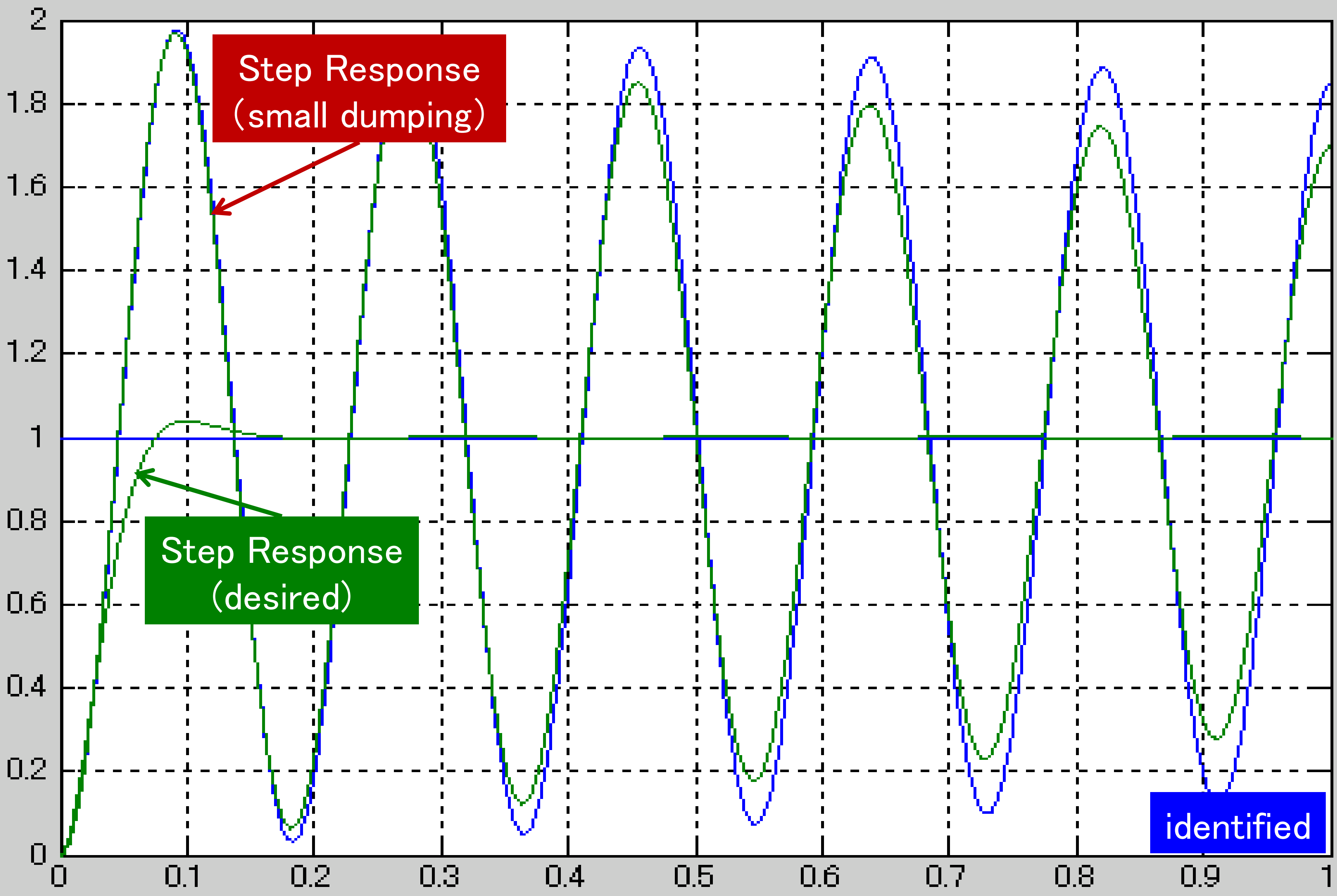


Position Control of a Cart

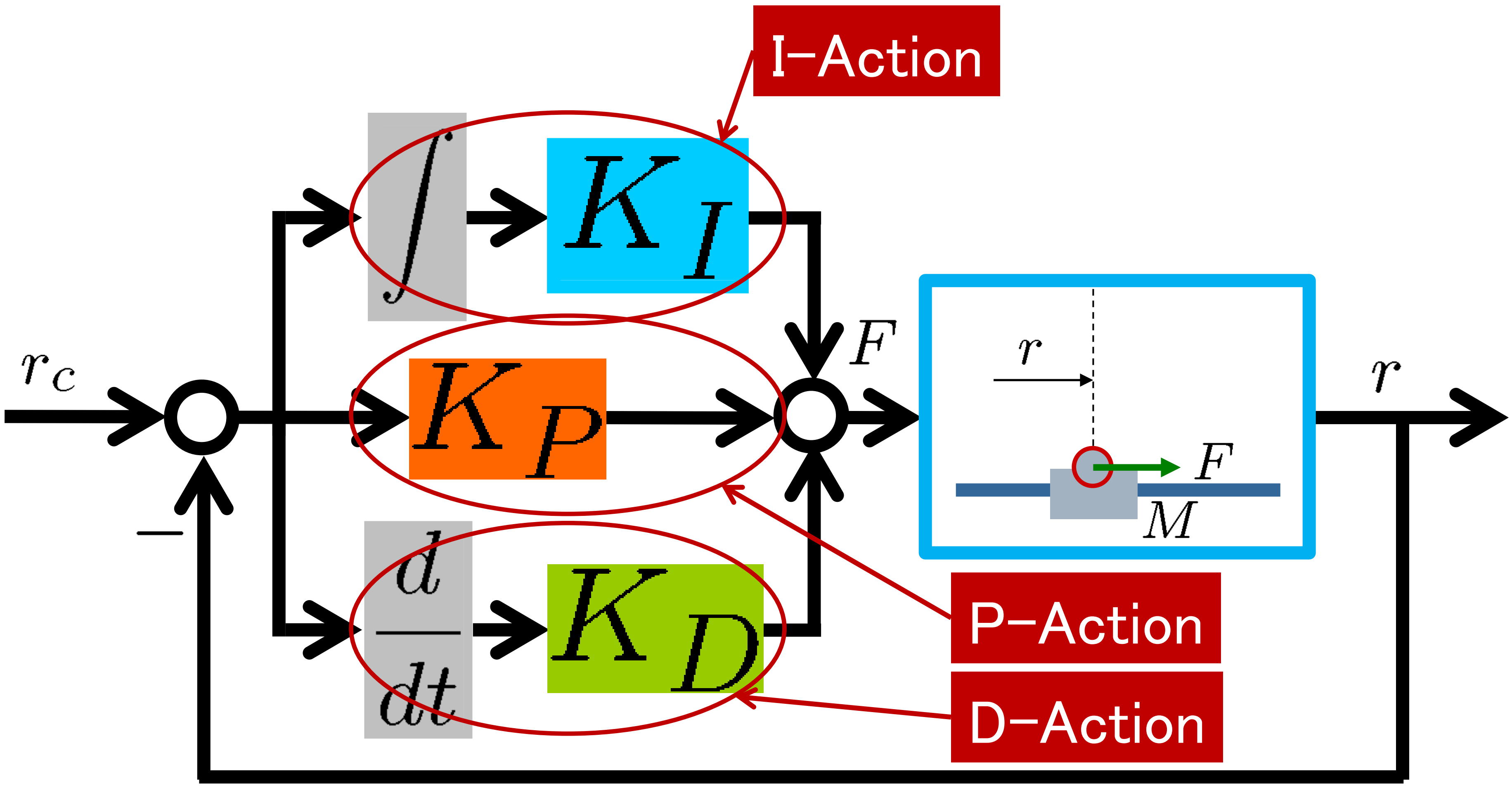


Unity feedback is used to correct the error between given command and current position.

How to Regulate the Oscillation?

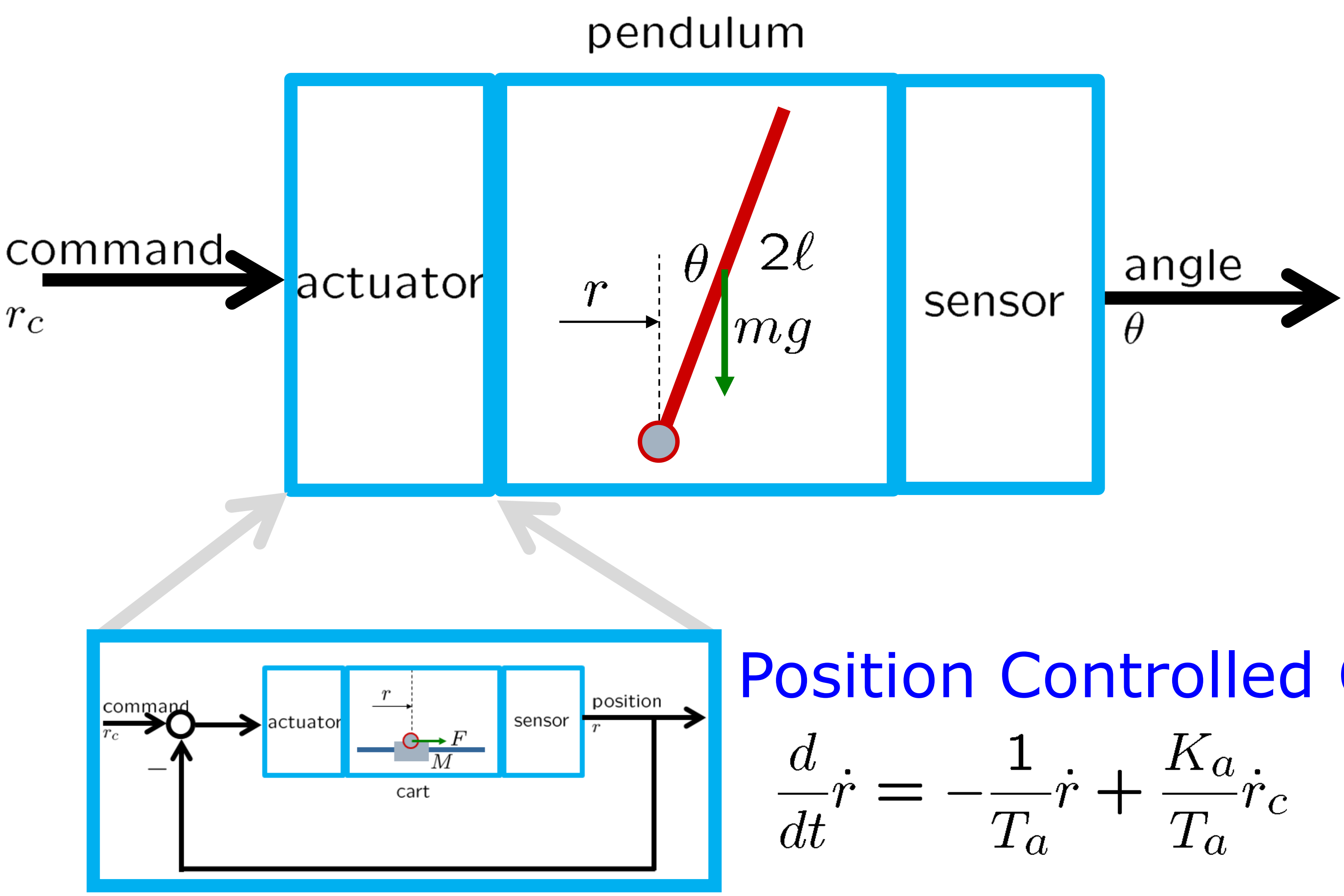


PID Control of a Cart (Motor/Ship)



Which action regulates the oscillation?

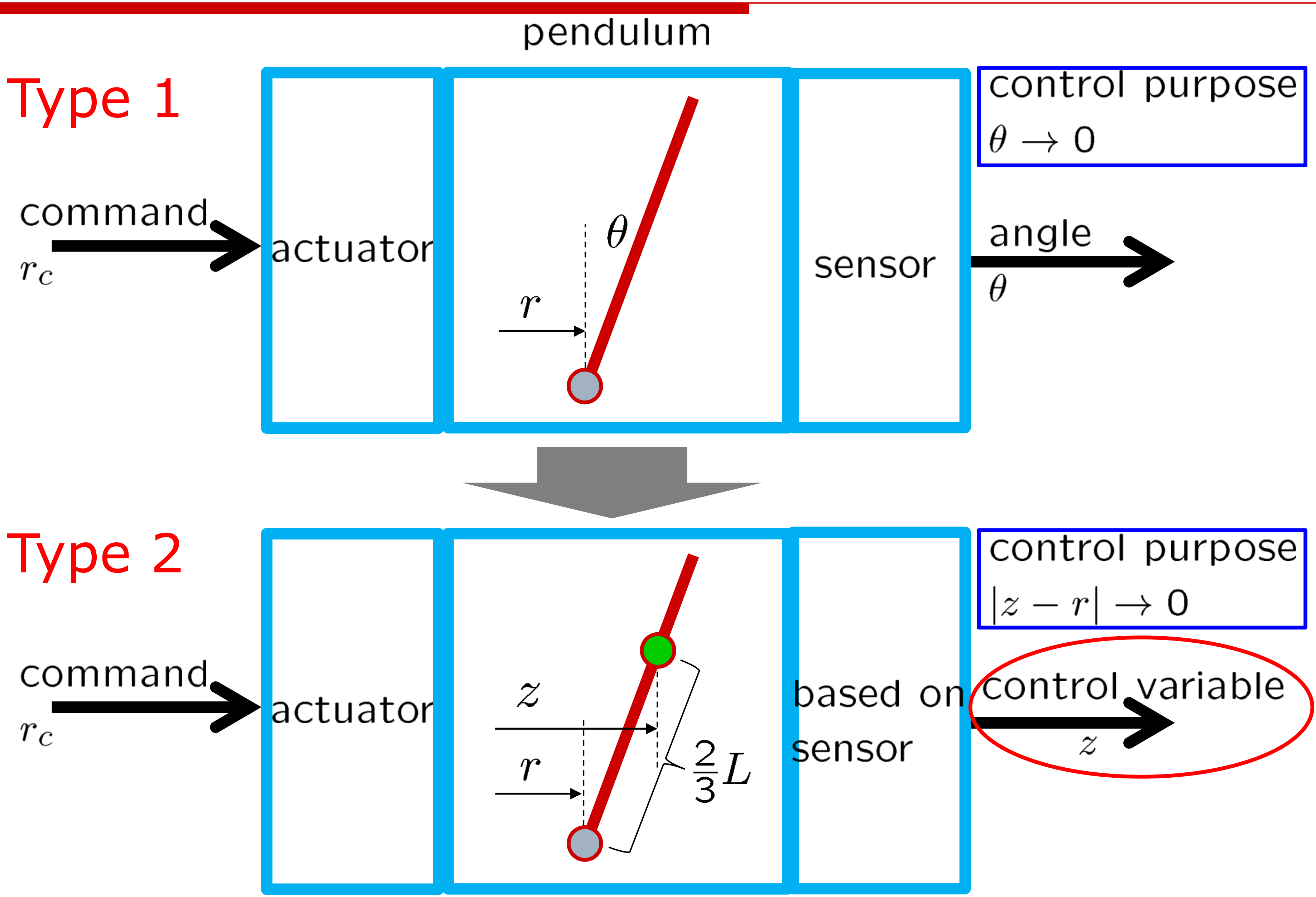
Andersen's Approach (Step 1)



Position Controlled Cart

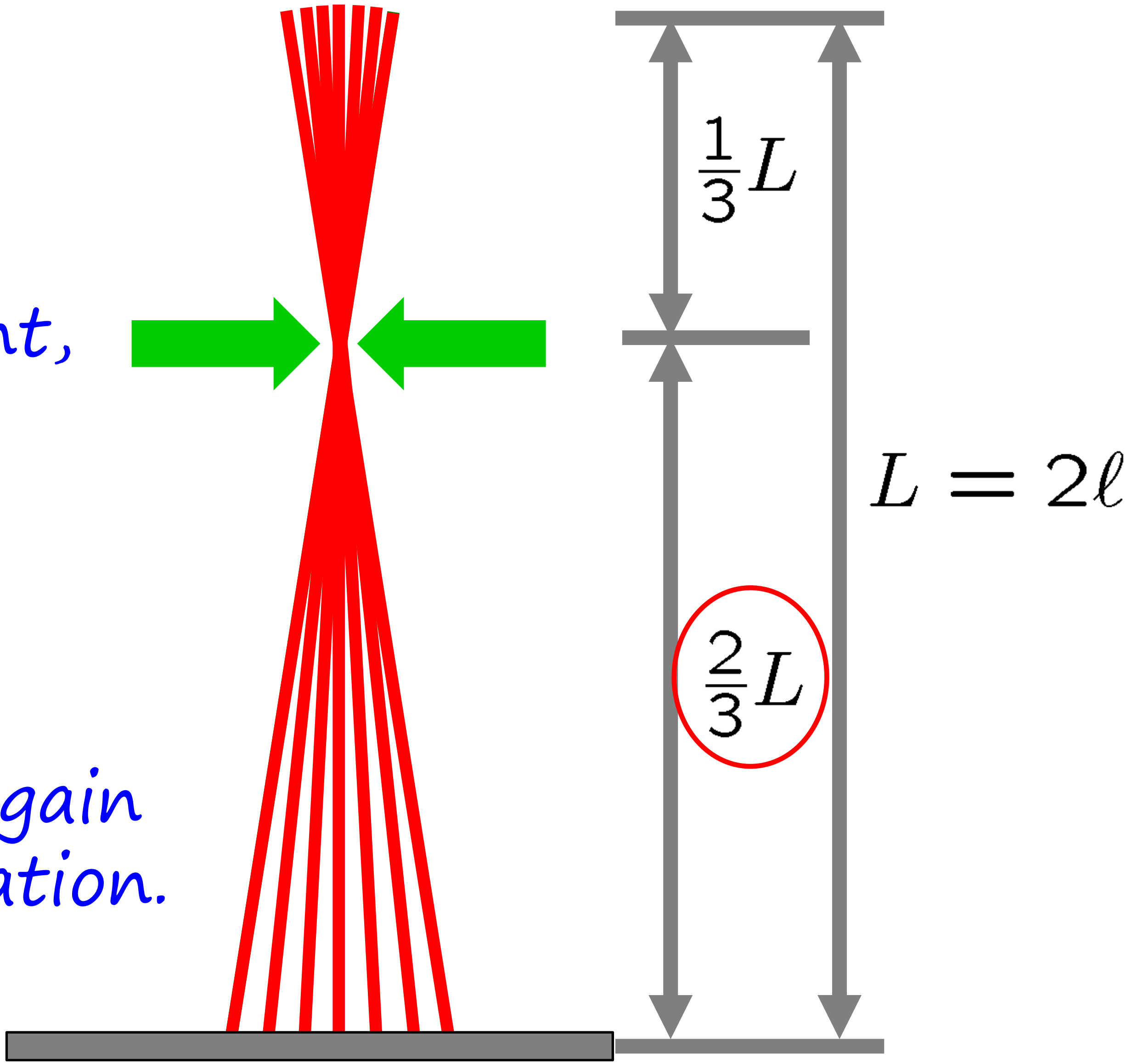
$$\frac{d}{dt}\dot{r} = -\frac{1}{T_a}\dot{r} + \frac{K_a}{T_a}\dot{r}_c$$

Which is easier to control?



Andersen’s Approach (Step 2)

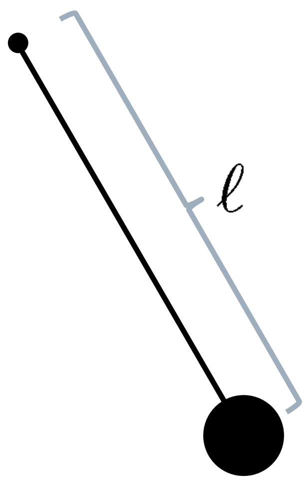
Pinching the point,



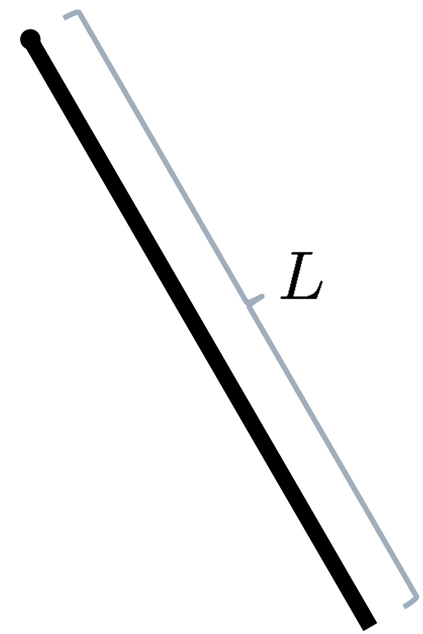
Adjust a control gain
to stop the oscillation.

The Same Period of Pendulums

Simple Pendulum



Rigid Pendulum



$$\ell = \frac{2}{3}L$$

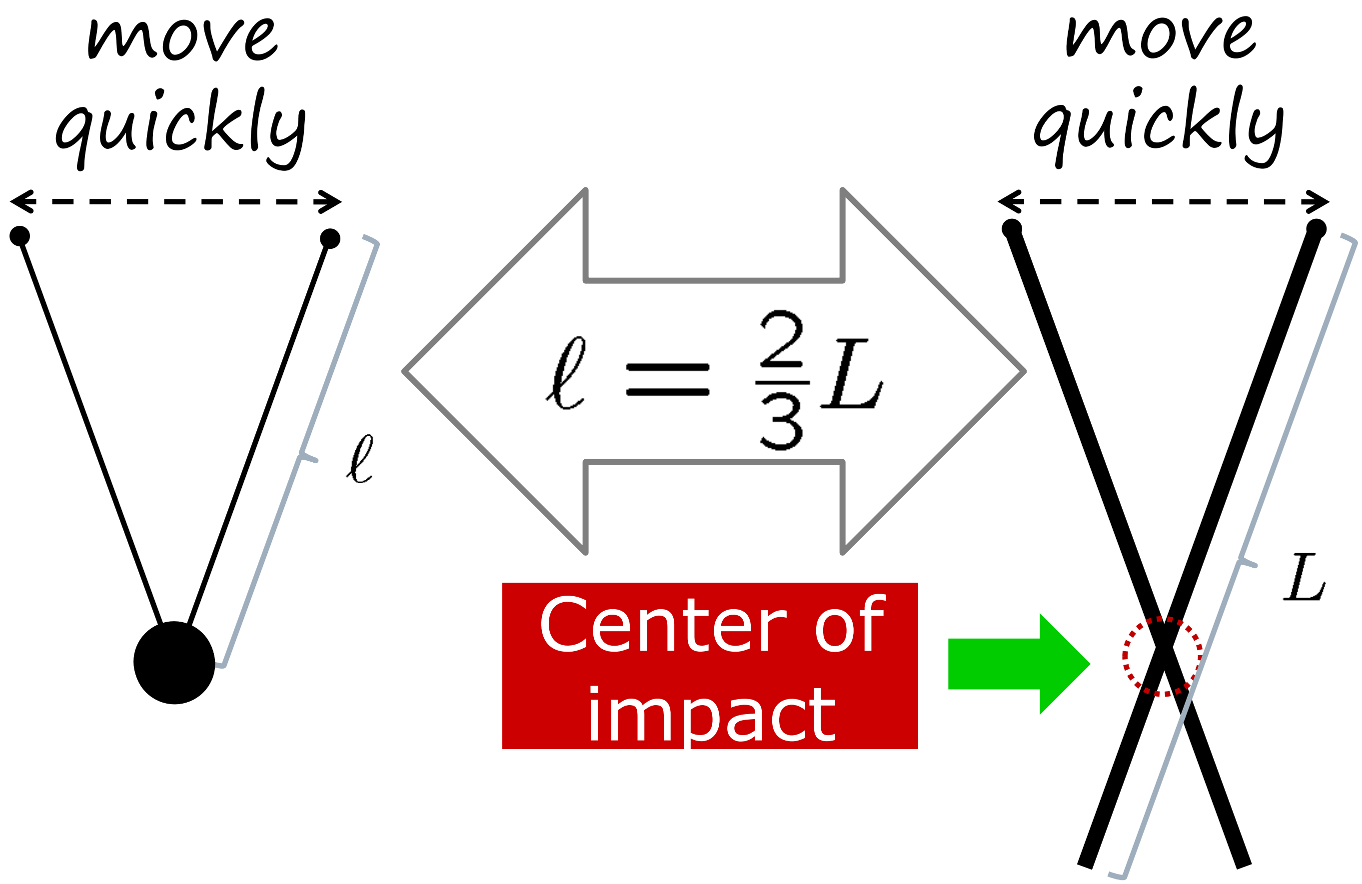
$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T = 2\pi\sqrt{\frac{2}{3}\frac{L}{g}}$$

Find the length ell to realize the same period as the rigid pendulum.

The Same Period of Pendulums

Center of Impact



The controlled variable should be such a physically unmovable point, what we call, a node.

Examples of Center of Impact

Hit a ball to get the minimum impact on hands.

Finding a Controlled Variable

$$\begin{cases} \sout{(M+m)\ddot{r} + m\ell\cos\theta\ddot{\theta} - m\ell\dot{\theta}^2\sin\theta = F} \\ m\ell\cos\theta\ddot{r} + \frac{4}{3}m\ell^2\ddot{\theta} = mg\ell\sin\theta \end{cases}$$

$$\Downarrow \quad \cos\theta \simeq 1, \sin\theta \simeq \theta$$

$$\underbrace{\ddot{r} + \frac{4\ell}{3}\ddot{\theta}}_{\frac{d^2}{dt^2}(r+\frac{4\ell}{3}\theta)} = g\theta \Rightarrow \frac{\Theta(s)}{R(s)} = \frac{-\frac{3}{4\ell}s^2}{s^2 - \frac{3g}{4\ell}}$$

non-minimum phase

$$\Downarrow \; z = r + \frac{2}{3}(2\ell)\theta$$

$$\ddot{z} = \frac{3g}{4\ell}(z - r) \Rightarrow \frac{Z(s)}{R(s)} = \frac{-\frac{3g}{4\ell}}{s^2 - \frac{3g}{4\ell}}$$

minimum phase

Velocity Input Model

- Motion equation

$$\ddot{z} = \frac{3g}{4\ell}(z - r), \quad \frac{d}{dt}\dot{r} = -\frac{1}{T_a}\dot{r} + \frac{K_a}{T_a}\dot{r}_c$$

- State equation

$$\underbrace{\begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dot{r} \\ \ddot{r} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3g}{4\ell} & 0 & -\frac{3g}{4\ell} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{T_a} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} z \\ \dot{z} \\ r \\ \dot{r} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_a}{T_a} \end{bmatrix}}_{B} \underbrace{\dot{r}_c}_{u}$$

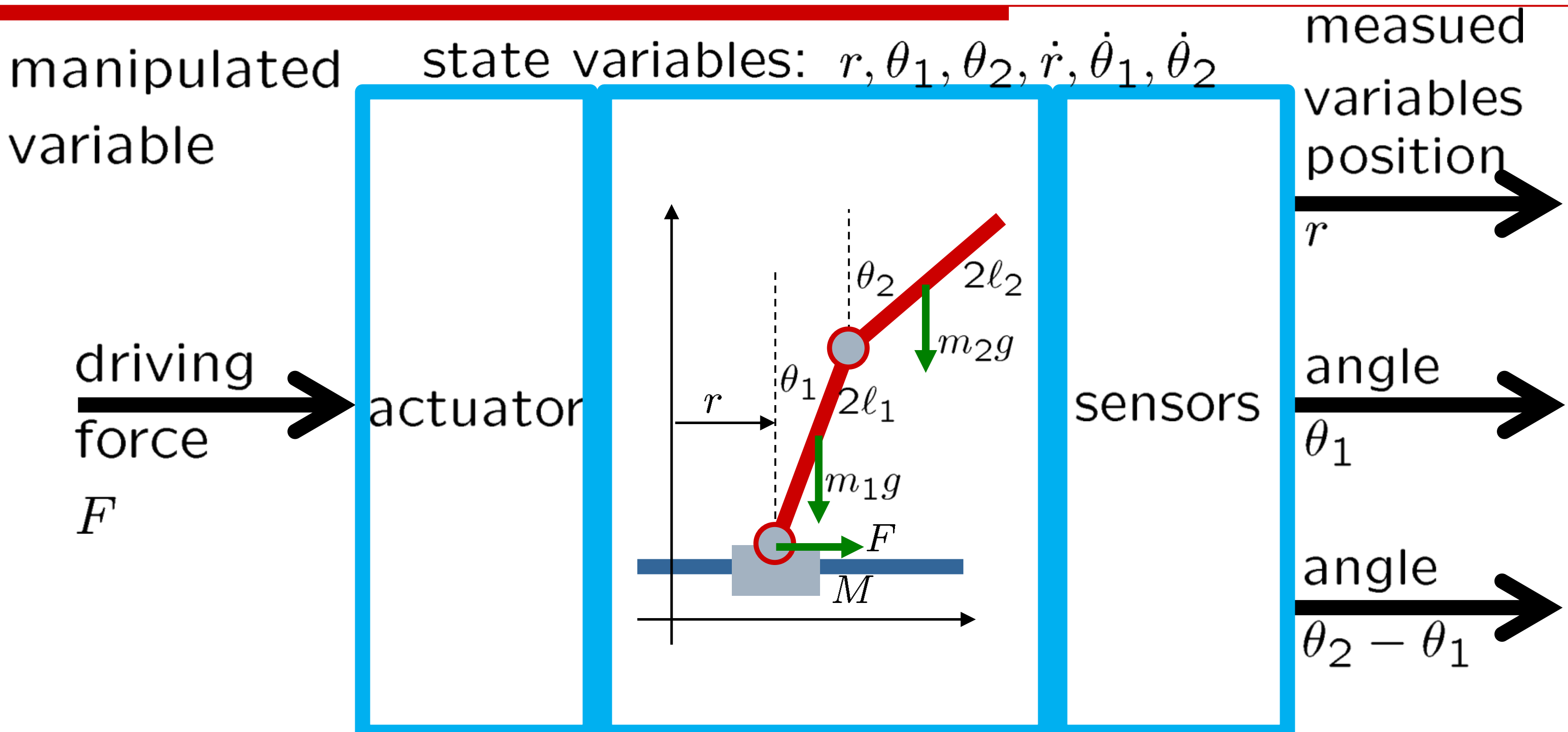
- Output equation

$$\underbrace{\begin{bmatrix} r \\ \theta \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{3}{4\ell} & 0 & -\frac{3}{4\ell} & 0 \end{bmatrix}}_{C_M} \underbrace{\begin{bmatrix} z \\ \dot{z} \\ r \end{bmatrix}}_{x}$$

velocity input

This model is obtained as a sub-product.
But the usefulness will be noticed later.

Double Inverted Pendulum

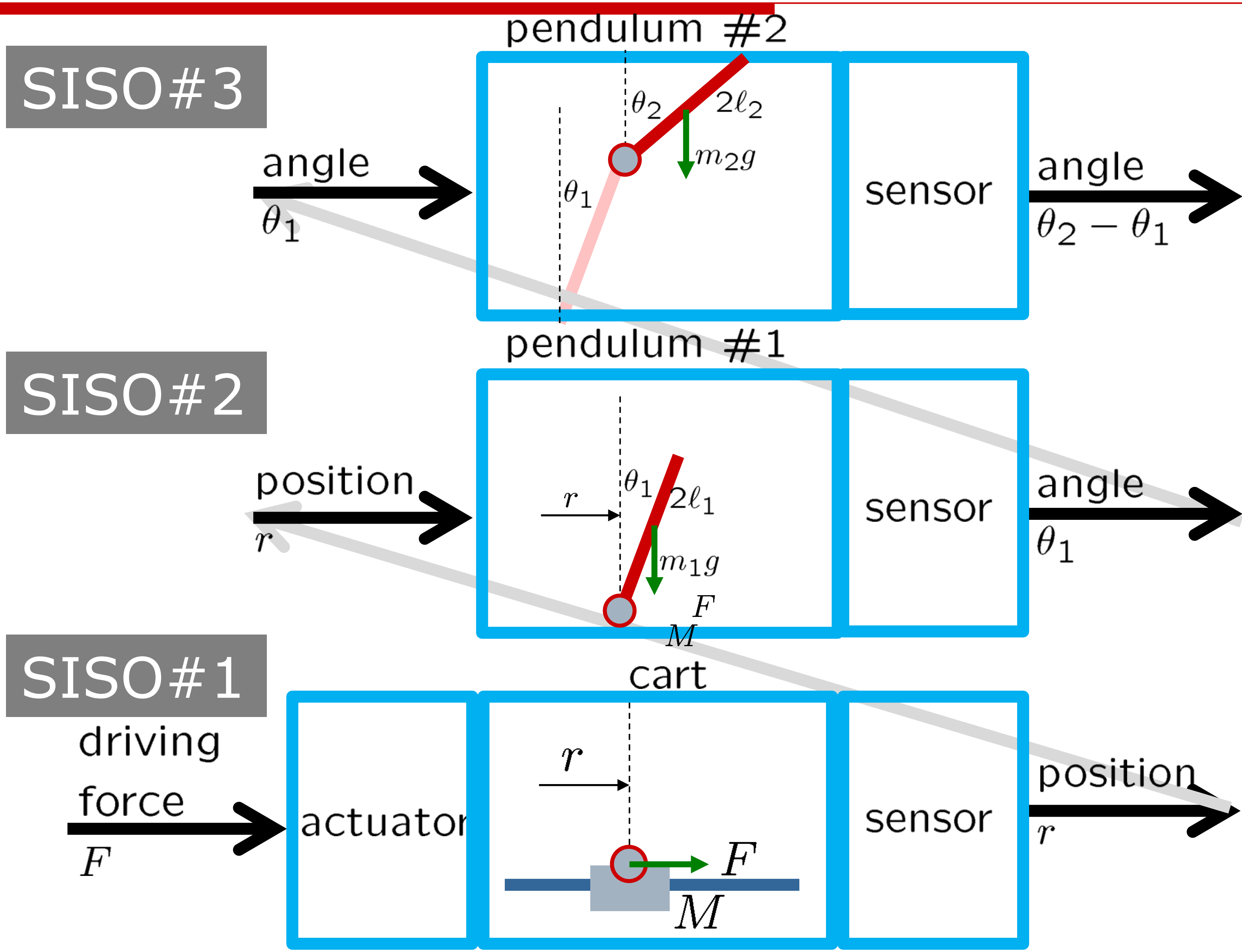


$$\underbrace{\begin{bmatrix} M+m_1+m_2 & (m_1+2m_2)\ell_1\cos\theta_1 & m_2\ell_2\cos\theta_2 \\ -(m_1+2m_2)\ell_1\cos\theta_1 & -(\frac{4}{3}m_1+4m_2)\ell_1^2 & -2m_2\ell_1\ell_2\cos(\theta_2-\theta_1) \\ m_2\ell_2\cos\theta_2 & 2m_2\ell_1\ell_2\cos(\theta_2-\theta_1) & \frac{4}{3}m_2\ell_2^2 \end{bmatrix}}_{M(\xi_1)} \underbrace{\begin{bmatrix} \ddot{r} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}}_{\dot{\xi}_2}$$

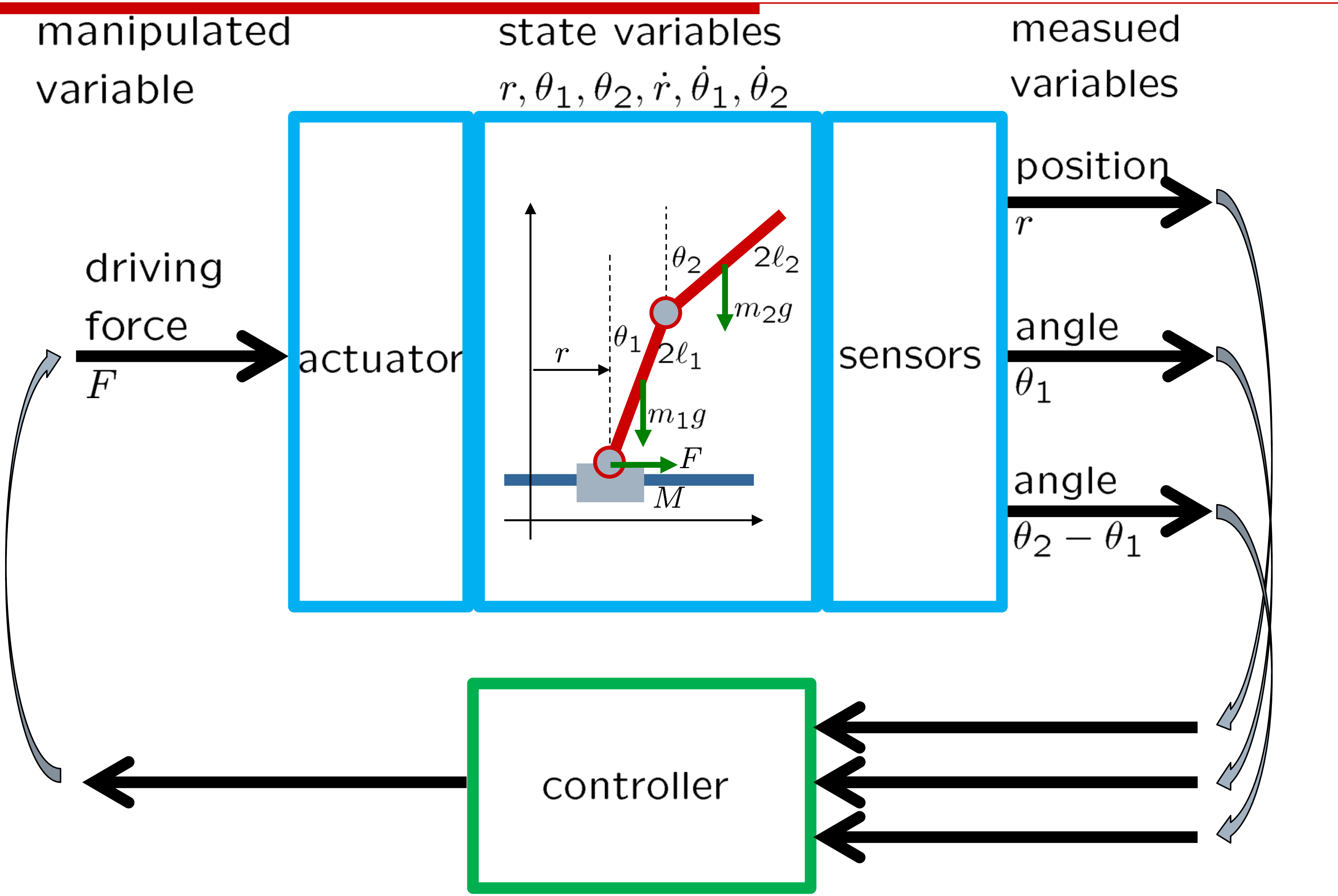
$$+ \underbrace{\begin{bmatrix} 0 & -(m_1+2m_2)\ell_1\sin\theta_1\dot{\theta}_1 & -m_2\ell_2\sin\theta_2\dot{\theta}_2 \\ 0 & 0 & 2m_2\ell_1\ell_2\sin(\theta_2-\theta_1)\dot{\theta}_2 \\ 0 & 2m_2\ell_1\ell_2\sin(\theta_2-\theta_1)\dot{\theta}_1 & 0 \end{bmatrix}}_{C(\xi_1,\xi_2)} \underbrace{\begin{bmatrix} \dot{r} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}_{\dot{\xi}_1} \underbrace{\begin{bmatrix} 0 \\ (m_1+2m_2)\ell_1 g\sin\theta_1 \\ -m_2\ell_2 g\sin\theta_2 \end{bmatrix}}_{G(\xi_1)} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{e_1} \underbrace{F}_{\zeta}$$

Under-actuated System

Andersen's Approach



Control System for DIP



LTI Model for DIP

- State Equation$(\ell_1 = \ell_2 = \ell, m_1 = m_2 = m)$

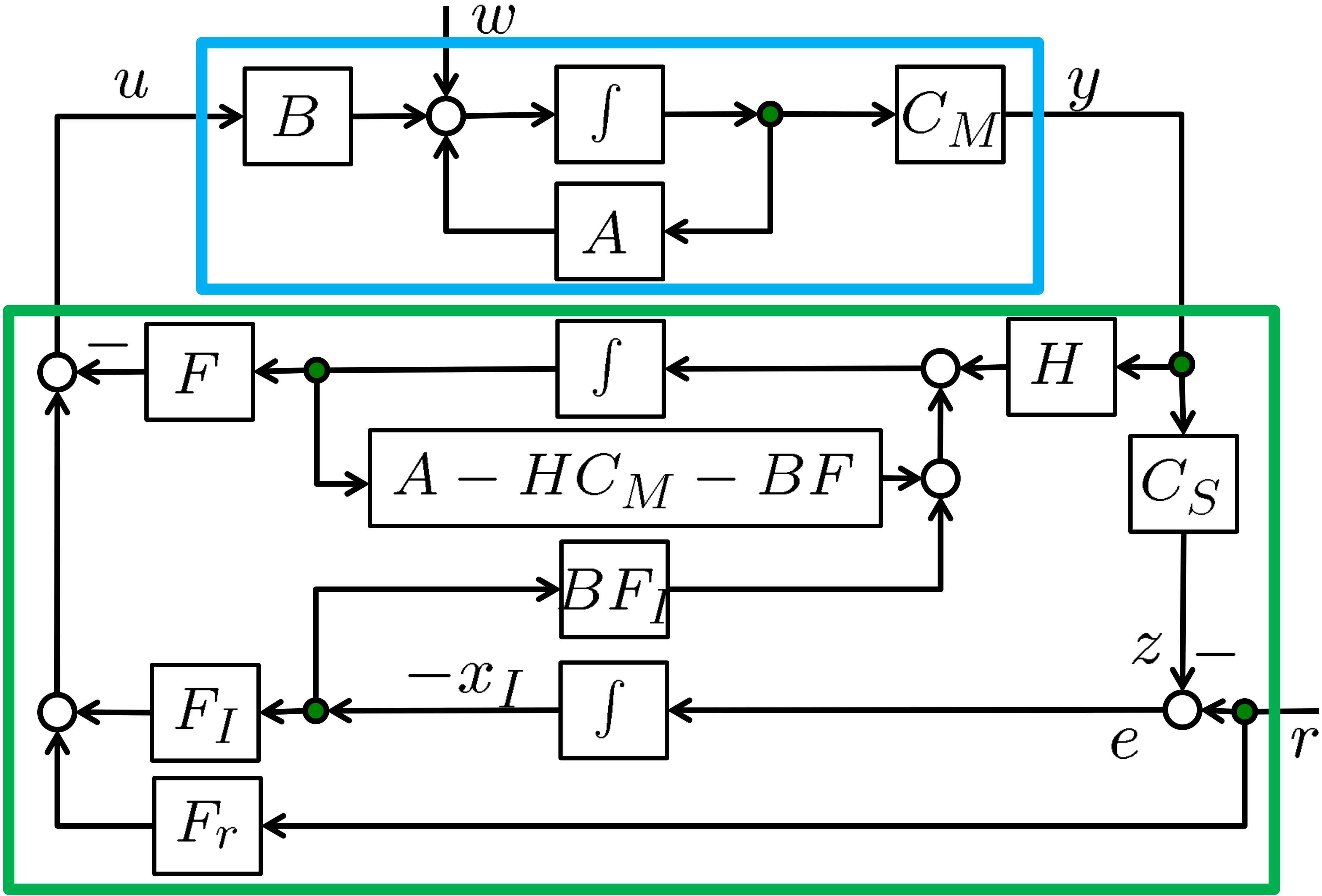
$$\dot{x} = Ax + Bu,\ x = \begin{bmatrix} r & \theta_1 & \theta_2 & \dot{r} & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix},\ u = F$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\dfrac{18\,g\,m}{7\,M + 2\,m} & \dfrac{3\,g\,m}{14\,M + 4\,m} & 0 & 0 & 0 \\ 0 & \dfrac{12\,g\,M + 15\,g\,m}{7\,ell\,M + 2\,ell\,m} & -\dfrac{18\,g\,M + 9\,g\,m}{28\,ell\,M + 8\,ell\,m} & 0 & 0 & 0 \\ 0 & -\dfrac{18\,g\,M + 9\,g\,m}{7\,ell\,M + 2\,ell\,m} & \dfrac{48\,g\,M + 15\,g\,m}{28\,ell\,M + 8\,ell\,m} & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{7}{7\,M + 2\,m} \\ -\dfrac{9}{14\,ell\,M + 4\,ell\,m} \\ \dfrac{3}{14\,ell\,M + 4\,ell\,m} \end{bmatrix}$$

- Output Equation

$$y = Cx,\ y = \begin{bmatrix} r \\ \theta_1 \\ \theta_2 - \theta_1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

LTI Control by LQI Design Method



OBC with an I-Action

- Controllable and Observable System

  $\dot{x} = Ax + Bu + w, \ \ y = C_M x$

  where w is a constant disturbance.

- Control Purpose (Controlled Variables)

  $z = C_S y = \underbrace{C_S C_M}_{C} x \to r \ \ (t \to \infty)$

- Assumption: $S = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is nonsingular.

  For any w, r, there exist x_∞, u_∞,

  $$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_\infty \\ u_\infty \end{bmatrix} = \begin{bmatrix} -w \\ r \end{bmatrix}$$

- Observer-Based Controller with an I-Action

$$\dot{\hat{x}} = (A - HC_M - BF)\hat{x} - BF_I x_I + Hy$$

$$\dot{x}_I = z - r$$

$$u = -F\hat{x} - F_I x_I$$

Steady State Analysis

- Stabilized Closed-loop System

$$\underbrace{\left[\begin{array}{c}\dot{x}\\ \dot{x}_I\\ \hline \dot{e}\end{array}\right]}_{\dot{x}'_E} = \underbrace{\left[\begin{array}{cc|c}A-BF & -BF_I & -BF\\ C & 0 & 0\\ \hline 0 & 0 & A-HC_M\end{array}\right]}_{A'_{EF}} \underbrace{\left[\begin{array}{c}x\\ x_I\\ \hline e\end{array}\right]}_{x'_E} + \underbrace{\left[\begin{array}{c}w\\ -r\\ -w\end{array}\right]}_{w'_E}$$

- Steady State

$$\underbrace{\left[\begin{array}{c}x\\ x_I\\ \hline e\end{array}\right]}_{x'_E} \to \underbrace{\left[\begin{array}{c|c}A_{EF}^{-1} & -A_{EF}^{-1}\left[\begin{array}{c}BF\\ 0\end{array}\right]\widehat{A}^{-1}\\ \hline 0 & \widehat{A}^{-1}\end{array}\right]}_{{A'_{EF}}^{-1}} \underbrace{\left[\begin{array}{c}-w\\ r\\ \hline w\end{array}\right]}_{-w'_E}$$

$$= \left[\begin{array}{c}x_\infty\\ -F_I^{-1}Fx_\infty - F_I^{-1}u_\infty - F_I^{-1}Fe_\infty\\ \hline e_\infty\end{array}\right]$$

$$\left\{\begin{array}{l}z = Cx \to Cx_\infty = r\\ -F_I x_I \to F(x_\infty + e_\infty) + u_\infty\\ e \to e_\infty := \widehat{A}^{-1}w\end{array}\right.$$

LQI Design Method (1)

- Step 1: Selection of Controlled Variables

  Determine a slection matrix C_S such that $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is nonsingular ($C = C_S C_M$).

- Step 2: Stabilization of the Error System

  For the error system:

$$\frac{d}{dt}\underbrace{\begin{bmatrix} x - x_\infty \\ u - u_\infty \end{bmatrix}}_{x_{E3}} = \underbrace{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}}_{A_{E3}} \underbrace{\begin{bmatrix} x - x_\infty \\ u - u_\infty \end{bmatrix}}_{x_{E3}} + \underbrace{\begin{bmatrix} 0 \\ I_m \end{bmatrix}}_{B_{E3}} \underbrace{\dot{u}}_{u_{E3}},$$

  determine a stabilizing state feedback

$$\dot{u} = -\begin{bmatrix} K & K_I \end{bmatrix} x_{E3}$$

  by minimizing a cost function

$$J = \int_0^\infty (x_{E3}^T Q_E x_{E3} + u_{E3}^T R_E u_{E3})\, dt,$$

  and then calculate

$$\begin{bmatrix} F & F_I \end{bmatrix} = \begin{bmatrix} K & K_I \end{bmatrix} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}.$$

LQI Design Method (2)

- Step 3: Calculation of State Observer

  To determine the gain H in a state observer

  $\dot{\hat{x}} = (A - HC)\hat{x} + Hy + Bu$

  by solving Riccati equation

  FARE : $\Gamma A^T + A\Gamma - \Gamma C^T V^{-1} C\Gamma + W = 0$

  on $\Gamma > 0$, and calculating

  $H = (C^T V^{-1} \Gamma)^T$.

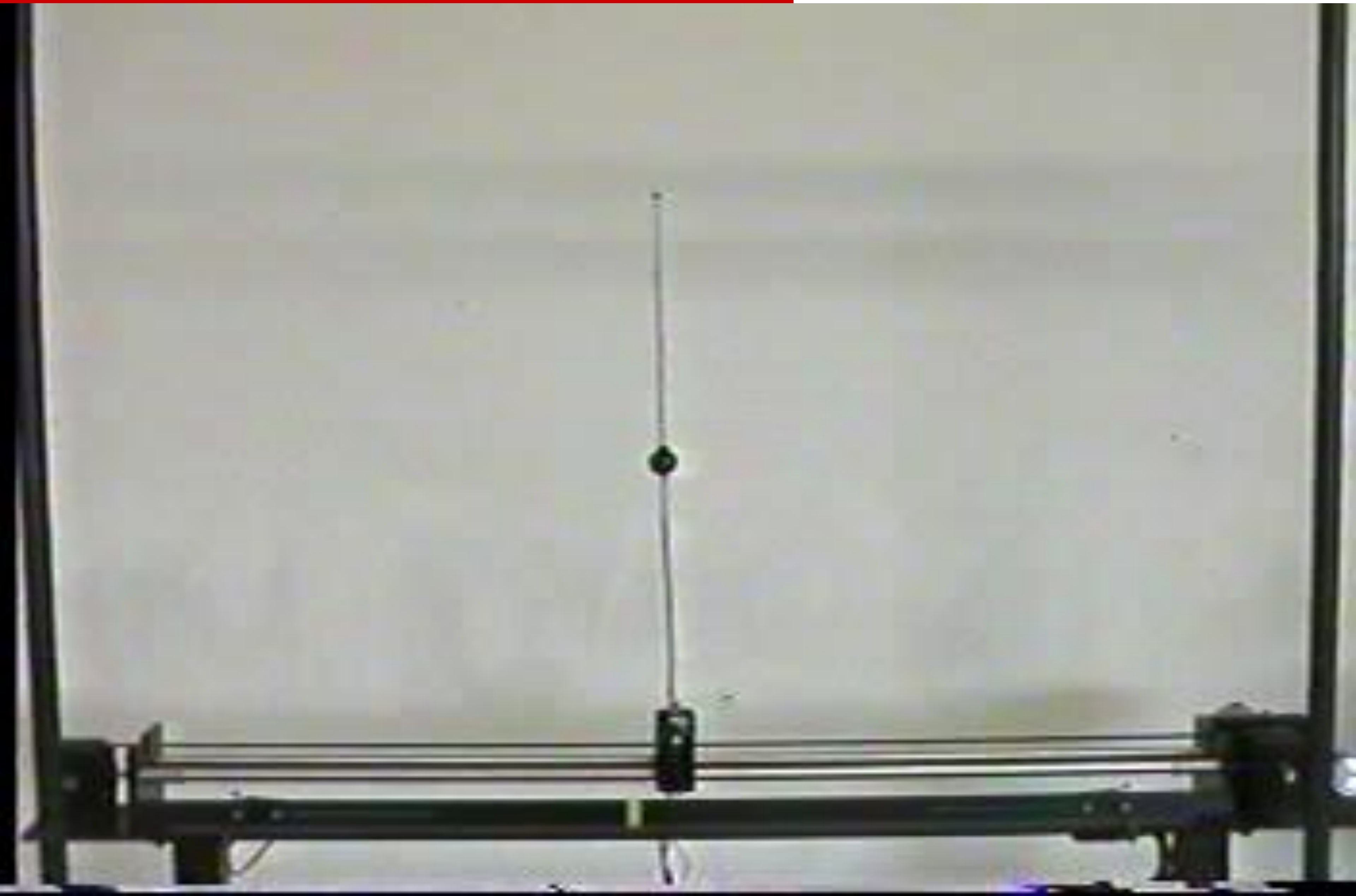
- Step 4: Calculation of LQI Controller

  Base on C_S, F, F_I, H obtained, calculate

$$\dot{x}_K = \underbrace{\begin{bmatrix} A - HC_M - BF & -BF_I \\ 0_{m\times n} & 0_{m\times m} \end{bmatrix}}_{A_K} x_K + \underbrace{\begin{bmatrix} H & 0_{n\times m} \\ C_S & -I_m \end{bmatrix}}_{B_K} \begin{bmatrix} y \\ r \end{bmatrix}$$

$$u = \underbrace{-\begin{bmatrix} F & F_I \end{bmatrix}}_{C_K} x_K.$$

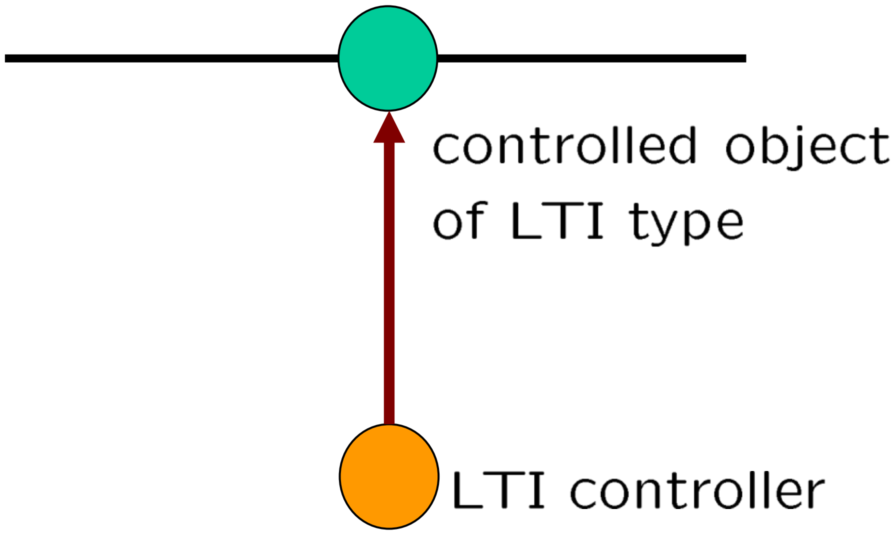
Double Inverted Pendulum (1979) [31]

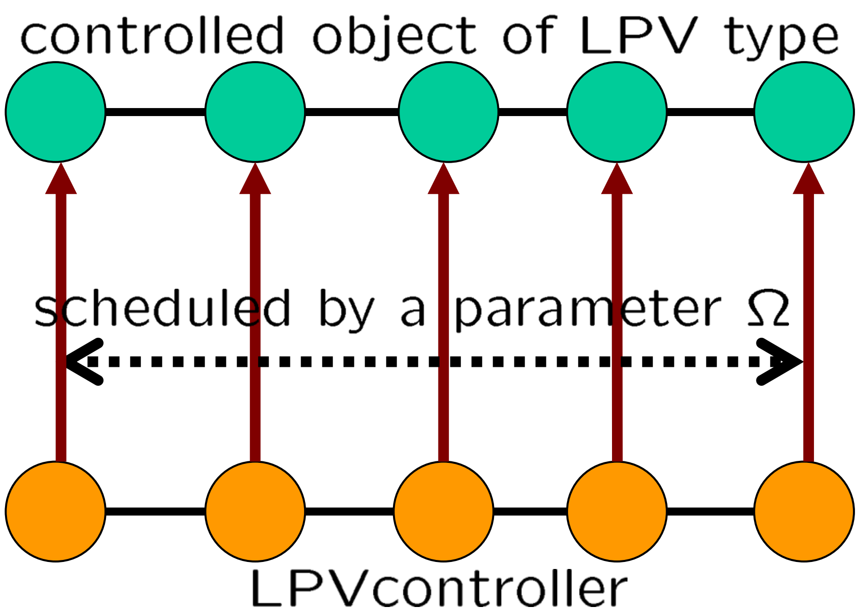


Outline

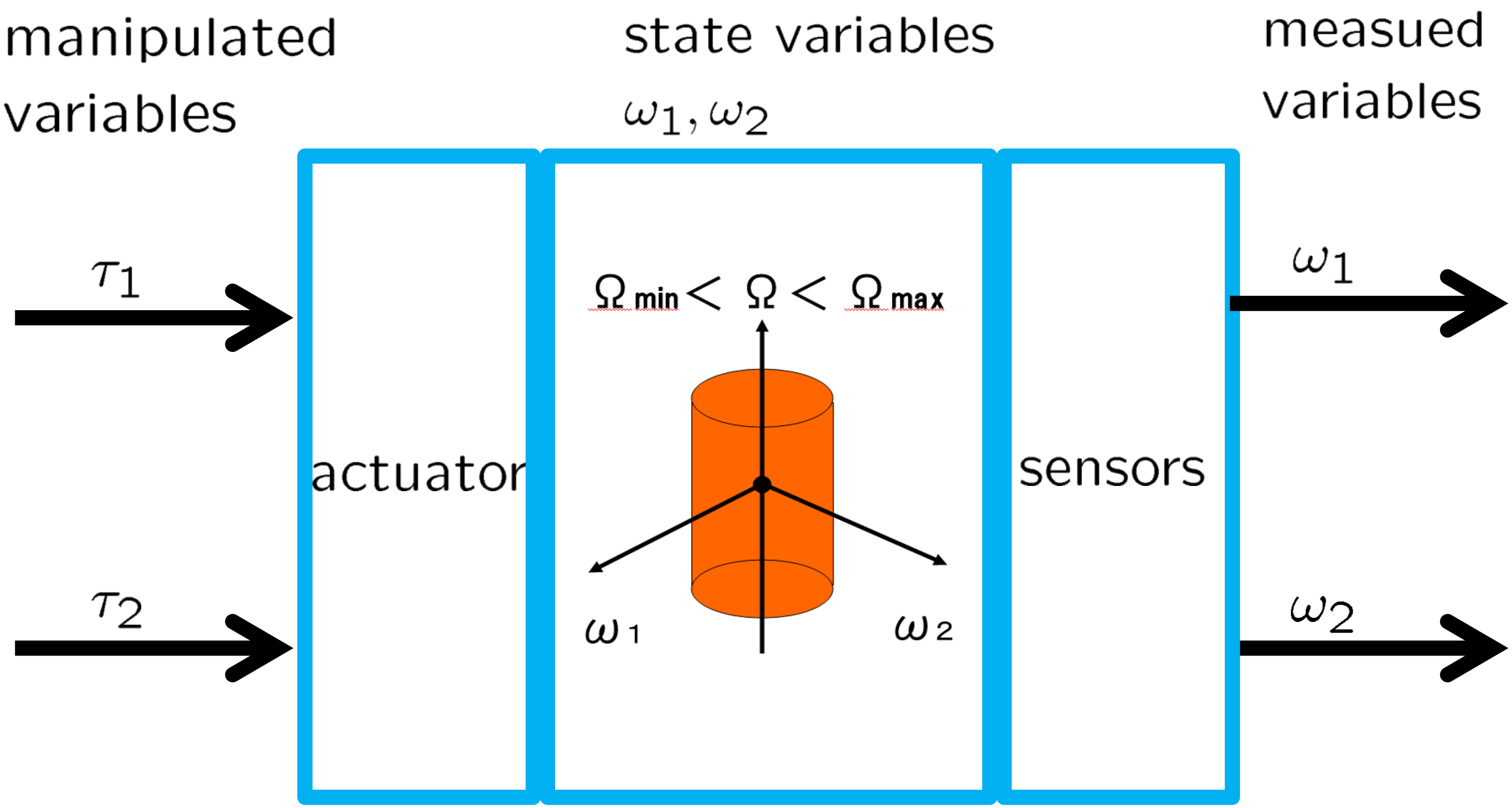
1 **LQI Control**
Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 **LPV Control**
Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control





Spinning Body



$$\begin{cases} J_1\dot{\omega}_1 - \omega_2\Omega(J_1 - J_3) = \tau_1 \\ J_1\dot{\omega}_2 - \omega_1\Omega(J_3 - J_1) = \tau_2 \end{cases}$$

Under the z-angular velocity variation, regulate the disturbed x&y-angular velocities.

LPV Model

- State equation: varying parameter

$$\underbrace{\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\Omega \begin{bmatrix} 0 & \frac{J_1 - J_3}{J_1} \\ -\frac{J_1 - J_3}{J_1} & 0 \end{bmatrix}}_{A(\Omega)} \underbrace{\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}}_{x} + \underbrace{\frac{1}{J_1}}_{B} \underbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}}_{u}$$

$(\Omega_{min} \leq \Omega \leq \Omega_{max})$

- Polytopic LPV Model:

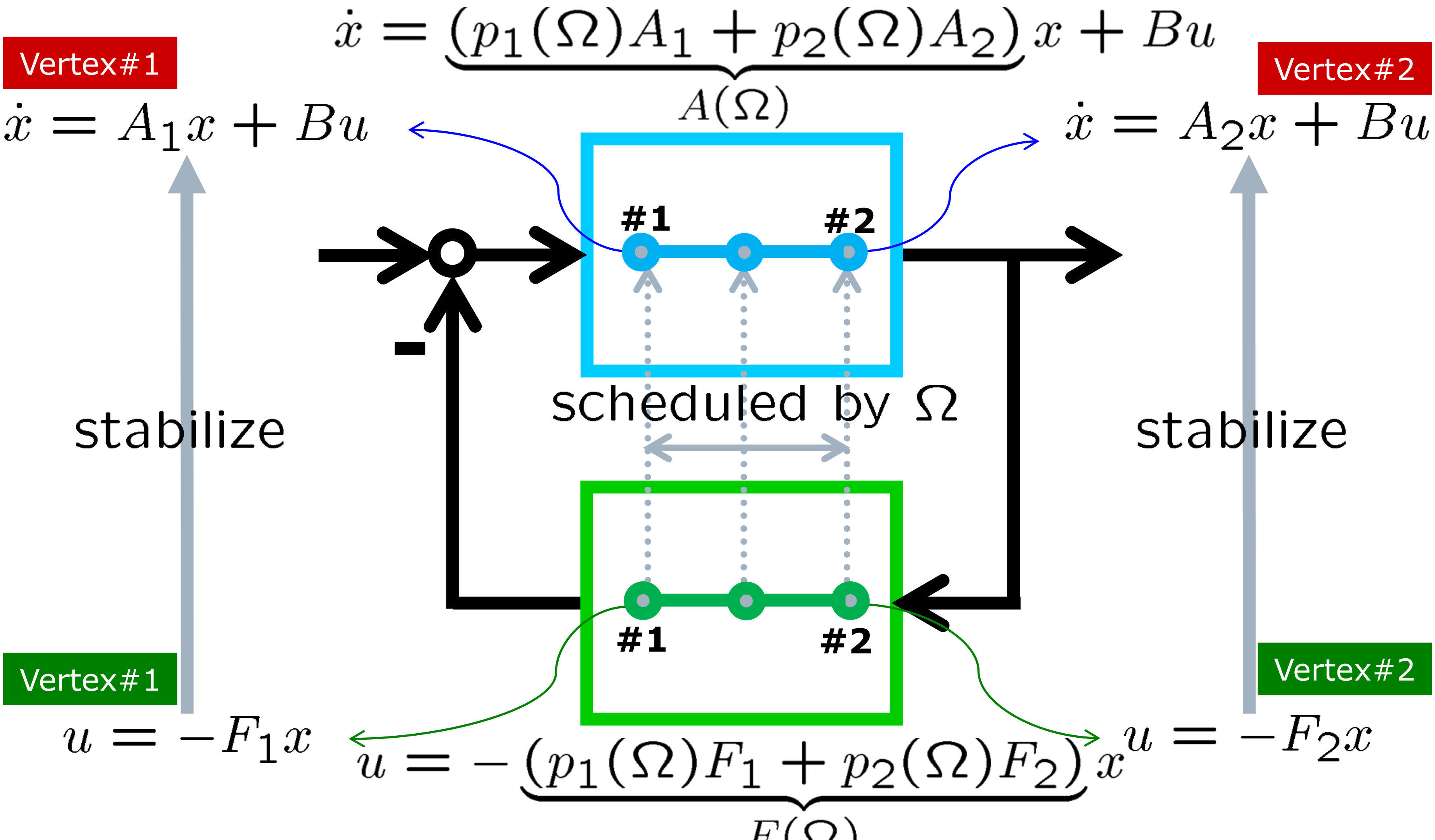
$$\dot{x} = \underbrace{(p_1(\Omega)A_1 + p_2(\Omega)A_2)}_{A(\Omega)} x + Bu$$

where $A_1 = A(\Omega_{min}), A_2 = A(\Omega_{max})$ and

$$p_1(\Omega) = \frac{\Omega_{max} - \Omega}{\Omega_{max} - \Omega_{min}}, p_2(\Omega) = \frac{\Omega - \Omega_{min}}{\Omega_{max} - \Omega_{min}}$$

satisfying $p_1(\Omega) + p_2(\Omega) = 1$

LPV Control

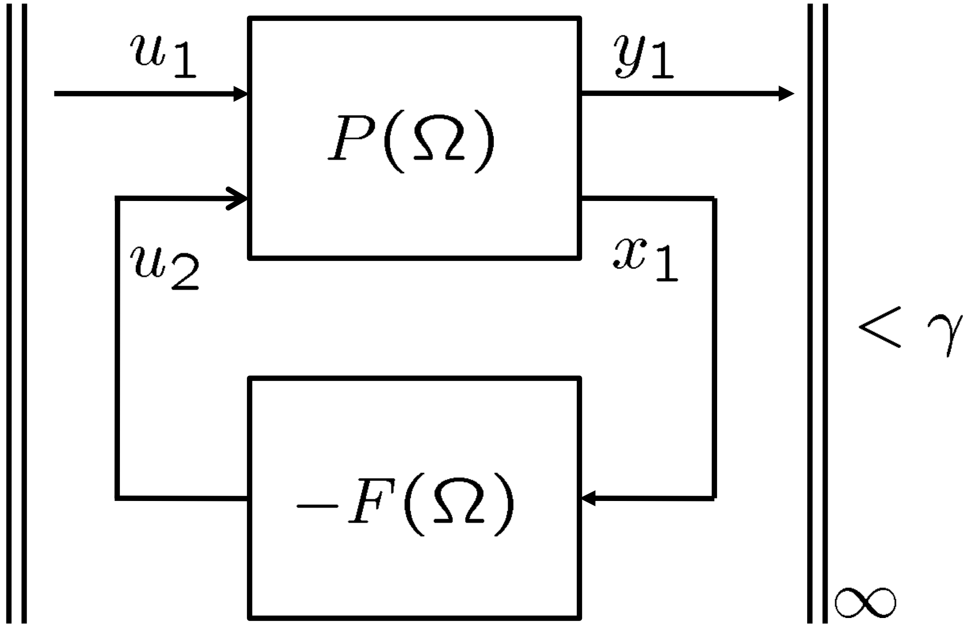


The interpolation of two vertex controllers doesn't guarantee the closed-loop stability.

Design Specification

- Spec.#1:
  The closed-loop system is internally stable.

- Spec.#2:
  The L_2-induced gain of the operator is bounded by γ.



CLPS by LPV SF

- 2-port representation

$$P(\Omega):\begin{cases}\dot{x} = A(\Omega)x + B_1u_1 + B_2u_2\\ \underbrace{\begin{bmatrix} y \\ u \end{bmatrix}}_{y_1} = \underbrace{\begin{bmatrix} C \\ 0 \end{bmatrix}}_{C_1} x + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D_{11}} u_1 + \underbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}_{D_{12}} u_2\\ y_2 = x\end{cases}$$

$$A(\Omega) = p_1(\Omega)A_1 + p_2(\Omega)A_2$$

$$\updownarrow \qquad \updownarrow$$

$$F(\Omega) = p_1(\Omega)F_1 + p_2(\Omega)F_2$$

- state feedback

$$u_2 = -F(\Omega)y_2$$

- closed-loop system

$$\begin{cases}\dot{x} = (A(\Omega) - B_2F(\Omega))x + B_1u_1\\ y_1 = \underbrace{\begin{bmatrix} C \\ -F(\Omega) \end{bmatrix}}_{C_1 - D_{12}F(\Omega)} x\end{cases}$$

LMI-Based Design of LPV SF

- Minimize γ on $Y = Y^T, Z_1, Z_2$
  subject to $Y > 0$ and
  LMI-SF1,2,3,4 for vertex1
  LMI-SF1,2,3,4 for vertex2
  (LMI: Lnear Matrix Inequalty)
- Determine the State Feedback gain
  for each vertex F_1, F_2 by
  $F_1 = Z_1 Y^{-1}$
  $F_2 = Z_2 Y^{-1}$

LMIs for SF

- **LMI-SF1**:

$$AY - B_2Z + (*)^T < -2\alpha Y$$

- **LMI-SF2**:

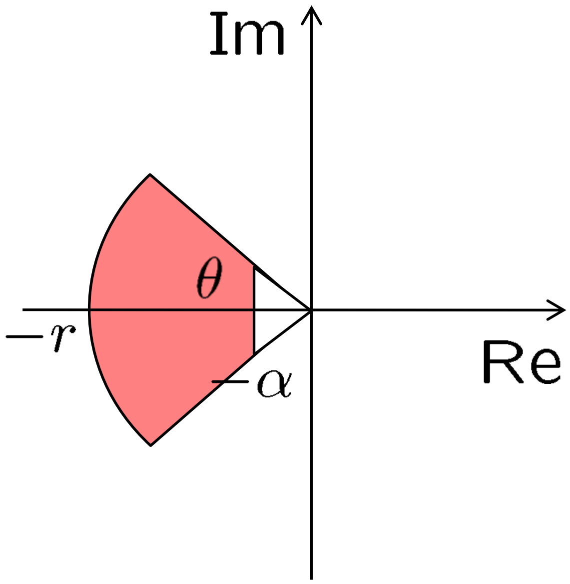
$$\begin{bmatrix} -rY & AY - B_2Z \\ (*)^T & -rY \end{bmatrix} < 0$$

- **LMI-SF3**:

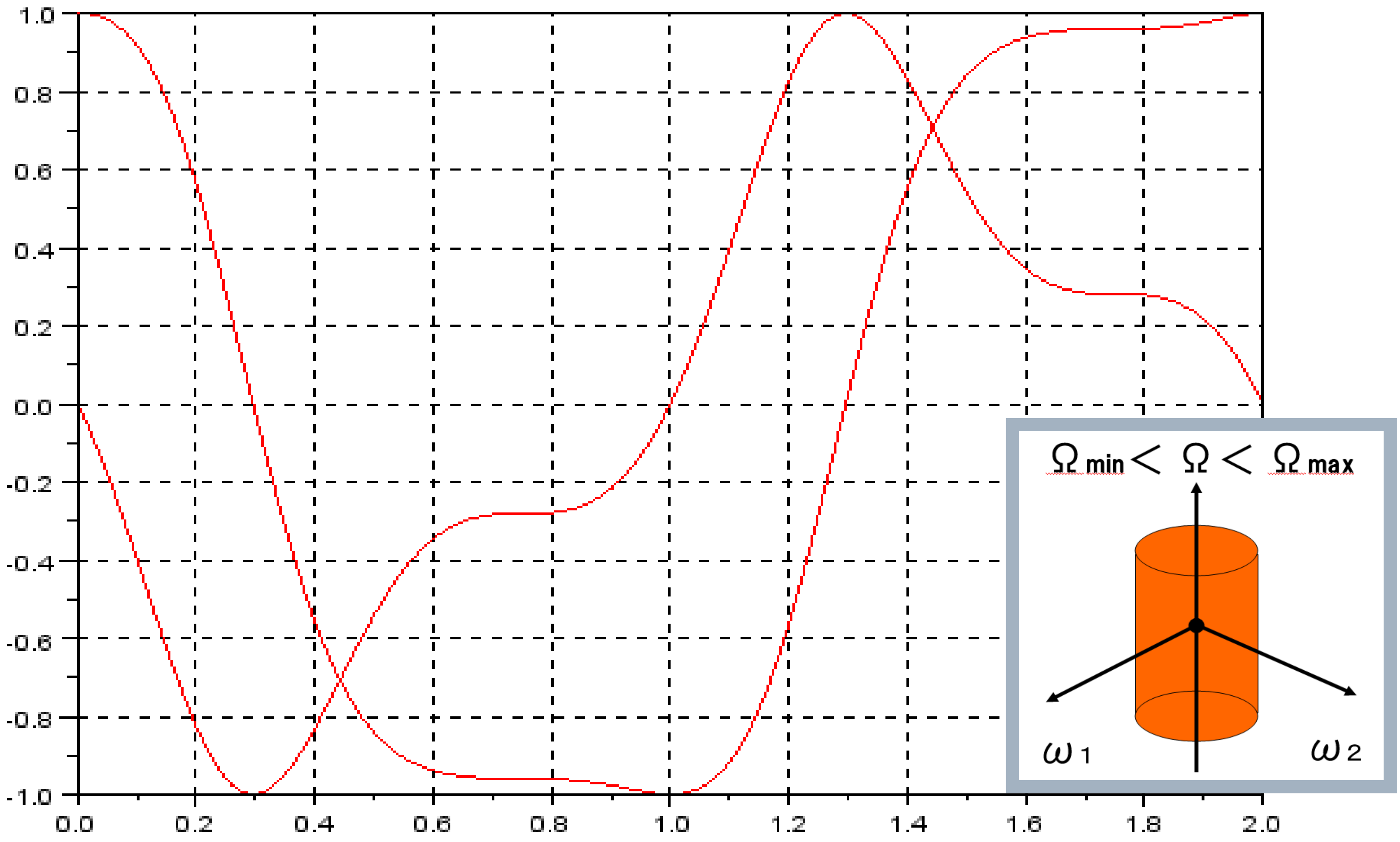
$$\begin{bmatrix} \sin\theta(AY - B_2Z + (*)^T) & \cos\theta(AY - B_2Z - (*)^T) \\ -\cos\theta(AY - B_2Z - (*)^T) & \sin\theta(AY - B_2Z + (*)^T) \end{bmatrix} < 0$$

- **LMI-SF4**:

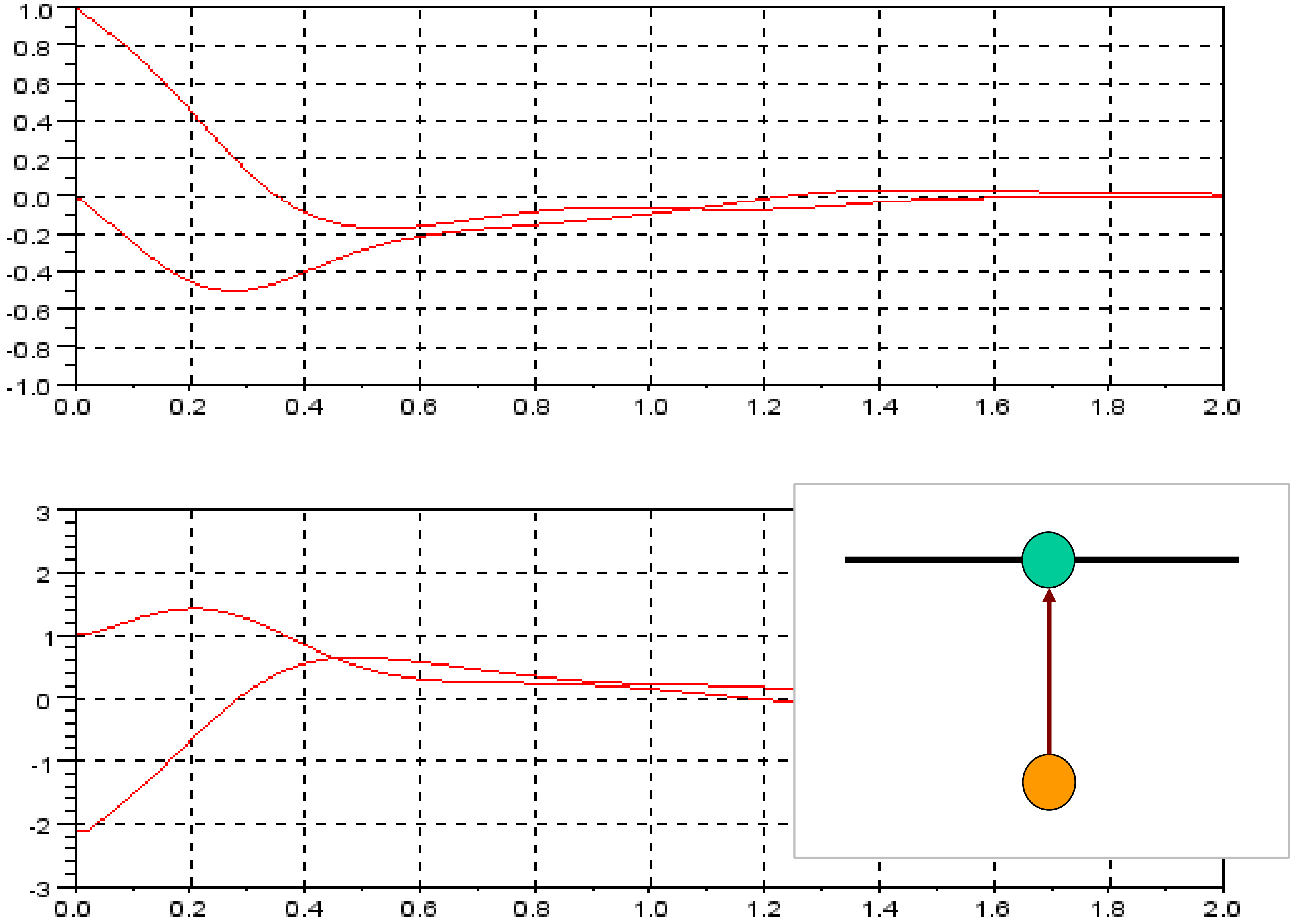
$$\begin{bmatrix} AY - B_2Z + (*)^T & B_1 & (*)^T \\ (*)^T & -\gamma^2 I & (*)^T \\ C_1Y - D_{12}Z & D_{11} & -I \end{bmatrix} < 0$$



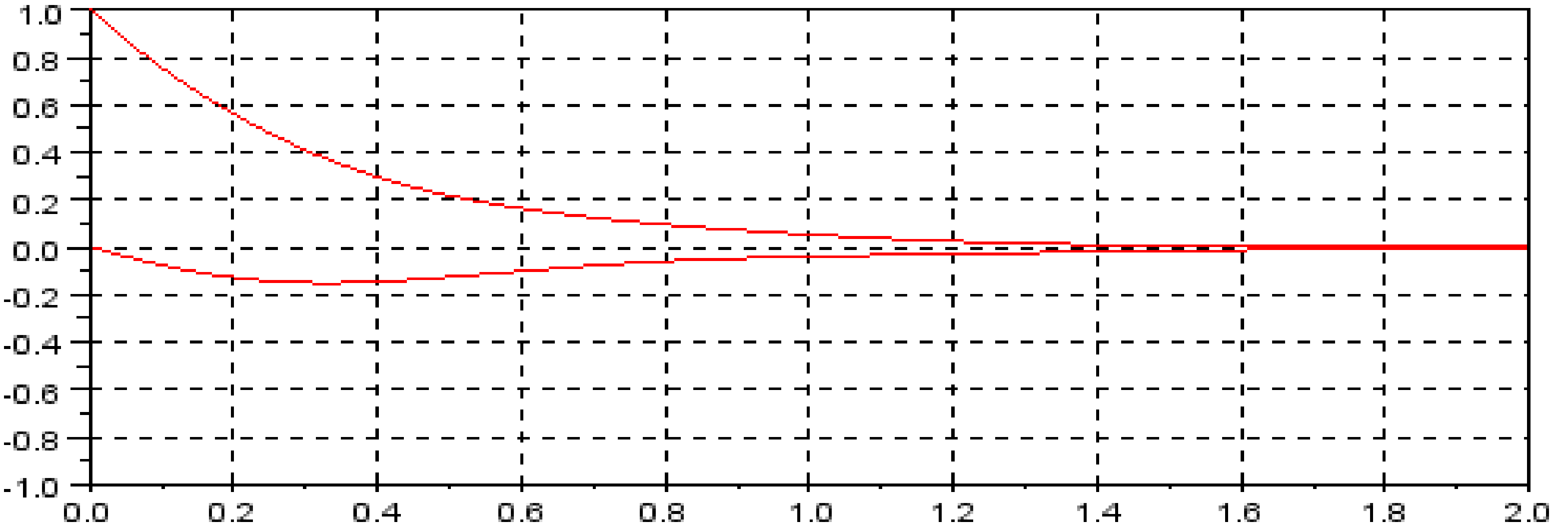
No Control

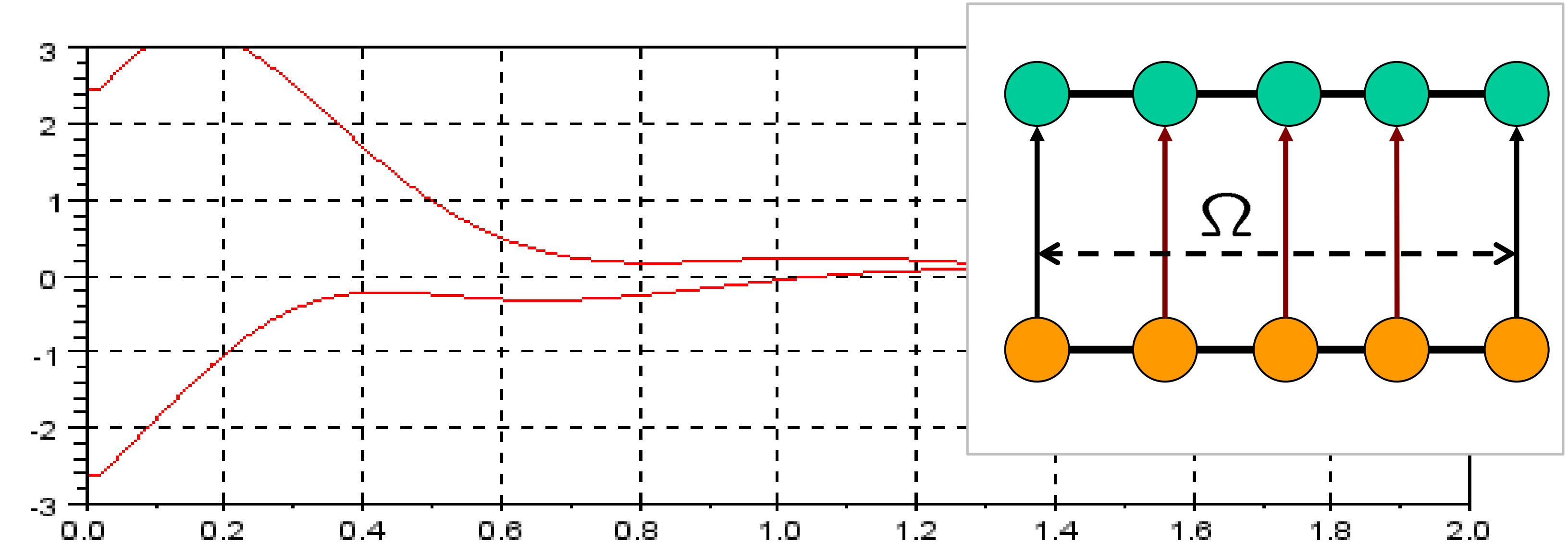

1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Ω min < Ω < Ω max
ω 1
ω 2

LTI Control (SF)

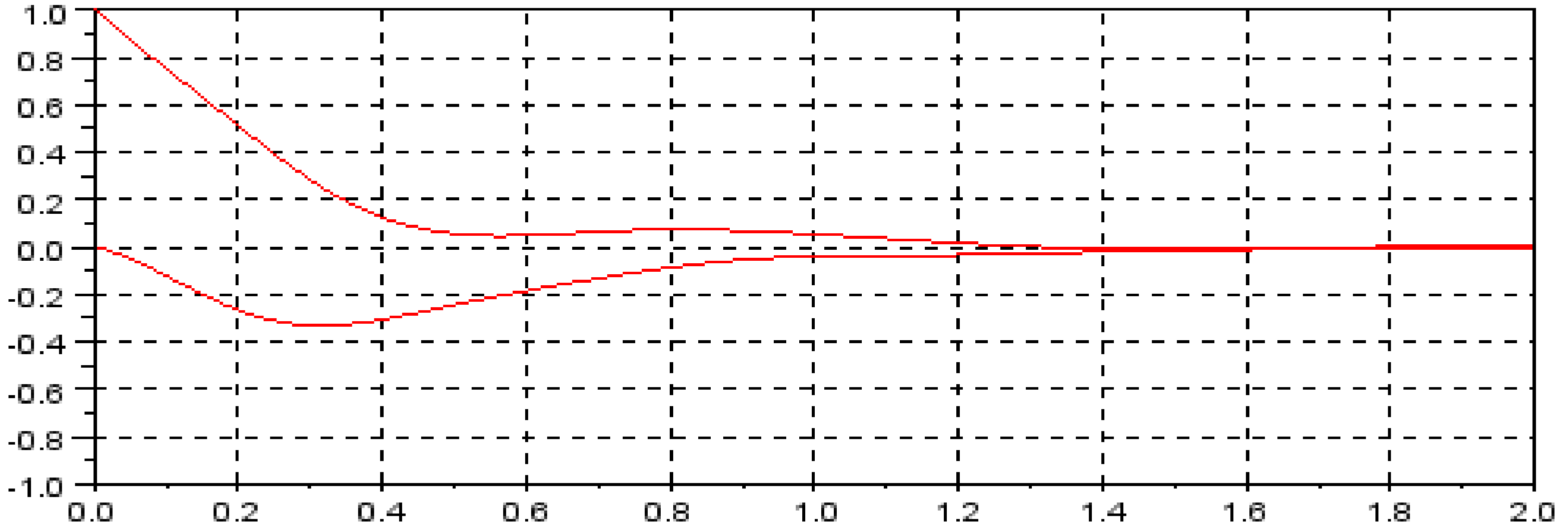


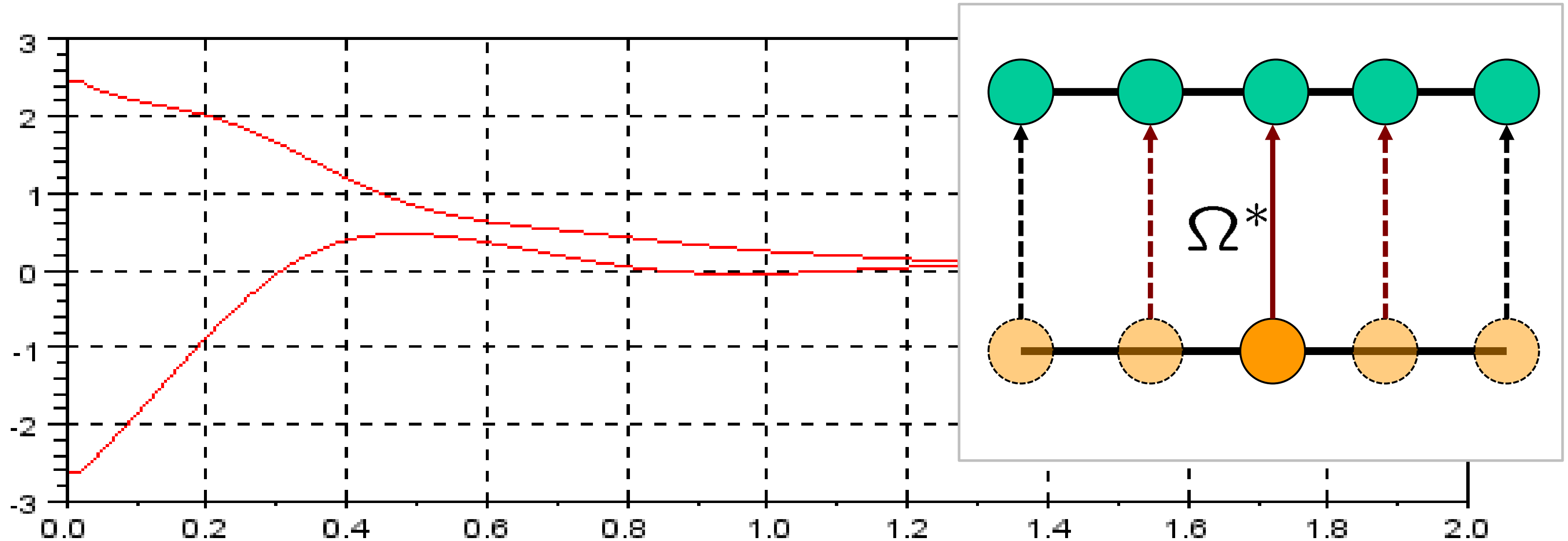
LPV Control (SF)





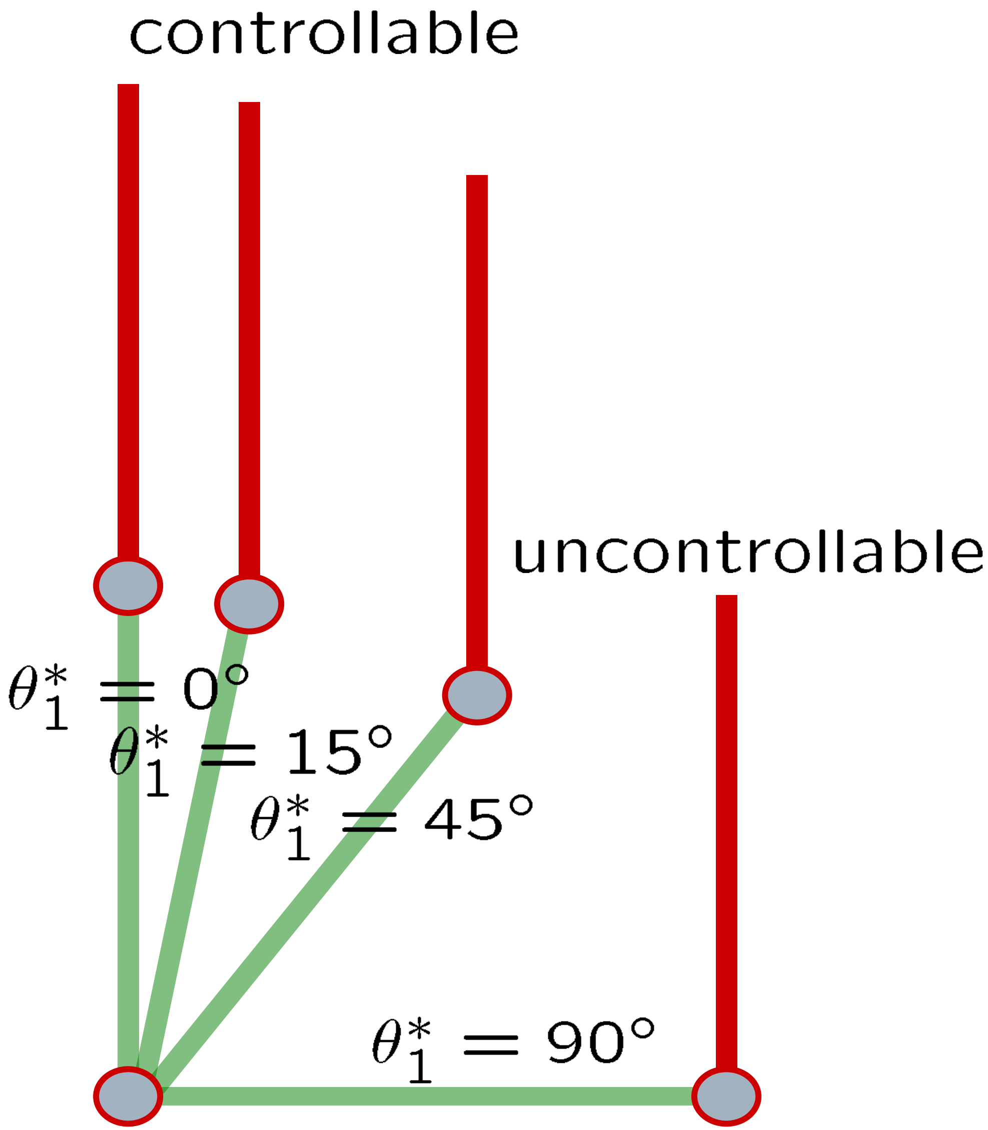
Quasi-LPV Control (SF)





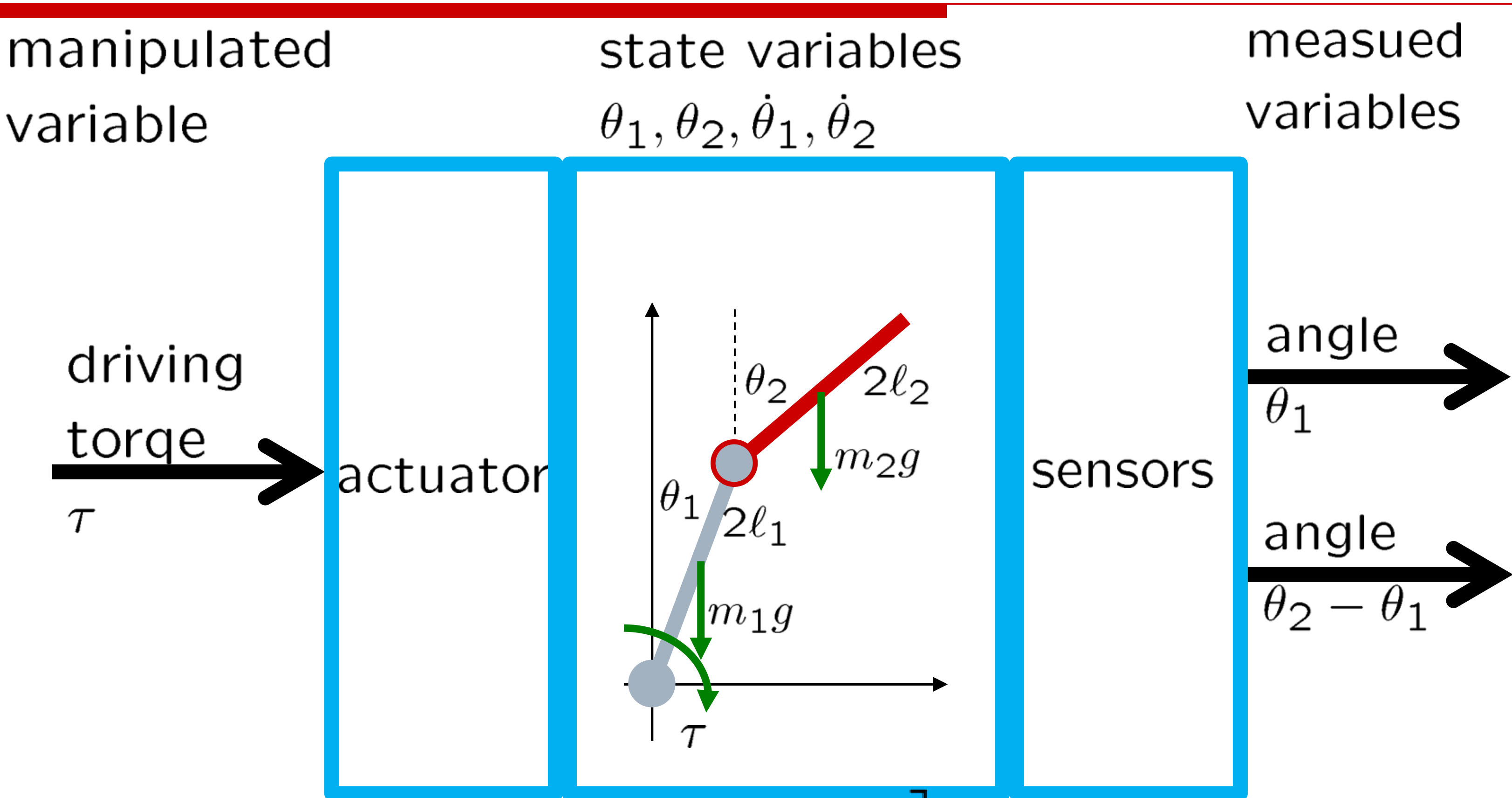
Arm-Driven IP (ADIP,1997)

Equilibrium States for ADIP



The horizontal movement is reduced.

Wide-range Stabilization of ADIP



$$\begin{bmatrix} \frac{4}{3}m_1\ell_1^2 + 4m_2\ell_1^2 & 2m_2\ell_1\ell_2\cos\theta_{21} \\ 2m_2\ell_1\ell_2\cos\theta_{21} & \frac{4}{3}m_2\ell_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$\begin{bmatrix} -2m_2\ell_1\ell_2\dot{\theta}_2^2\sin\theta_{21} \\ 2m_2\ell_1\ell_2\dot{\theta}_1^2\sin\theta_{21} \end{bmatrix} + \begin{bmatrix} -(m_1 + 2m_2)\ell_1 g\sin\theta_1 \\ -m_2\ell_2 g\sin\theta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\tau$$

Under-actuated System

LTI Model for ADIP

- State Equation($\theta_1^* = 0$)

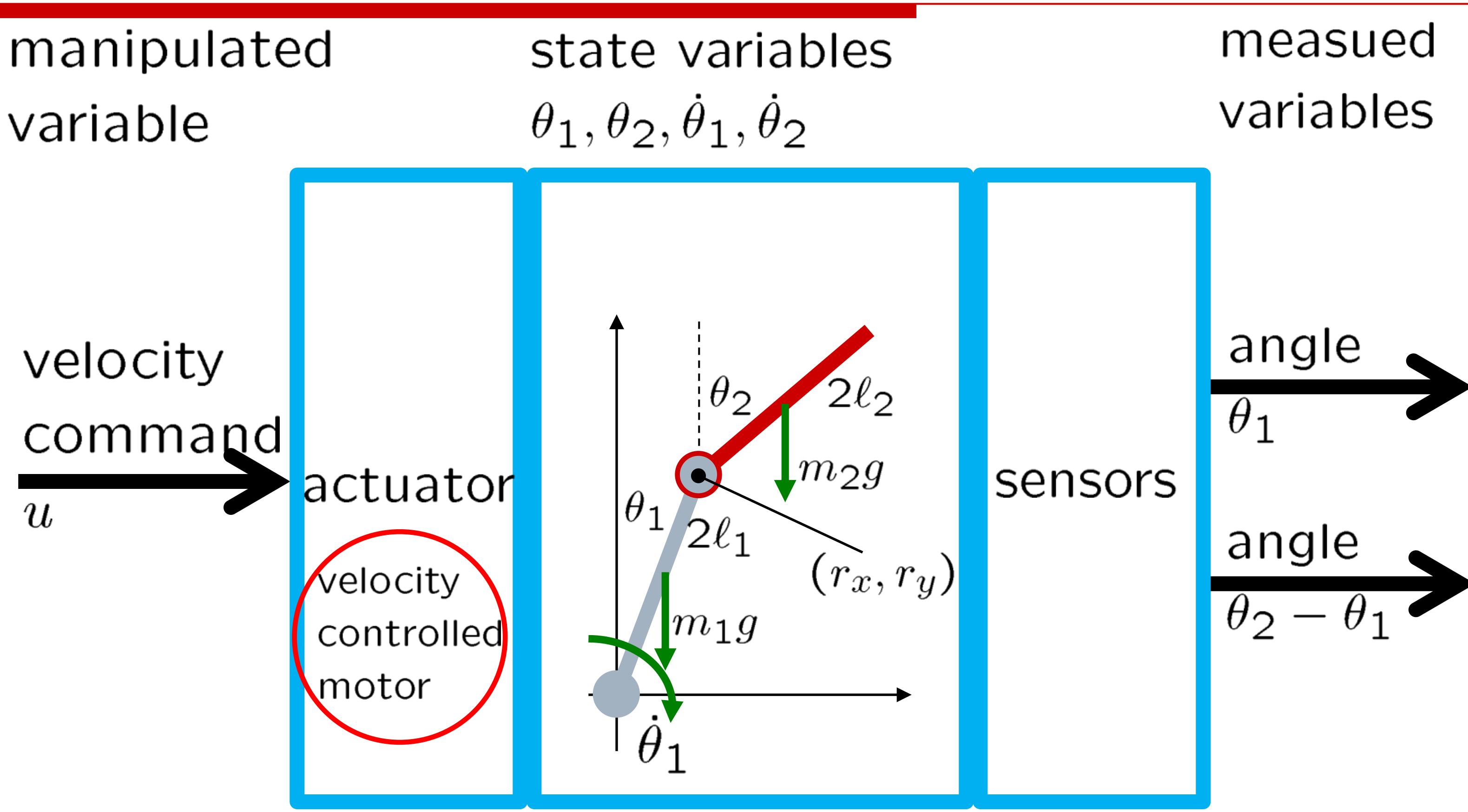
$$\dot{x} = Ax + Bu,\ x = \begin{bmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T,\ u = \tau$$

$$\left[\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{12\, ell1\, g\, m2 + 6\, ell1\, g\, m1}{-6\, ell1^2\, m2 - 8\, ell1^2\, m1} & \dfrac{9\, ell1\, g\, m2}{-6\, ell1^2\, m2 - 8\, ell1^2\, m1} & 0 & 0 \\ \dfrac{18\, ell1\, g\, m2 + 9\, ell1\, g\, m1}{-6\, ell1\, ell2\, m2 - 8\, ell1\, ell2\, m1} & \dfrac{-18\, ell1\, g\, m2 - 6\, ell1\, g\, m1}{-6\, ell1\, ell2\, m2 - 8\, ell1\, ell2\, m1} & 0 & 0 \end{matrix}\right] \left[\begin{matrix} 0 \\ 0 \\ -\dfrac{6}{-6\, ell1^2\, m2 - 8\, ell1^2\, m1} \\ \dfrac{9}{-6\, ell1\, ell2\, m2 - 8\, ell1\, ell2\, m1} \end{matrix}\right]$$

- Output Equation

$$y = Cx,\ y = \begin{bmatrix} \theta_1 \\ \theta_2 - \theta_1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}$$

Velocity Controlled Actuator



$$\frac{d}{dt}\dot\theta_1 = -\frac{1}{T_a}\dot\theta_1 + \frac{K_a}{T_a}u \quad \begin{pmatrix} r_x = 2\ell_1 \sin\theta_1, & \dot r_x = r_y\dot\theta_1, & \ddot r_x = \dot r_y\dot\theta_1 + r_y\ddot\theta_1 \\ r_y = 2\ell_1 \cos\theta_1, & \dot r_y = -r_x\dot\theta_1, & \ddot r_y = -\dot r_x\dot\theta_1 - r_x\ddot\theta_1 \end{pmatrix}$$

$$\boxed{\underbrace{\cos\theta_2}_{1}\ddot r_x + \frac{4}{3}\ell_2\ddot\theta_2 = (g + \underbrace{\ddot r_y}_{0})\underbrace{\sin\theta_2}_{\theta_2}} \quad \underset{z = r_x + \frac{4\ell_2}{3}\theta}{\Rightarrow} \quad \boxed{\ddot z = \frac{3g}{4\ell_2}(z - r_x)}$$

LPV Model for ADIP

- Motion equation

$$\ddot{z} = \frac{3g}{4\ell_2}(z - r_x), \dot{r}_x = r_y\dot{\theta}_1, \frac{d}{dt}\dot{\theta}_1 = -\frac{1}{T_a}\dot{\theta}_1 + \frac{K_a}{T_a}u$$

- State equation

$$\underbrace{\frac{d}{dt}\begin{bmatrix} z \\ \dot{z} \\ r_x \\ \dot{\theta}_1 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3g}{4\ell_2} & 0 & -\frac{3g}{4\ell_2} & 0 \\ 0 & 0 & 0 & r_y \\ 0 & 0 & 0 & -\frac{1}{T_a} \end{bmatrix}}_{A(r_y)} \underbrace{\begin{bmatrix} z \\ \dot{z} \\ r_x \\ \dot{\theta}_1 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_a}{T_a} \end{bmatrix}}_{B} \dot{\theta}_1$$

$$(\underline{r_y} \le r_y \le \overline{r_y},\ \ r_y = 2\ell_1 \cos\theta_1)$$

varying parameter

velocity input

- Polytopic LPV Model:

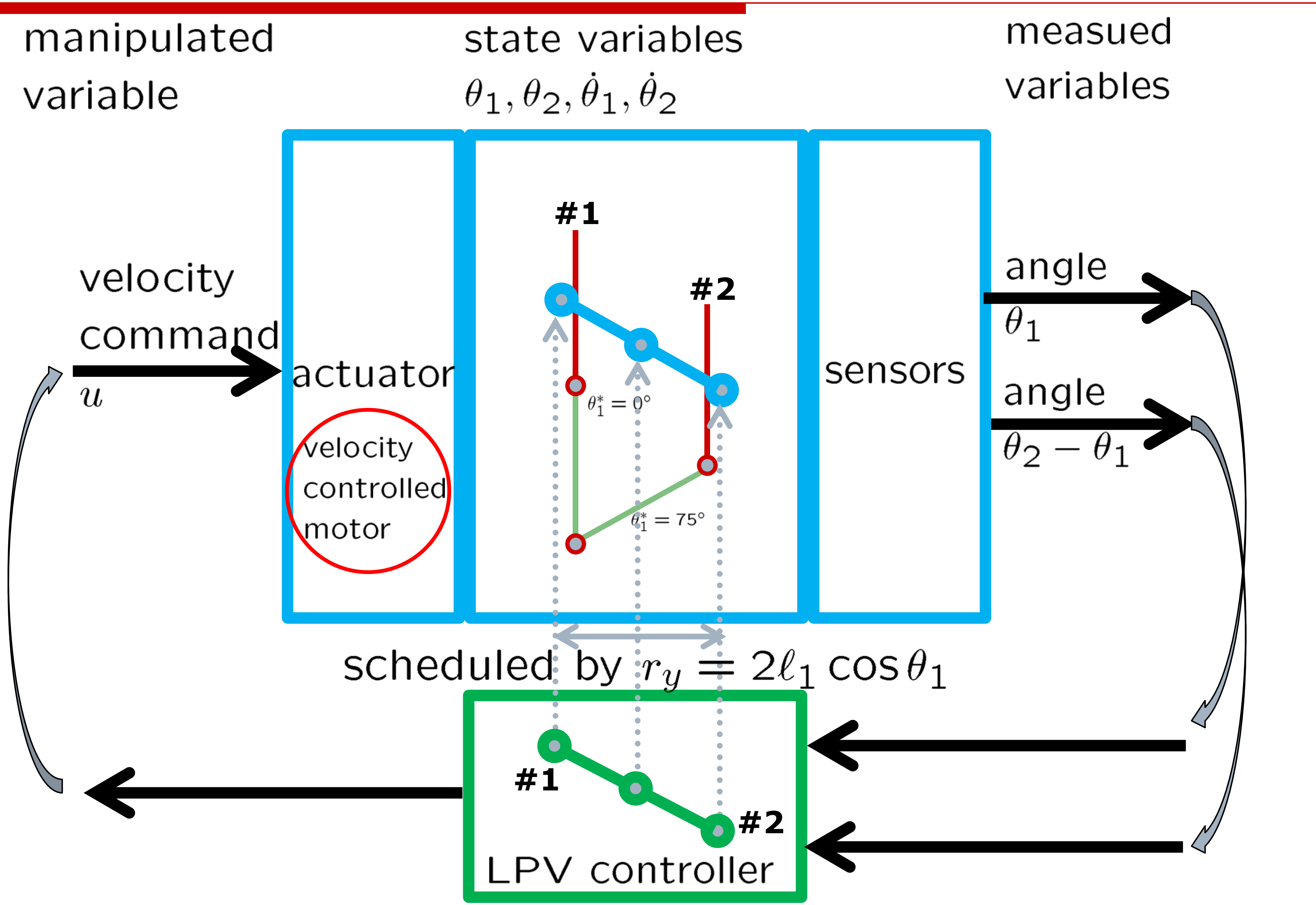
$$\dot{x} = \underbrace{(p_1(r_y)A_1 + p_2(r_y)A_2)}_{A(r_y)} x + Bu$$

where $A_1 = A(\underline{r_y}), A_2 = A(\overline{r_y})$ and

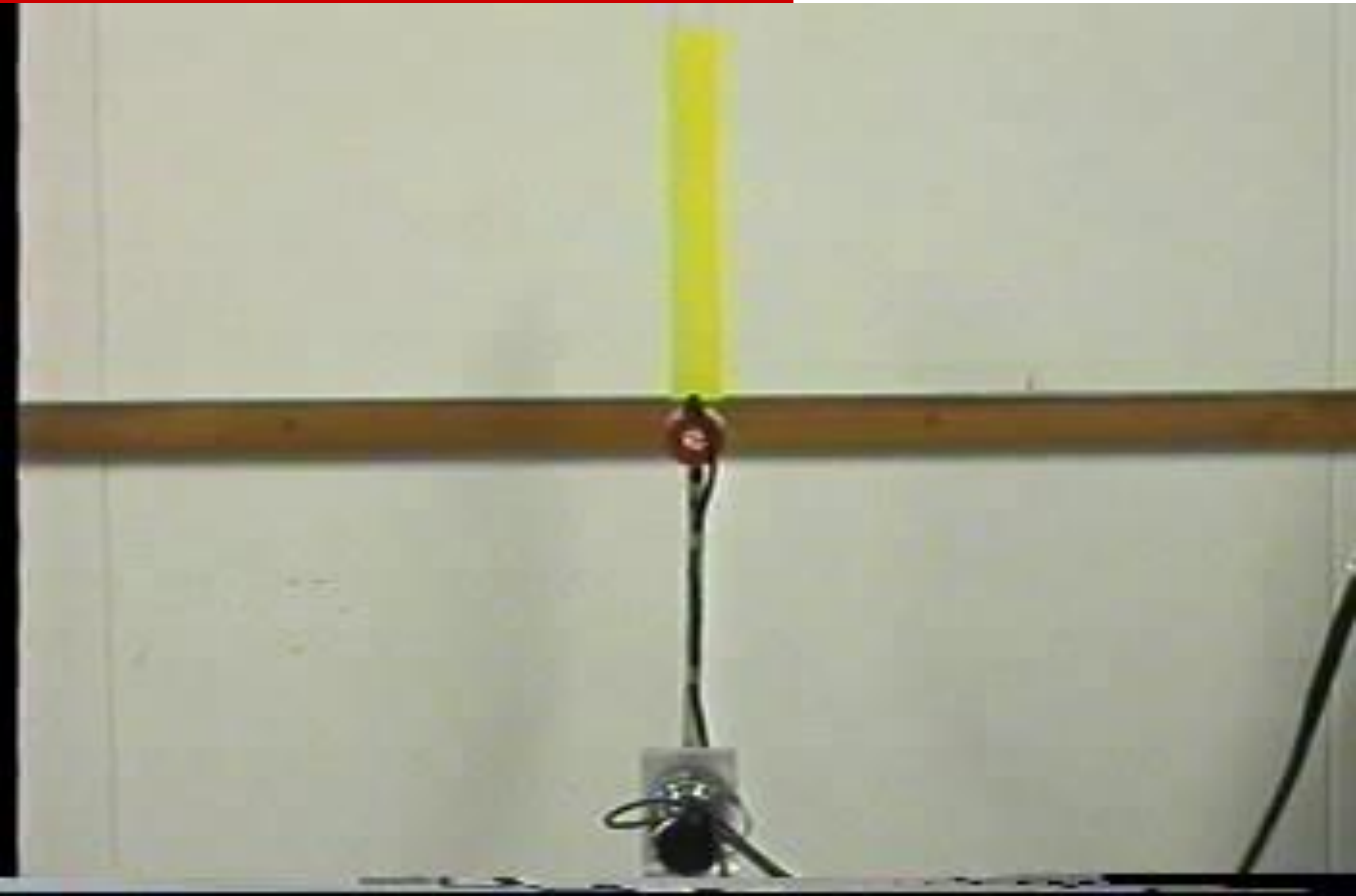
$$p_1(r_y) = \frac{\overline{r_y} - r_y}{\overline{r_y} - \underline{r_y}}, p_2(r_y) = \frac{r_y - \underline{r_y}}{\overline{r_y} - \underline{r_y}}$$

satisfying $p_1(r_y) + p_2(r_y) = 1$

Control System for ADIP



Arm-Driven IP (1997)



Outline

1 **LQI Control**

Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 **LPV Control**

Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control

Applications

3 Underwater Vehicle

4 Flexible Riser

5 Azimuth thrusters

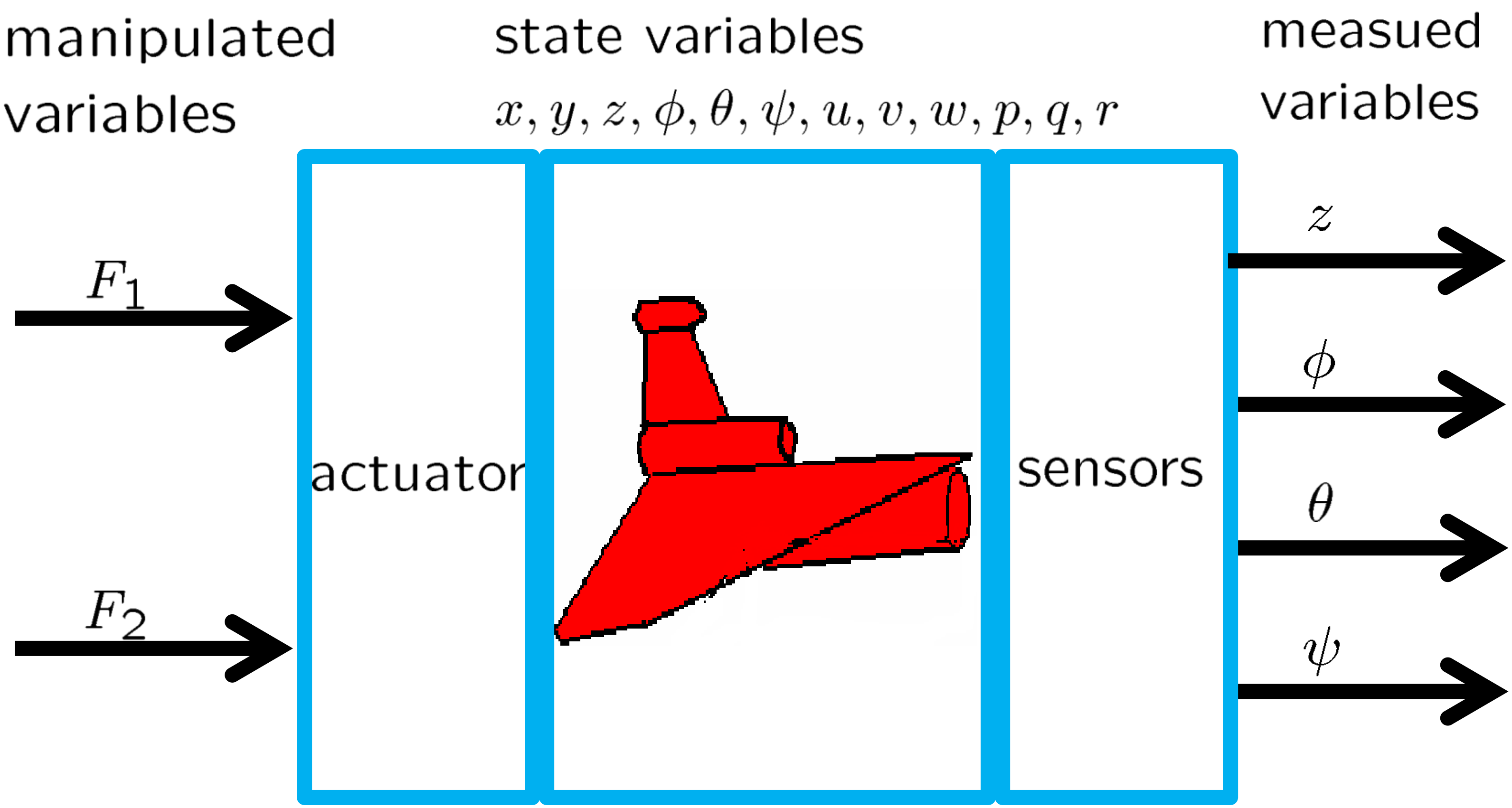
6 Nomoto's Model

7 Wind Turbine

Underwater Vehicle: DELTA

Wide-area survey:
Towing mode for wide scanning
Self-propulsive mode for local investigation

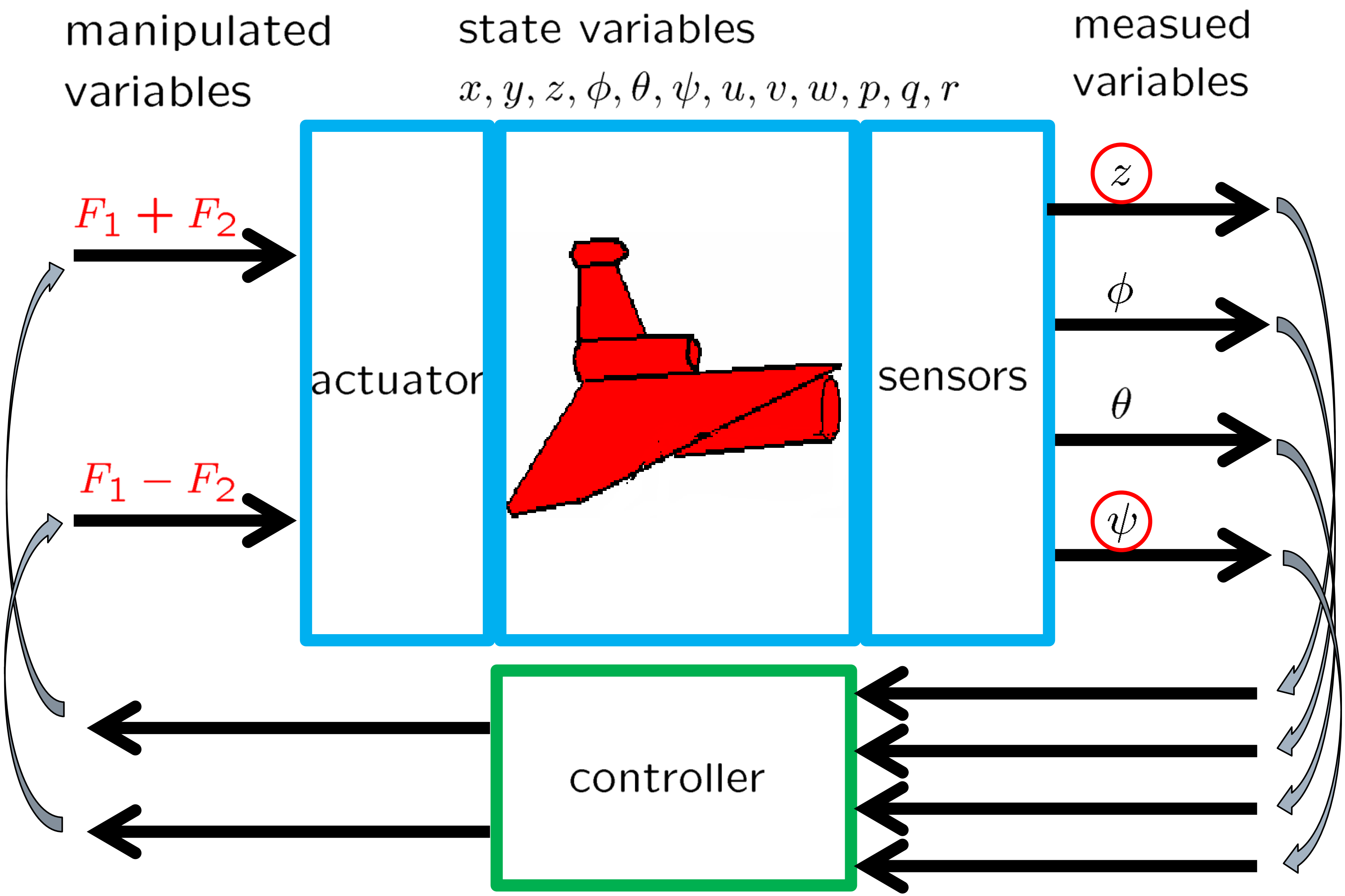
Underwater Vehicle (DELTA)



$$\dot{\xi}_E = J(\xi_E)\xi_B \quad \left(\xi_E = \begin{bmatrix} x, y, z, \phi, \theta, \psi \end{bmatrix}^T,\ \xi_B = \begin{bmatrix} u, v, w, p, q, r \end{bmatrix}^T\right)$$

$$M\dot{\xi}_B + (C(\xi_B) + D(\xi_B))\xi_B + G(\xi_E) = F$$

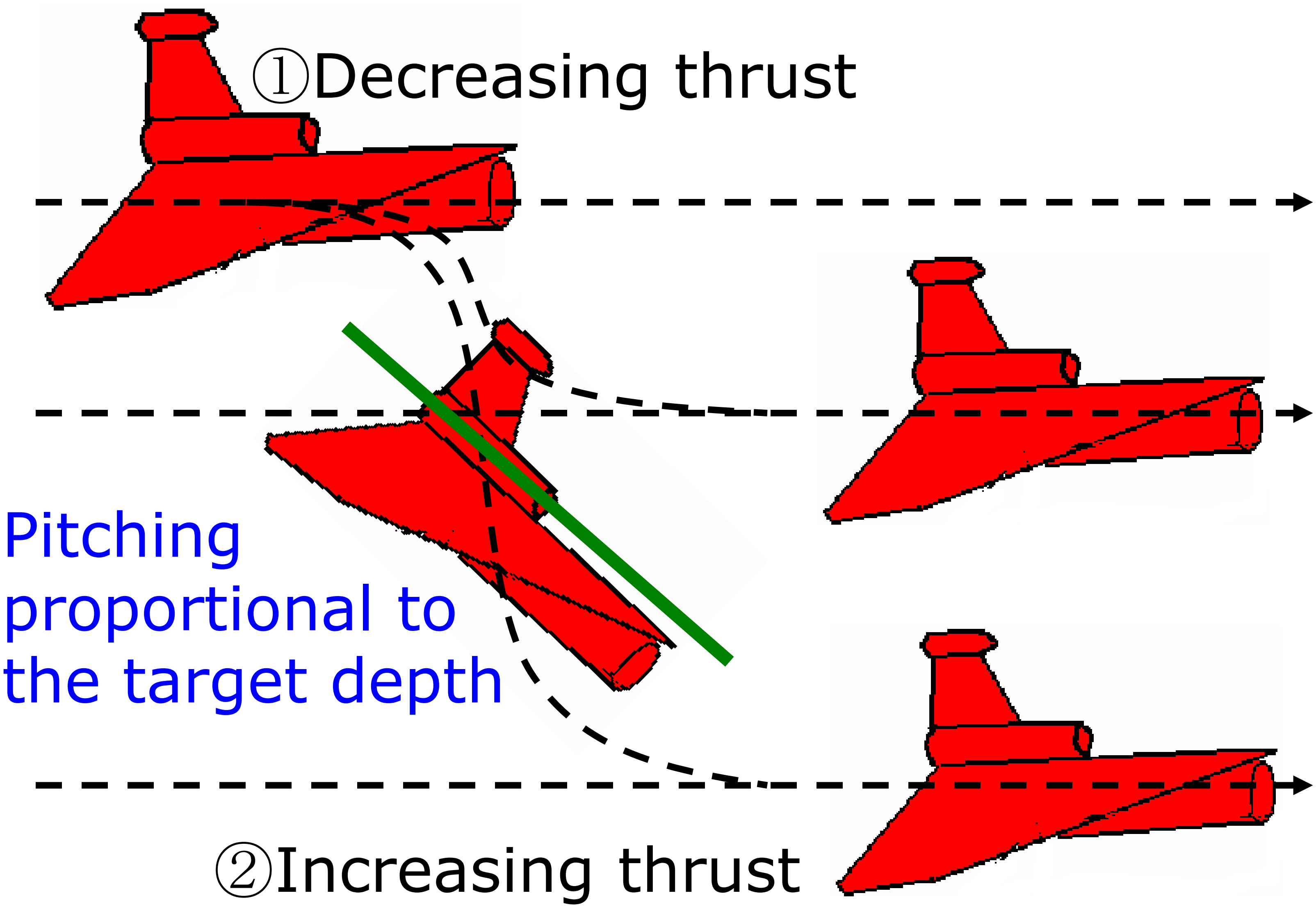
Control System for DELTA



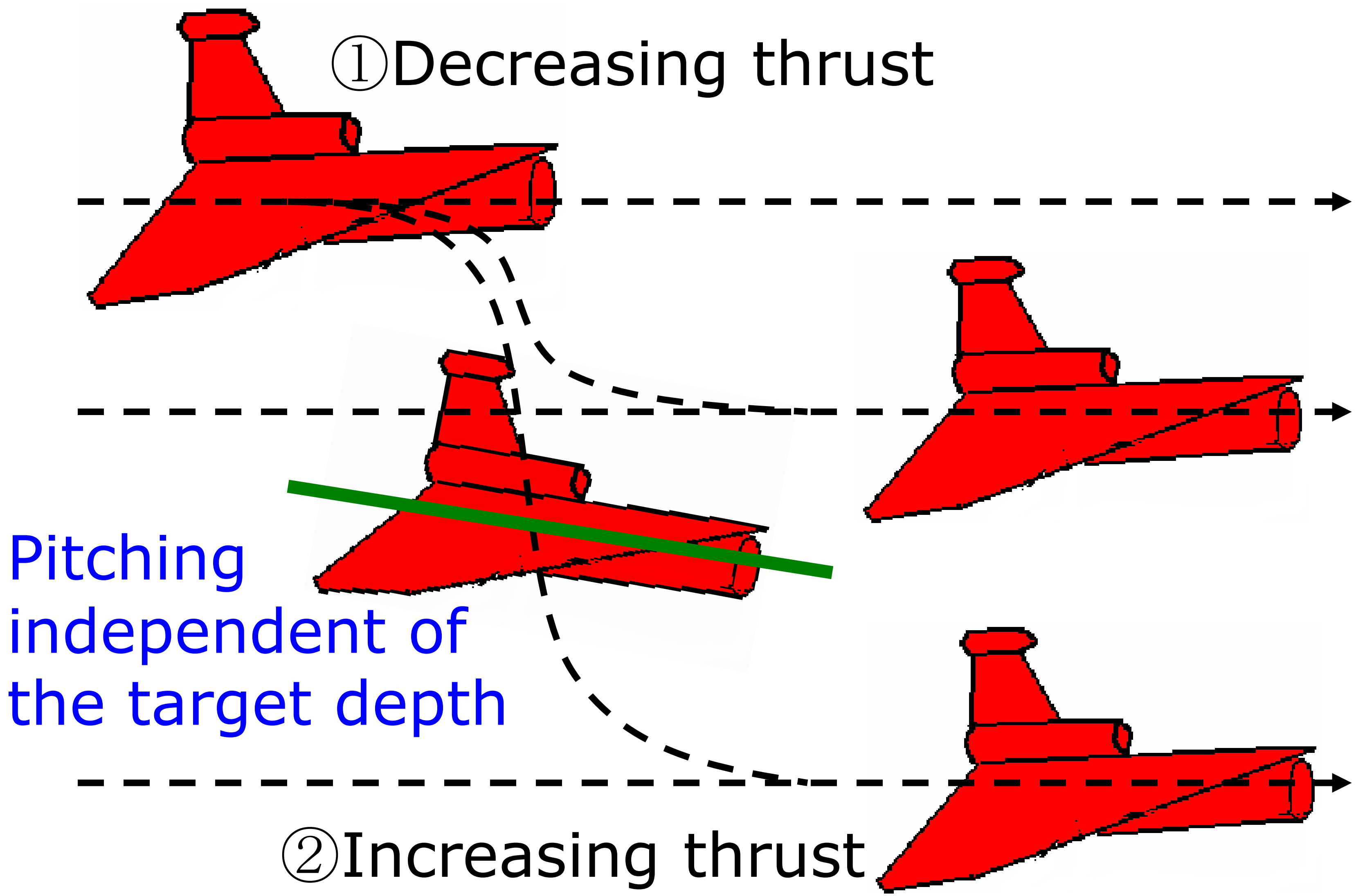
Exp#1 (LQI Control, 1992)

Exp#2 (Manual and LQI Controls)

Diving by **Linear** Control



Diving by **Scheduling** Control



Physical Parameters of DELTA

$$L=1.13,\ d=0.185,\ \rho=102,\ g=9.8$$

$$\nabla=53.24/1000,\ x_T=L\times(-0.088495),\ z_T=L\times(-0.04956)$$

$$x_B=-0.10555+0.0015,\ z_B=-0.04254$$

$$m=(51.82+w_1+w_2)/9.8,\ dm=0.5\rho L^2 d\times 0.02434$$

$$x_{G**}=(-5.1359+0.4787w_1-0.4368w_2)/(48.9469+w_1+w_2)$$

$$x_G=(m-dm)/m\,x_{G**}+dm/m\,x_{dm},\quad z_G=(-1.5536+0.075w_1+0.075w_2)/(51.82+w_1+w_2)$$

$$I_{xx}=0.32323+0.075^2(w_1+w_2)/9.8$$

$$I_{yy*}=0.54778+((0.4787^2+0.075^2)w_1+(0.4368^2+0.075^2)w_2)/9.8,\ I_{yy}=I_{yy*}+dm\,x_{dm}^2$$

$$I_{zz*}=0.86779+(0.4787^2w_1+0.4368^2w_2)/9.8,\ I_{zz}=I_{zz*}+dm\,x_{dm}^2,\ I_{xz}=0$$

Hydrodynamic Coefficients

$$A_{11}=0.5\rho L^2 d\times 0.1278,\quad A_{22}=0.5\rho L^2 d\times 0.0,\quad A_{33}=0.5\rho L^2 d\times 0.5981$$

$$A_{44}=0.5\rho L^4 d\times 0.0843,\quad A_{55}=0.5\rho L^4 d\times 0.4499,\quad A_{66}=0.5\rho L^4 d\times 0.0$$

$$X_{uu}=0.5\rho Ld\times(-0.4062),\quad X_{uuk}=0.5\rho Ld\times(-0.173\times 1.0)),\quad X_v=0.5\rho ULd\times 0.4944,\quad X_{ww}=0.5\rho Ld\times 0.2017$$

$$Y_v=0.5\rho ULd\times(-9.901),\quad Y_{vv}=0.5\rho Ld\times 7.88,\quad Y_p=0.5\rho UL^2d\times 0.24618,\quad Y_r=0.5\rho UL^2 d\times 4.7369,\quad Y_{rr}=0.5\rho L^3 d\times 17.695$$

$$Z_w=0.5\rho ULd\times(-7.726)\times 0.9,\quad Z_q=0.5\rho UL^2d\times Z_{q*}$$

$$K_v=0.5\rho UL^2d\times(-0.3254),\quad K_p=0.5\rho UL^3d\times(-0.3336),\quad K_r=0.5\rho UL^3d\times 0.0029953$$

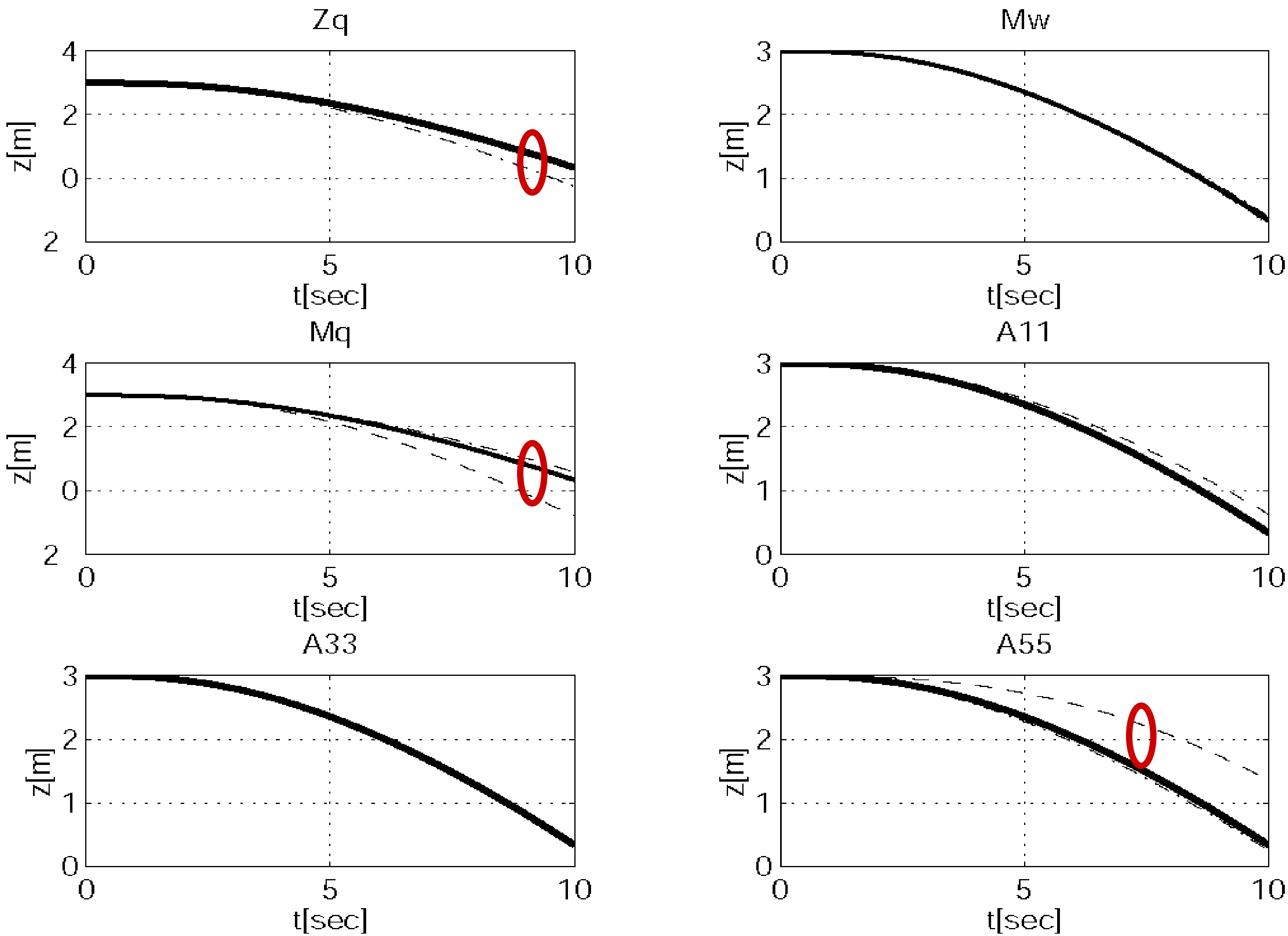
$$M_w=0.5\rho UL^2d\times(-0.8822),\quad M_q=0.5\rho UL^3d\times M_{q*}$$

$$N_v=0.5\rho UL^2d\times 0.9939,\quad N_{vv}=0.5\rho L^2d\times(-6.9564),\quad N_r=0.5\rho UL^3d\times(-0.7028),\quad N_{rww}=0$$

$$\ell_{pTHy}=0.306,\quad \ell_z=L\times(-0.1211),\quad \ell_k=L\times(-0.1593),\quad z_{TH}=L\times 0.08496,\quad \ell_{pTHx}=L\times(-0.1209),\quad y_{TH}=0.306$$

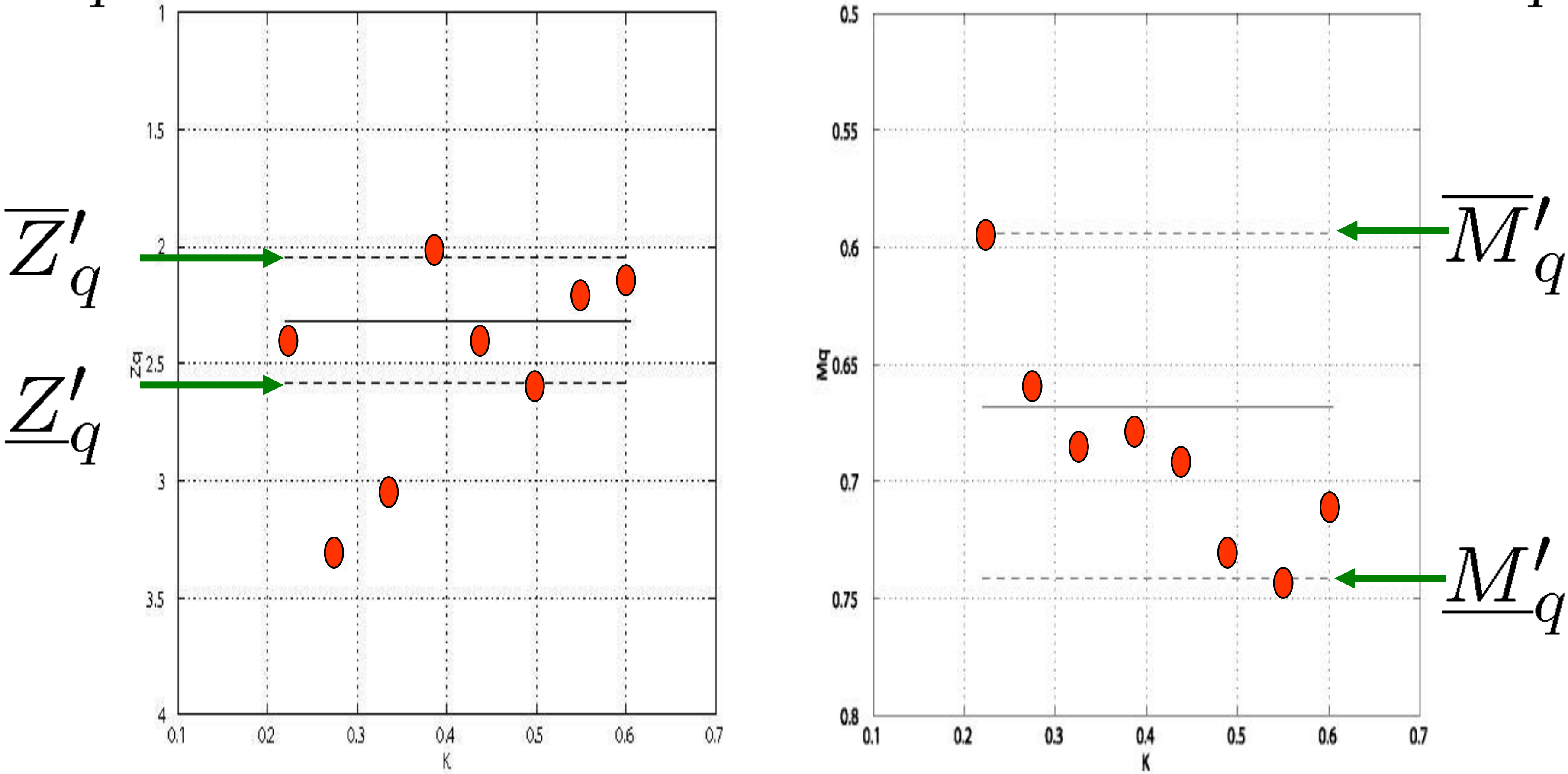
$$b_t=-0.4352,\quad c_t=0.9383,\quad e_t=0.8662,\quad f_t=-0.1054;$$

Parameter Sensitivities



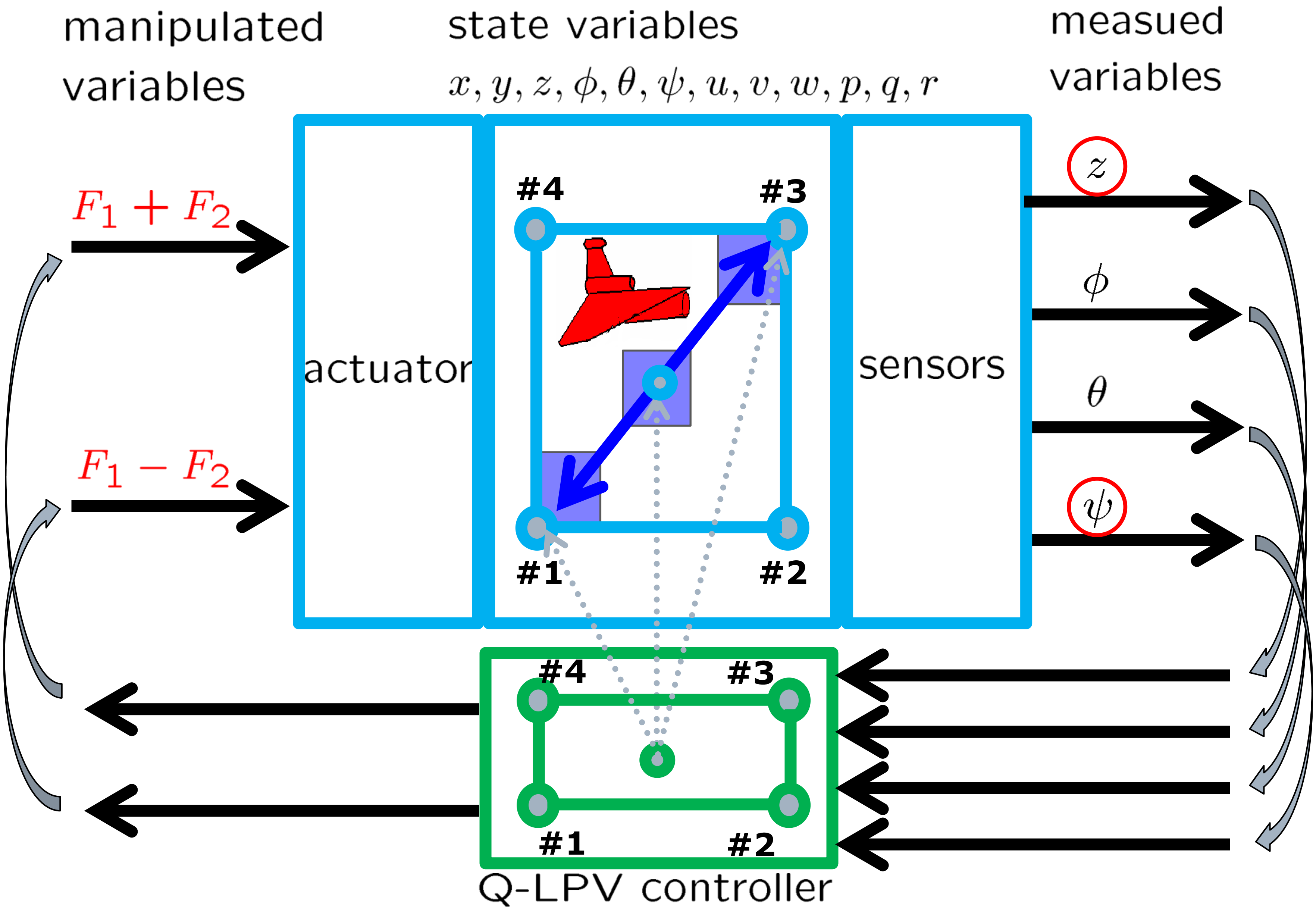
Parameter Uncertainties

Z'_q $$Z_q = (\tfrac{1}{2}\rho U^* L^2) Z'_q, \quad M_q = (\tfrac{1}{2}\rho U^* L^3) M'_q$$ M'_q



Reduced Freq. Reduced Freq.

Control System for DELTA



Exp#3 (LQI Control, 1m to 3m)

Exp#4 (Q-LPV Control, 1m to 3m)

Exp#5 (LQI Control, 1m to 4m)

Exp#6 (Q-LPV Control, 1m to 4m)

Exp#7 (LQI Control, 5m to 1m)

Exp#8 (Q-LPV Control, 5m to 1m)

Exp#9 (Q-LPV Control)

Outline

1 **LQI Control**

Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 **LPV Control**

Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control

Applications

3 Underwater Vehicle

4 Flexible Riser

5 Azimuth thrusters

6 Nomoto’s Model

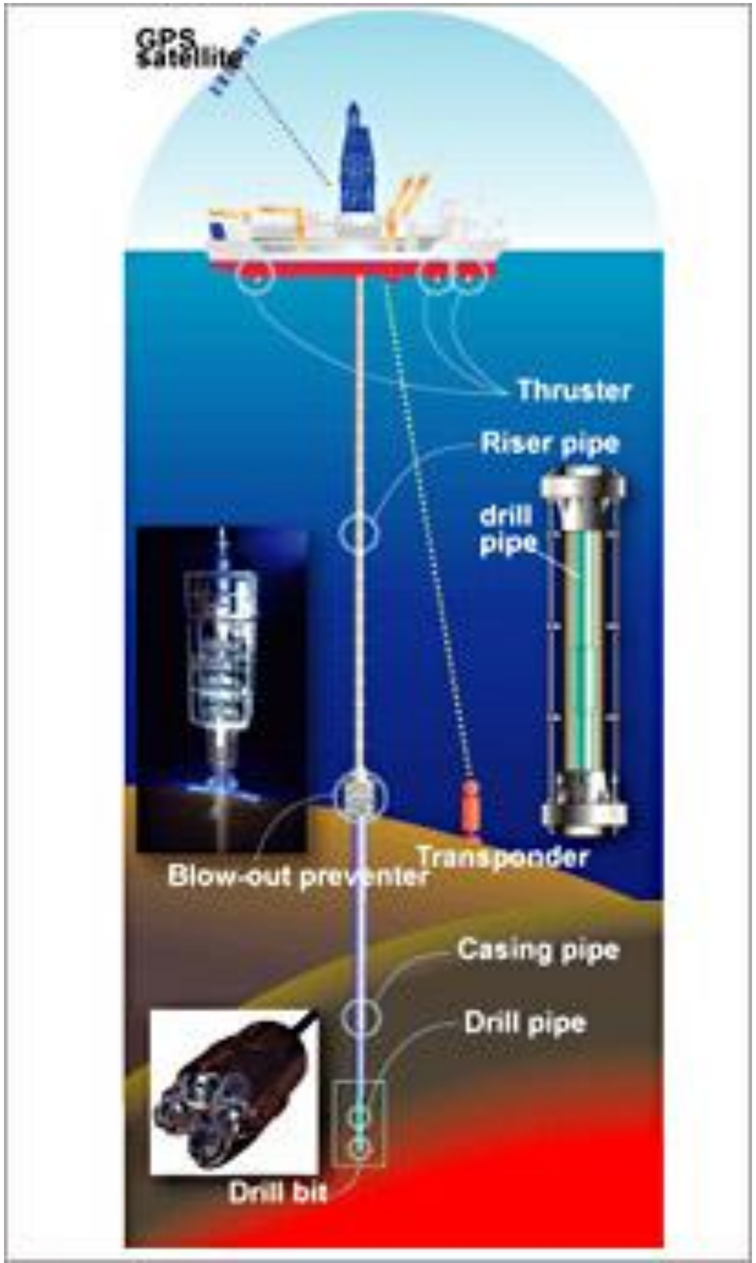
7 Wind Turbine

CHIKYU: Deep-sea Drilling Vessel

Open the new frontier of earth and life science for future of mankind by revealing the system of major earthquakes, global changes, origin of life

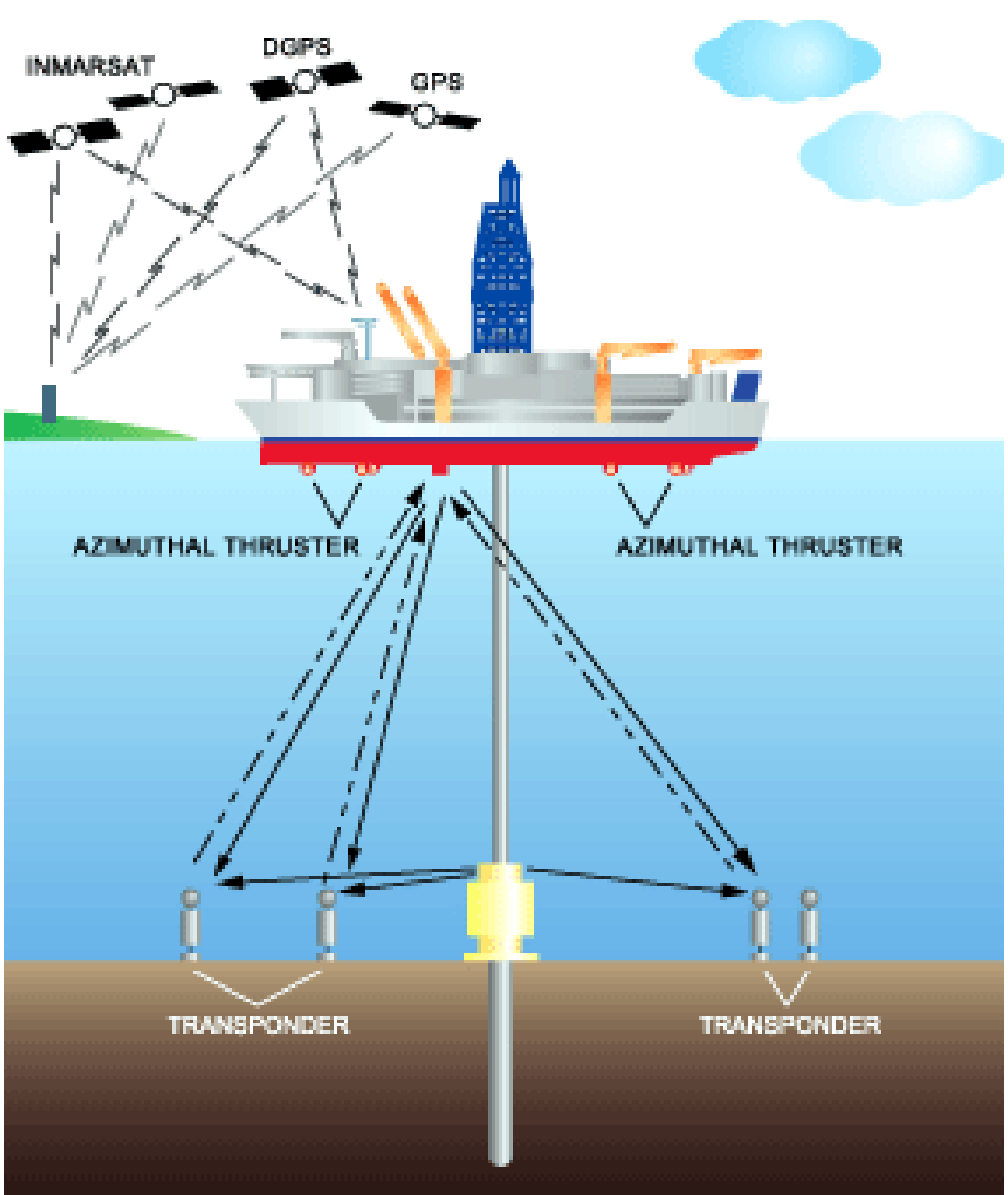
Length : 210m
Breadth : 38m
Gross tonnage : 57,087tons

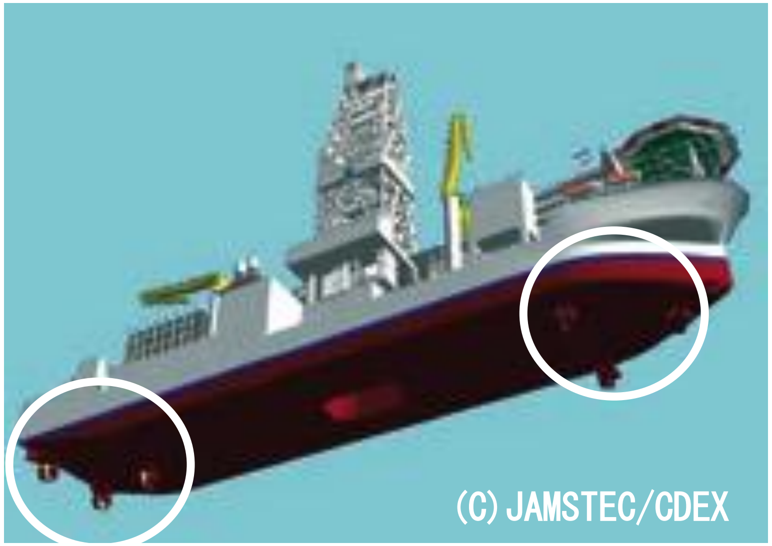
CHIKYU: Riser Pipe Units & BOP



Length :27m
Diameter :1.2m

CHIKYU: DPS & Azimuth Thrusters





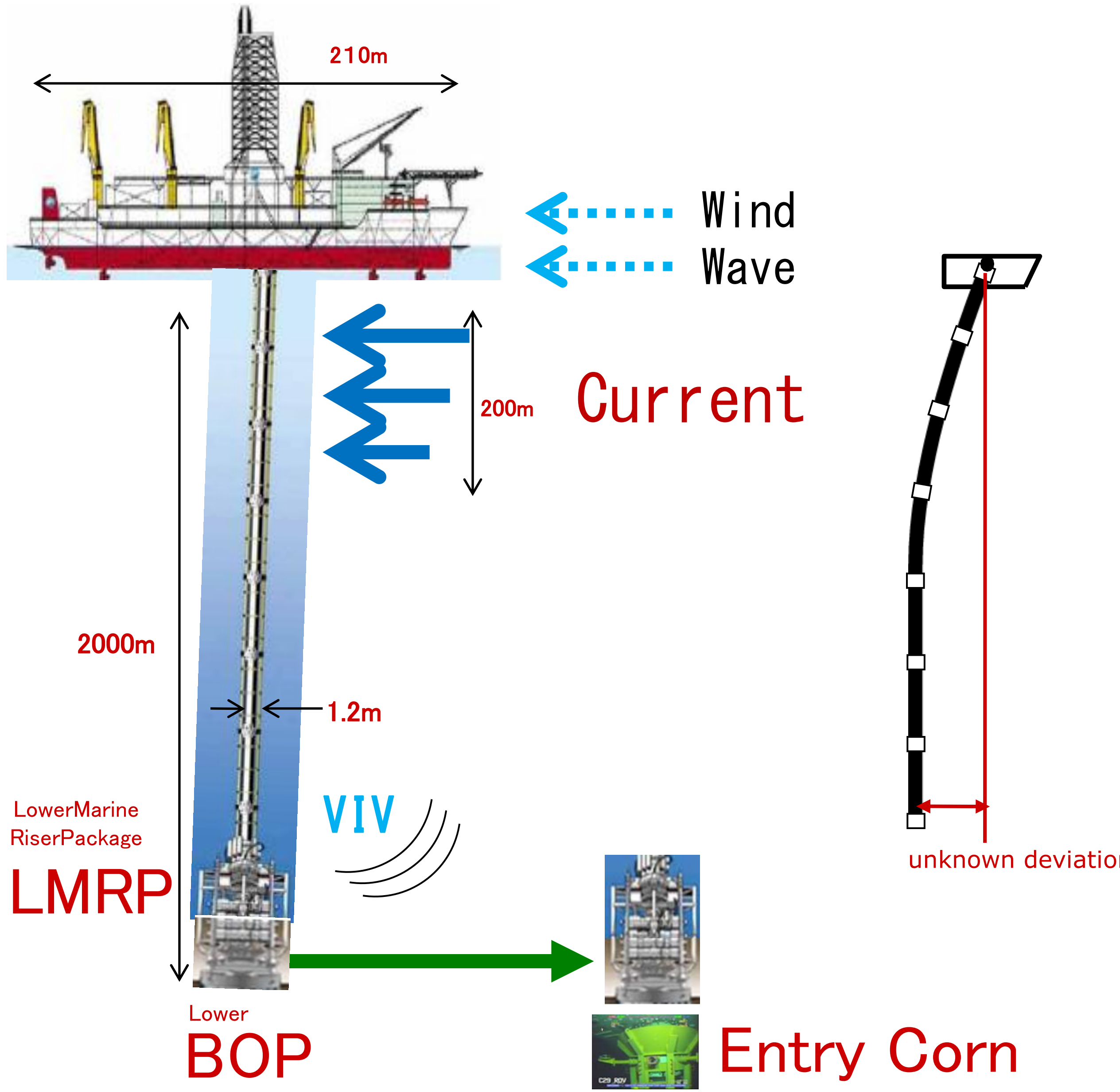
CHIKYU: Drill House

ドリルフロアにある金網で保護されたドリラーズハウス。いくつもの掘削装置を動かすコントロールルームです。
降下中のドリルビットを海中の無人探査機(ROV)でモニタリングしています。

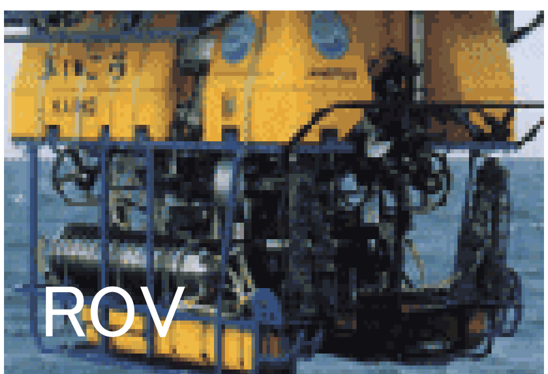
The drill house, a room within a protective steel cage on the drill floor, is where many drilling operations are controlled. Here the controllers are monitoring the drill bit during its descent to the sea floor, using the Remotely Operated Vehicle (ROV).

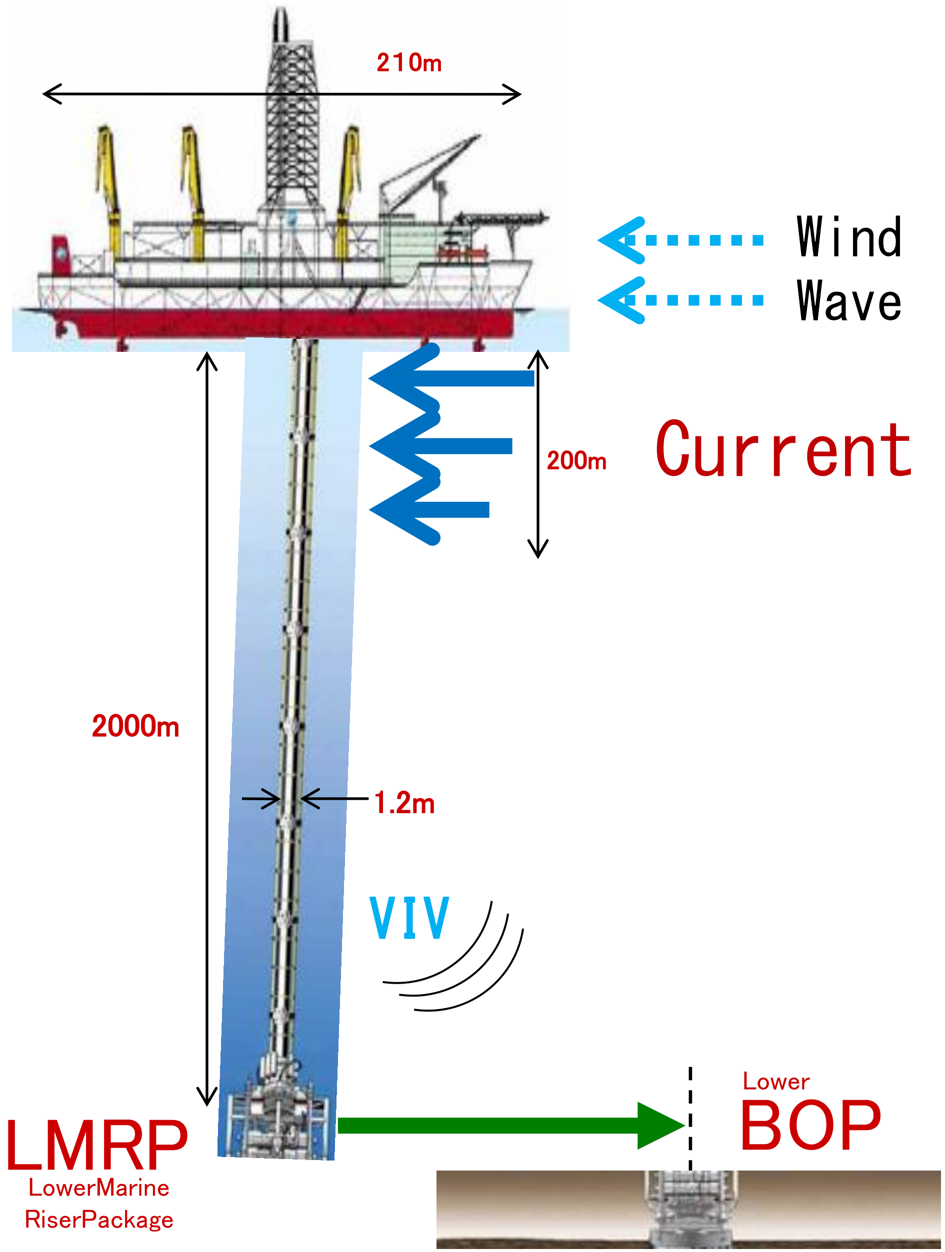
CHIKYU: Operations

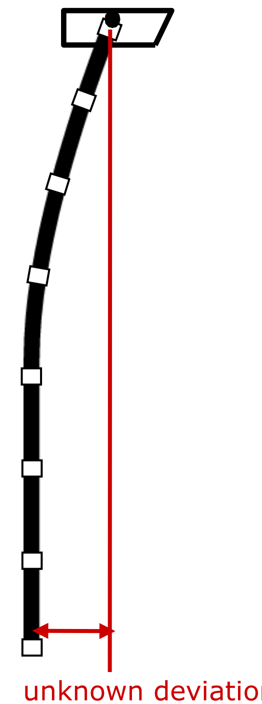
CHIKYU: Landing Operation



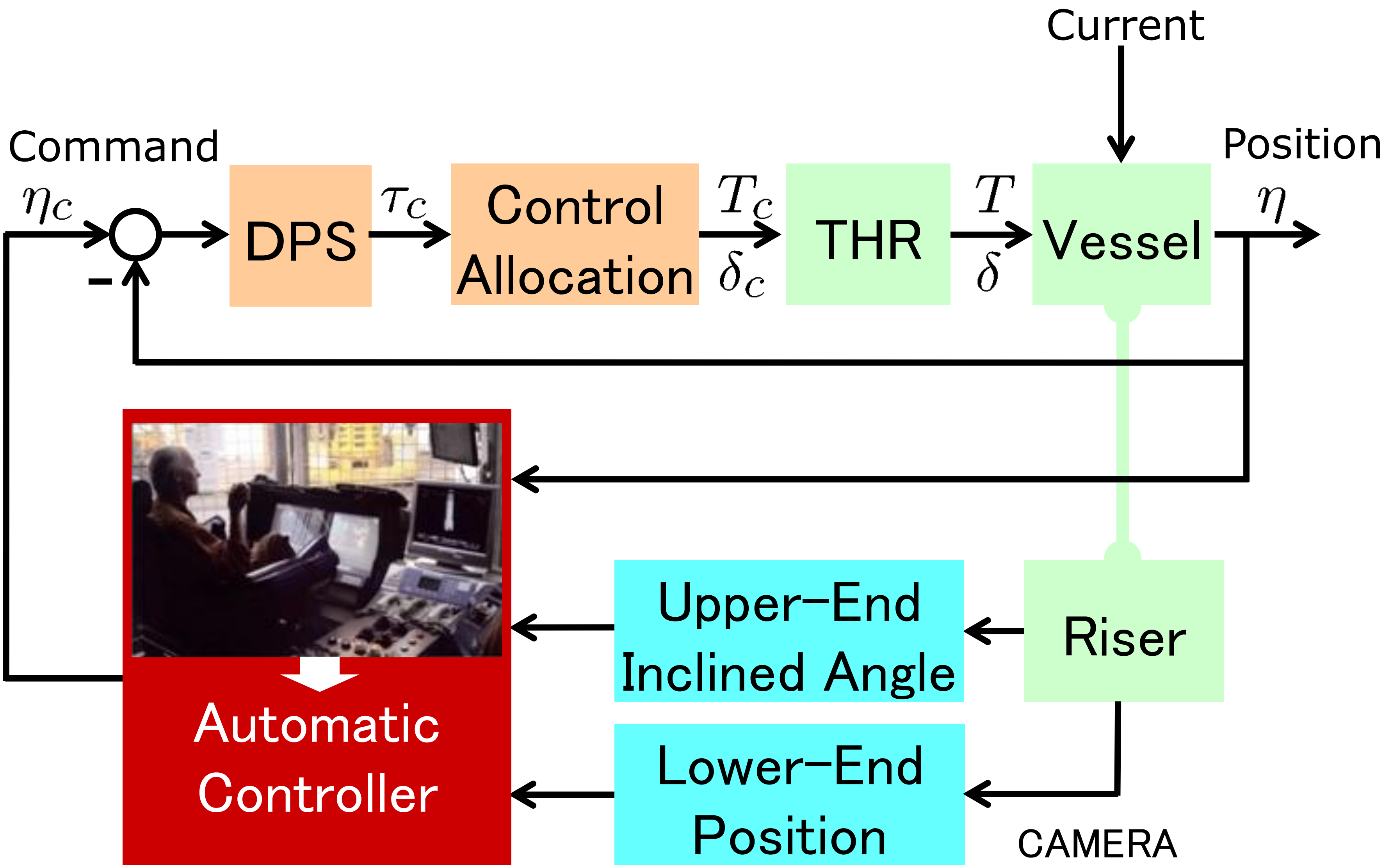
CHIKYU: Reentry Operation



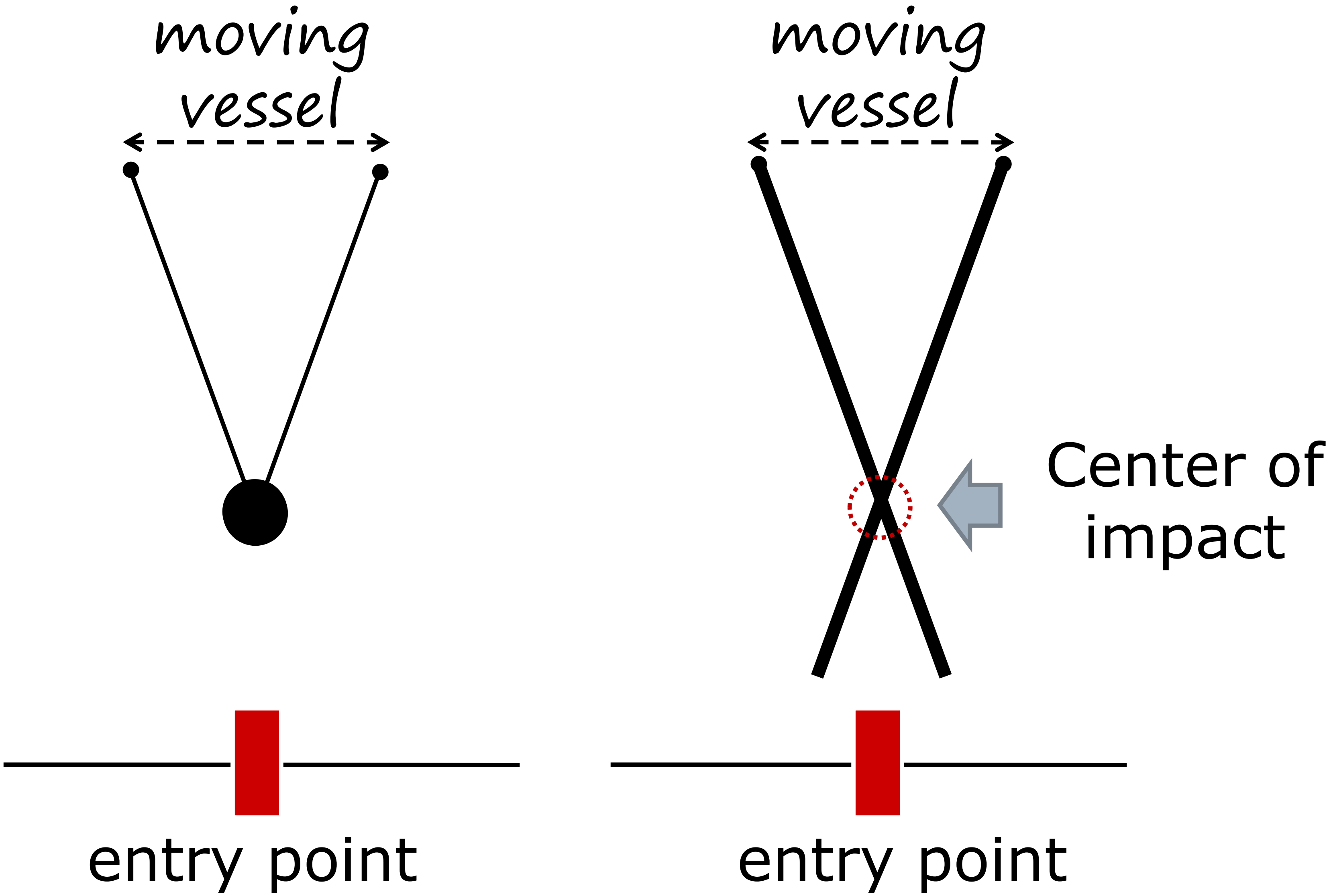




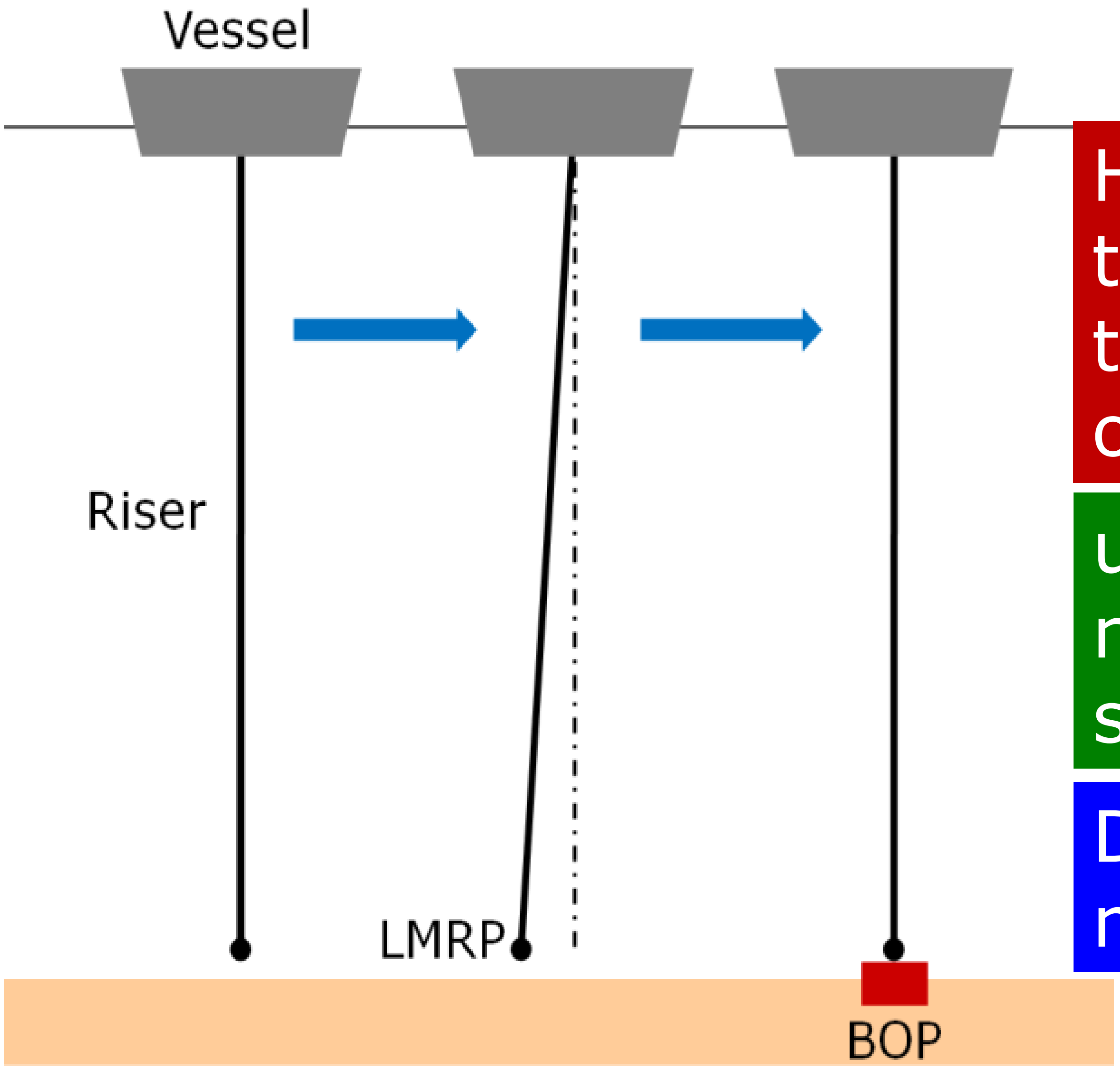
CHIKYU: Reentry Control System



Why the reentry op is difficult?



Reentry Control Problem

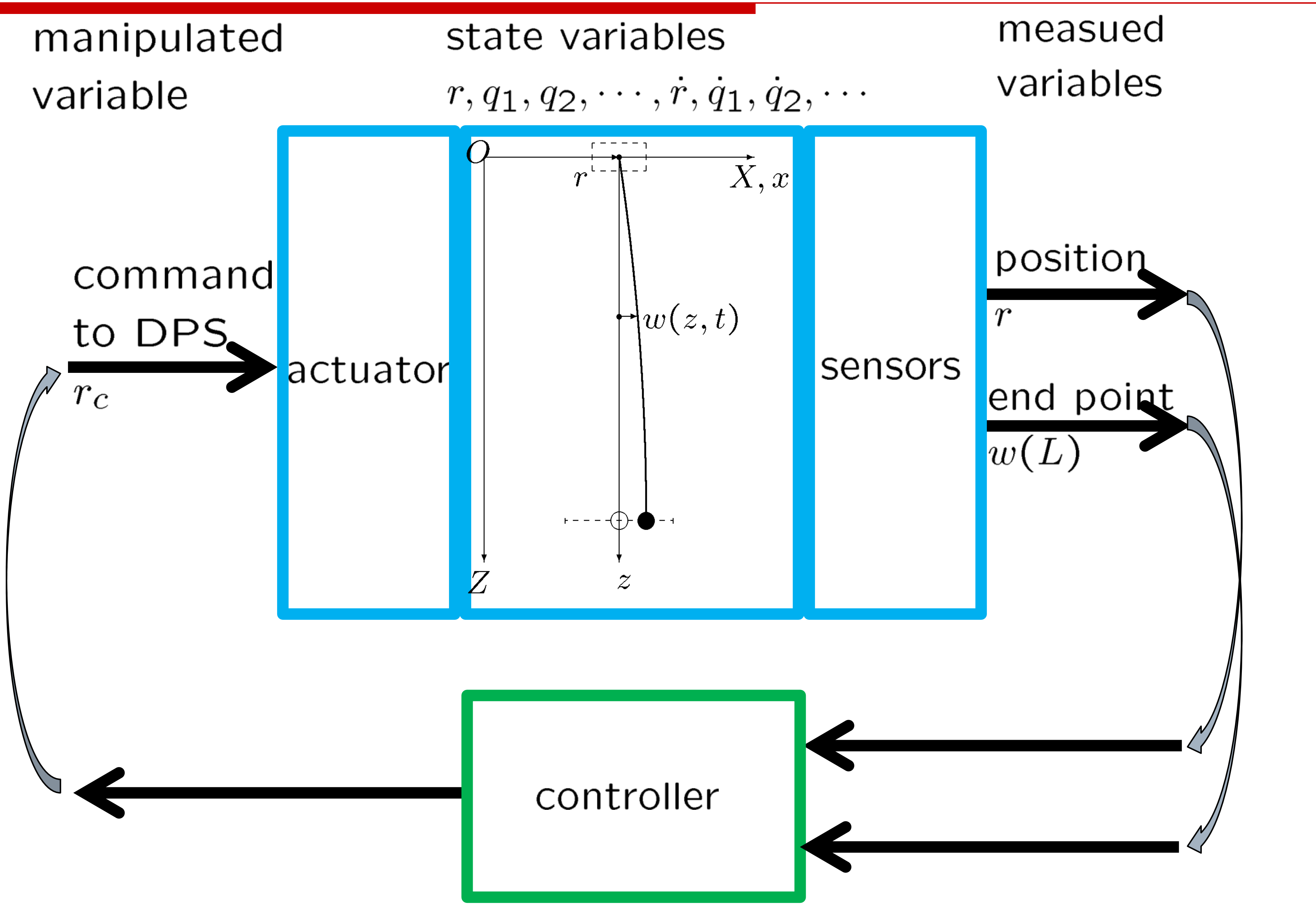


How to realize the stability and the performance of reentry control

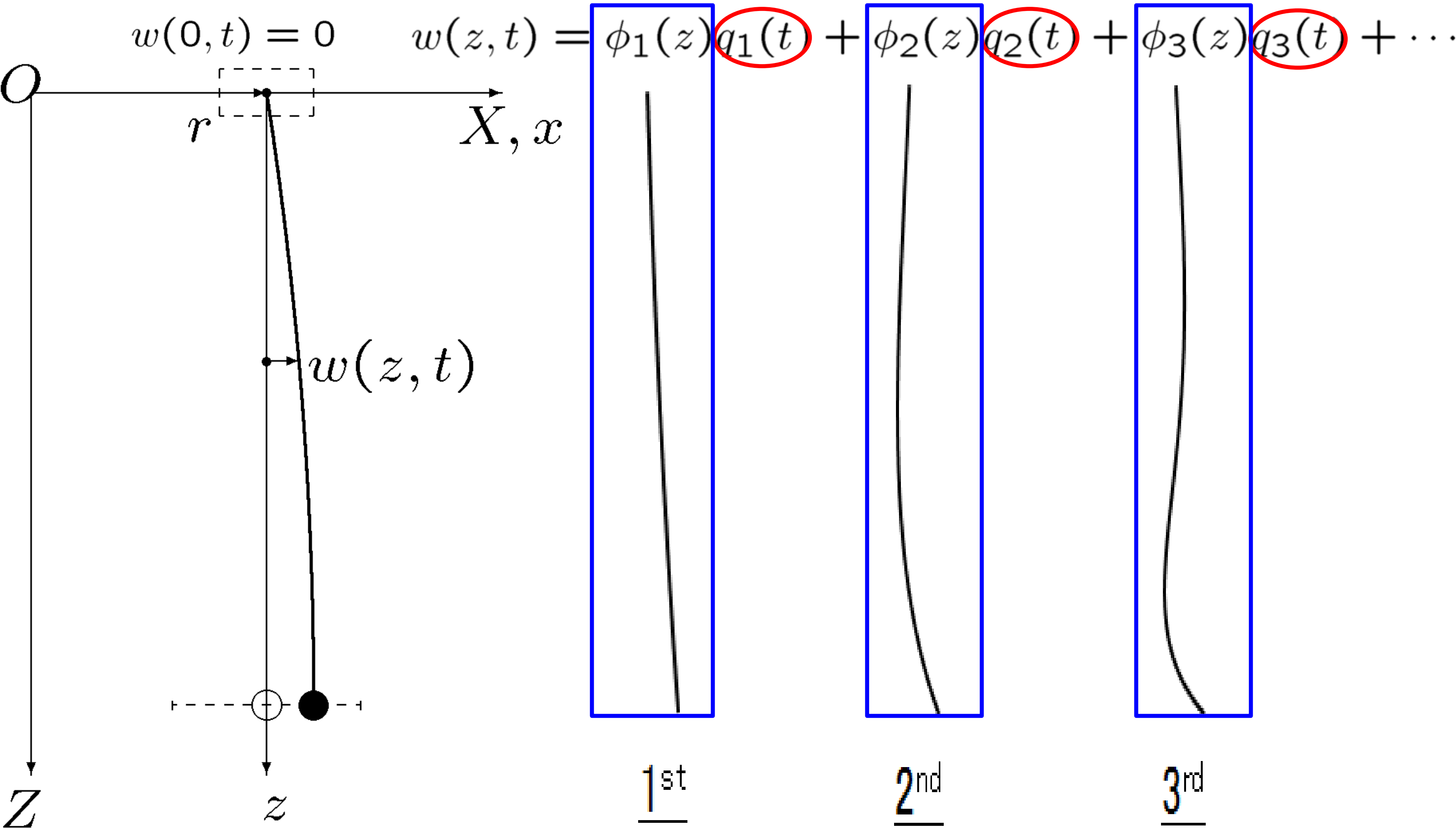
under no wind, no current and surface current

DPS not to be modified

Control System for a Riser



State Variables q1,q2,...



Motion Equation

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{r} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \dot{r} \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{22} \end{bmatrix} \begin{bmatrix} r \\ q \end{bmatrix} = \begin{bmatrix} F_r \\ F_q \end{bmatrix}$$

$$M_{11} = M_s + M_b + (\tilde{m} + m_a)L$$

$$M_{12} = \int_0^L (\tilde{m} + m_a)\phi^T(z)dz + M_b\phi^T(L)$$

$$M_{21} = \int_0^L (\tilde{m} + m_a)\phi(z)dz + M_b\phi(L)$$

$$M_{22} = \int_0^L (\tilde{m} + m_a)\phi(z)\phi^T(z)dz + M_b\phi(L)\phi^T(L)$$

$$D_{11} = \int_0^L \zeta_d|V_{rel}|dz$$

$$D_{12} = \int_0^L \zeta_d|V_{rel}|\phi^T(z)dz \qquad (V_{rel}(z) = \dot{r} + \dot{w}(z) - V_c(z))$$

$$D_{21} = \int_0^L \zeta_d|V_{rel}|\phi(z)dz$$

$$D_{22} = \int_0^L \zeta_d|V_{rel}|\phi(z)\phi^T(z)dz$$

$$K_{22} = M_b g\phi(L)\phi'^T(L) + \int_0^L \phi(z)(\mu(z - \tilde{L})\phi'^T(z))'dz$$

Velocity Input Model

$$M_{21}\ddot{r} + M_{22}\ddot{q} + D_{21}\dot{r} + D_{22}\dot{q} + K_{22}q = F_q$$

$$\Downarrow \qquad \mathcal{M}_{21} = \mathrm{diag}\{M_{21}(1), \cdots, M_{21}(N)\}, \mathbf{1}_N = \begin{bmatrix} 1, \cdots, 1 \end{bmatrix}^T$$

$$\underbrace{\mathbf{1}_N\ddot{r} + \mathcal{M}_{21}^{-1}M_{22}\ddot{q}}_{\ddot{\xi}} + \underbrace{\mathcal{M}_{21}^{-1}D_{22}M_{22}^{-1}\mathcal{M}_{21}}_{-A_{22}}\underbrace{(\mathbf{1}_N\dot{r} + \mathcal{M}_{21}^{-1}M_{22}\dot{q})}_{\dot{\xi}}$$

$$+\underbrace{\mathcal{M}_{21}^{-1}K_{22}M_{22}^{-1}\mathcal{M}_{21}}_{-A_{21}}\boxed{\underbrace{(\mathbf{1}_N r + \mathcal{M}_{21}^{-1}M_{22}q)}_{\xi}} - \underbrace{\mathcal{M}_{21}^{-1}K_{22}M_{22}^{-1}\mathcal{M}_{21}\mathbf{1}_N}_{A_{23}} r$$

$$= \underbrace{(\mathcal{M}_{21}^{-1}D_{22}M_{22}^{-1}\mathcal{M}_{21}\mathbf{1}_N - \mathcal{M}_{21}^{-1}D_{21})}_{B_2}\dot{r} + \underbrace{\mathcal{M}_{21}^{-1}F_q}_{w_2}$$

$$\ddot{\xi} = A_{21}\xi + A_{22}\dot{\xi} + A_{23}r + B_2\dot{r} + w_2$$

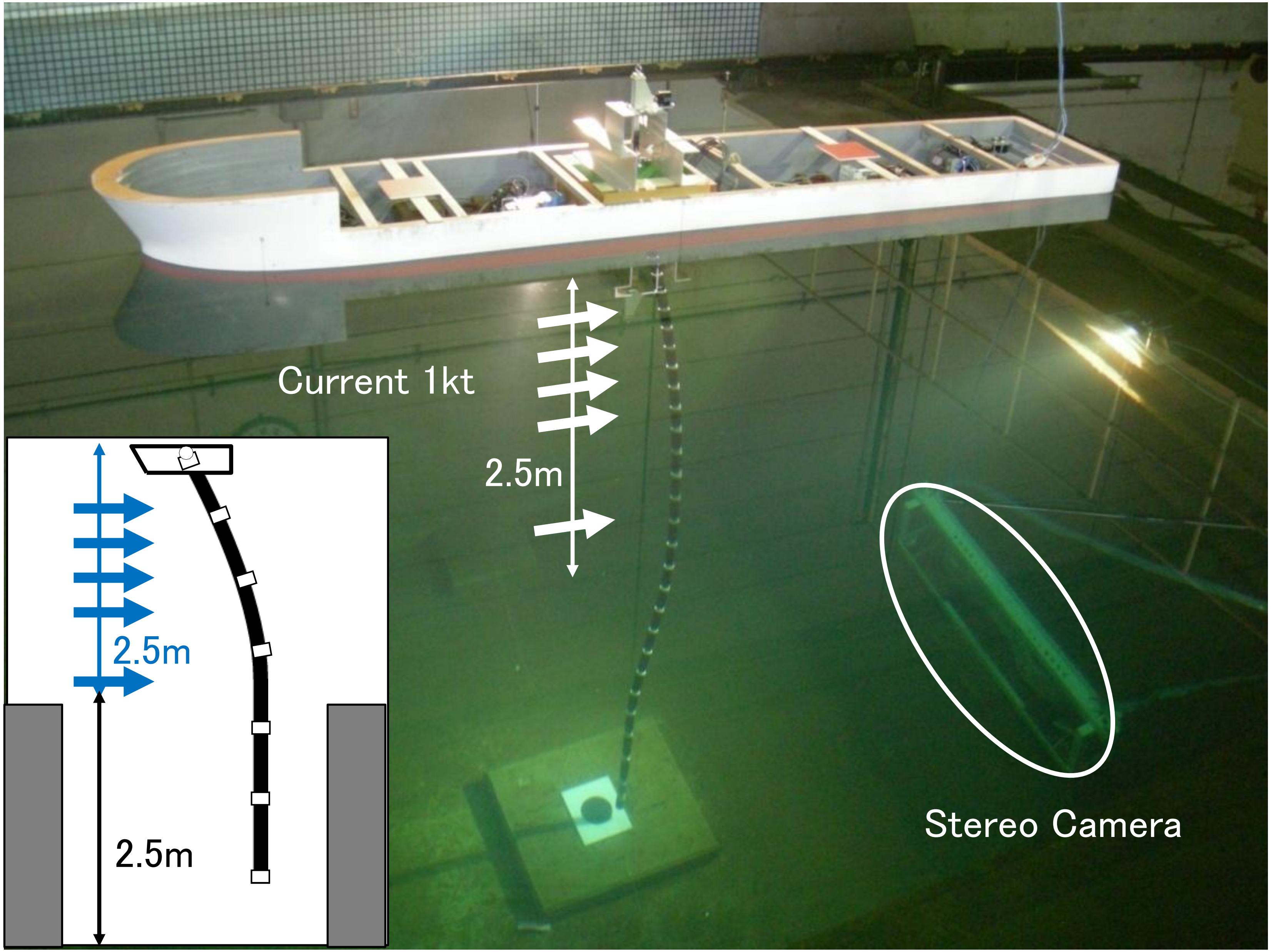
$$\Downarrow \qquad q = M_{22}^{-1}\mathcal{M}_{21}(\xi - \mathbf{1}_N r)$$

velocity input

varying parameter

$$\underbrace{\frac{d}{dt}\begin{bmatrix} \xi \\ \dot{\xi} \\ r \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & I_N & 0 \\ A_{21} & A_{22}(|V_{rel}|) & A_{23} \\ 0 & 0 & 0 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} \xi \\ \dot{\xi} \\ r \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ B_2(|V_{rel}|) \\ 1 \end{bmatrix}}_{B}\underbrace{\dot{r}}_{u} + \underbrace{\begin{bmatrix} 0 \\ w_2 \\ 0 \end{bmatrix}}_{w}$$

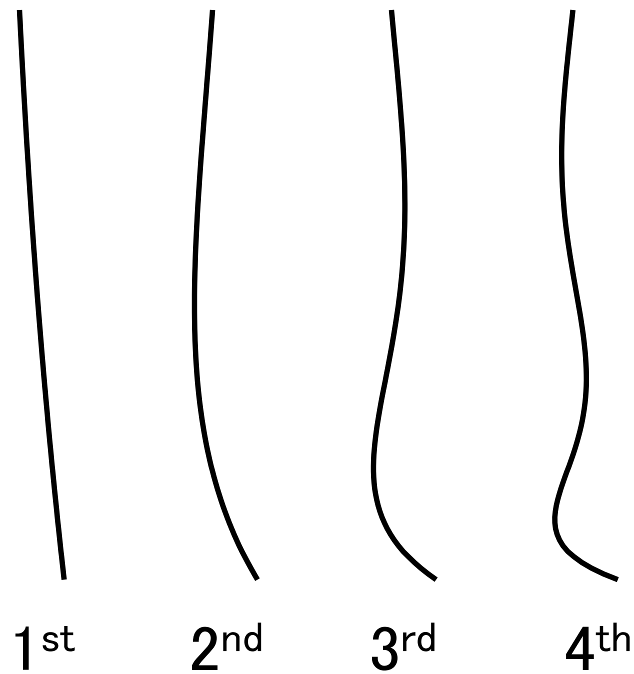
Experimental Set Up (5m)



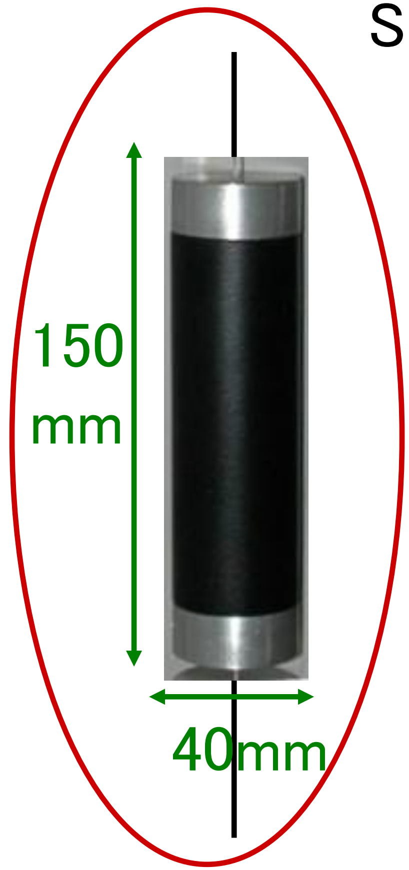
Riser Pipe Unit Model (5m)

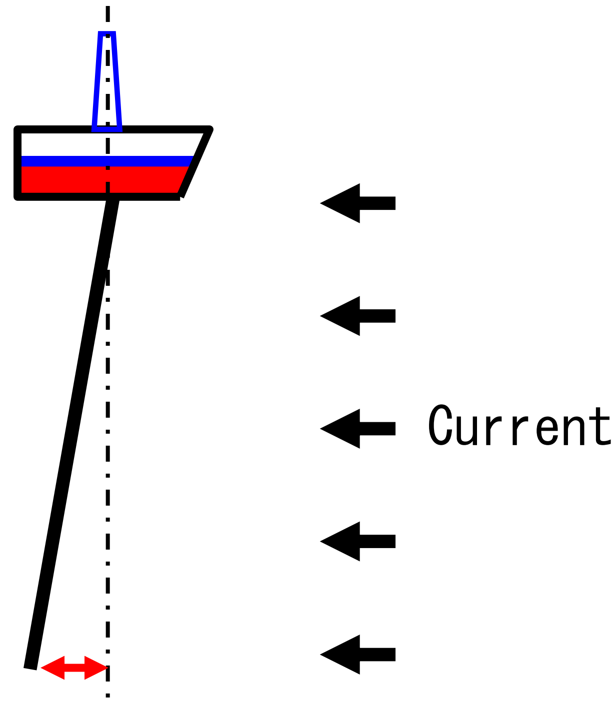
CHIKYU: 210 m length
Vessel Model: 3.8 m length
Scaling factor : 1/55

Riser: 2500 m length
Riser Model: 4.8 m length
Scaling factor : 1/500



$1/\sqrt{55}$ period



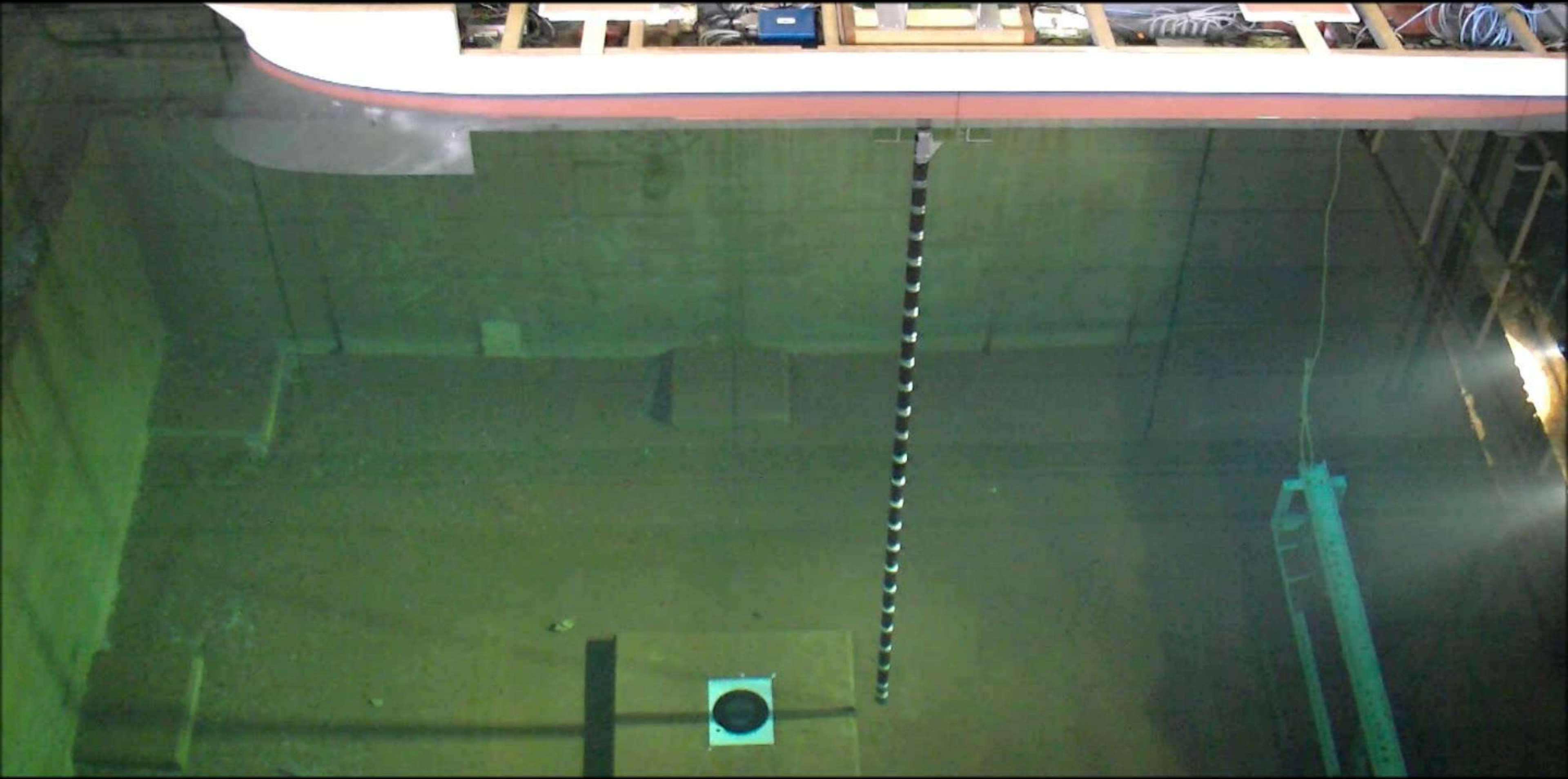


1/55 deviation

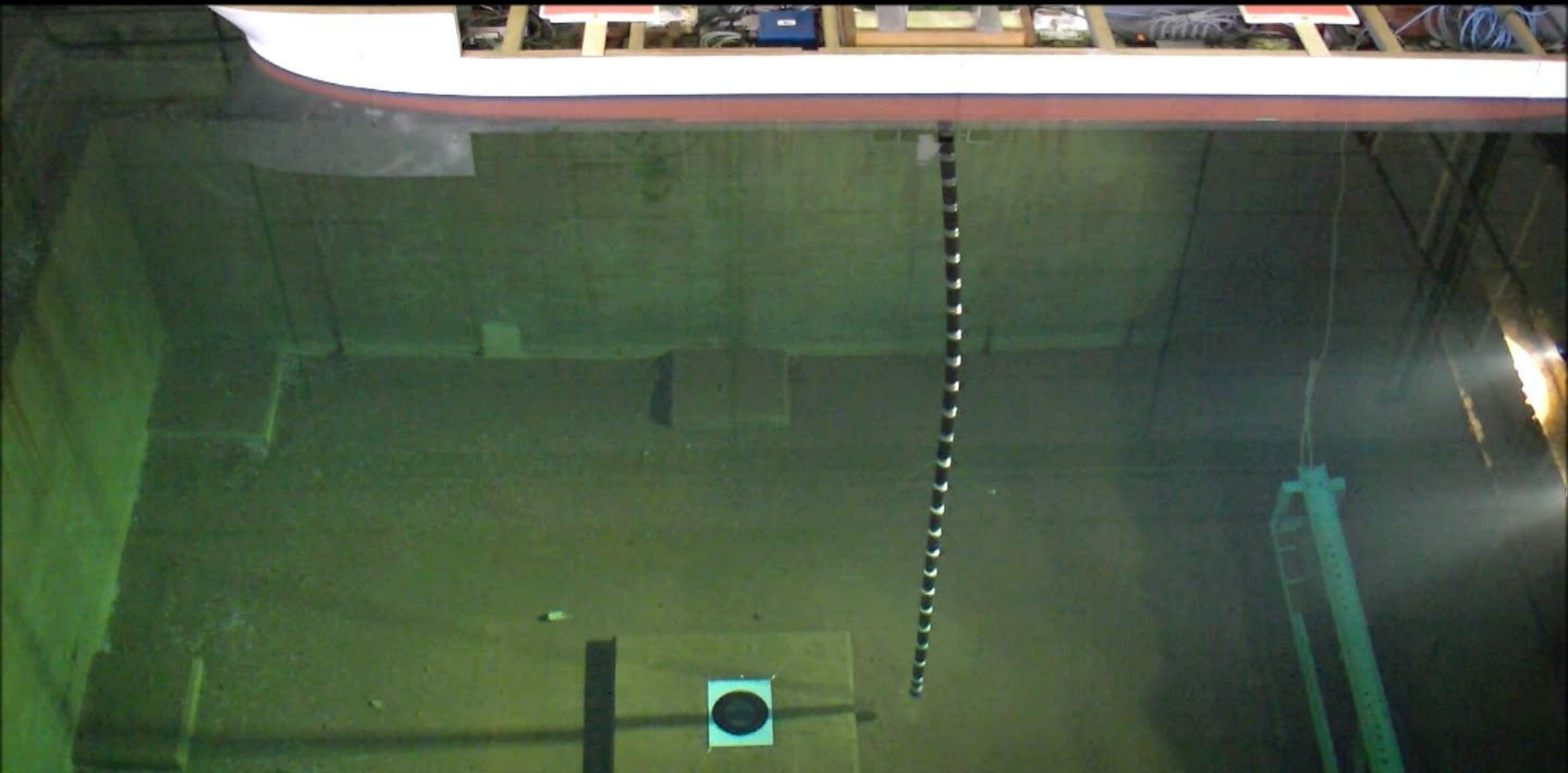
Dynamic Similarity

Geometric Similarity

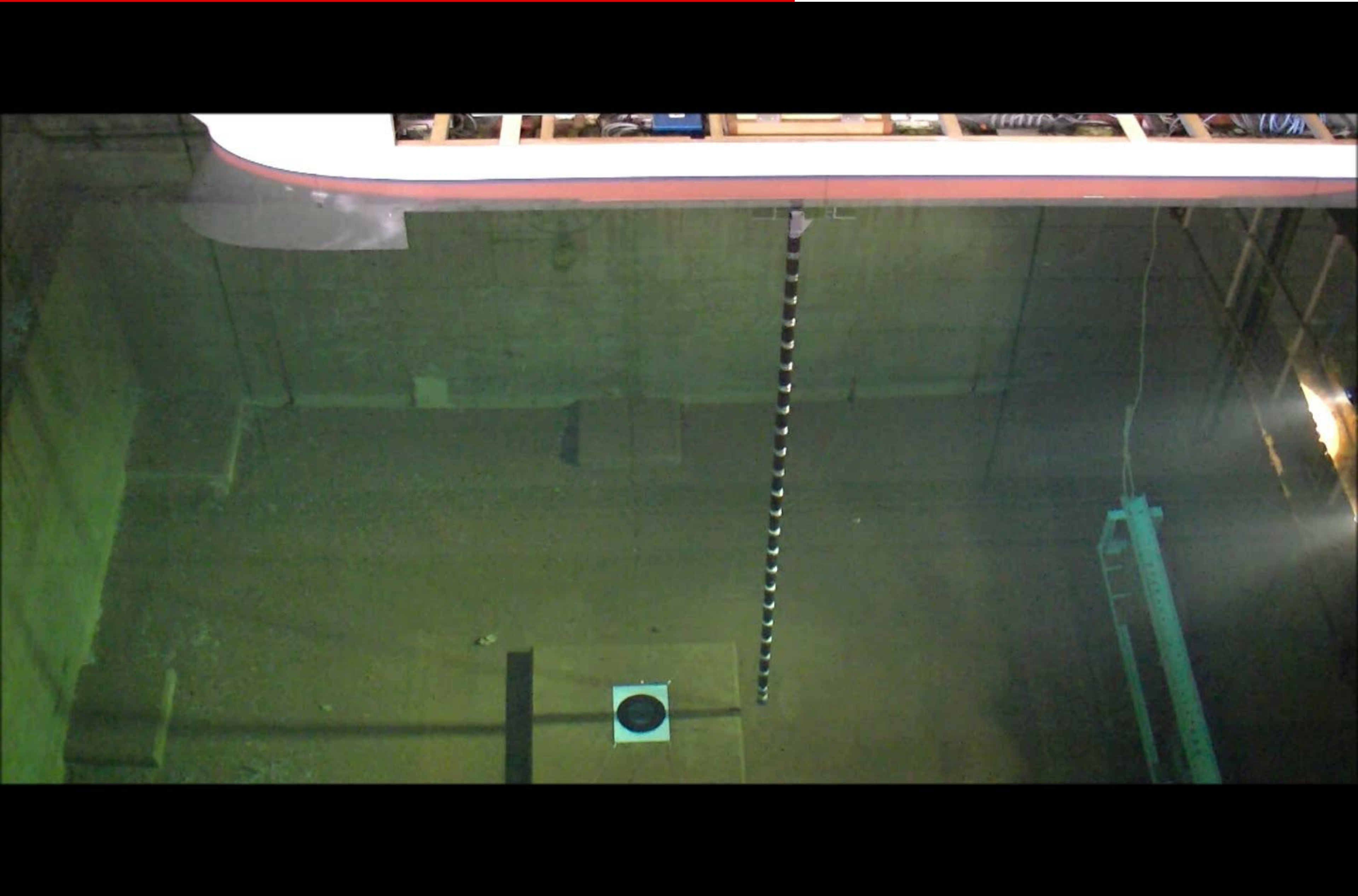
Exp#1 (No Control for Riser)



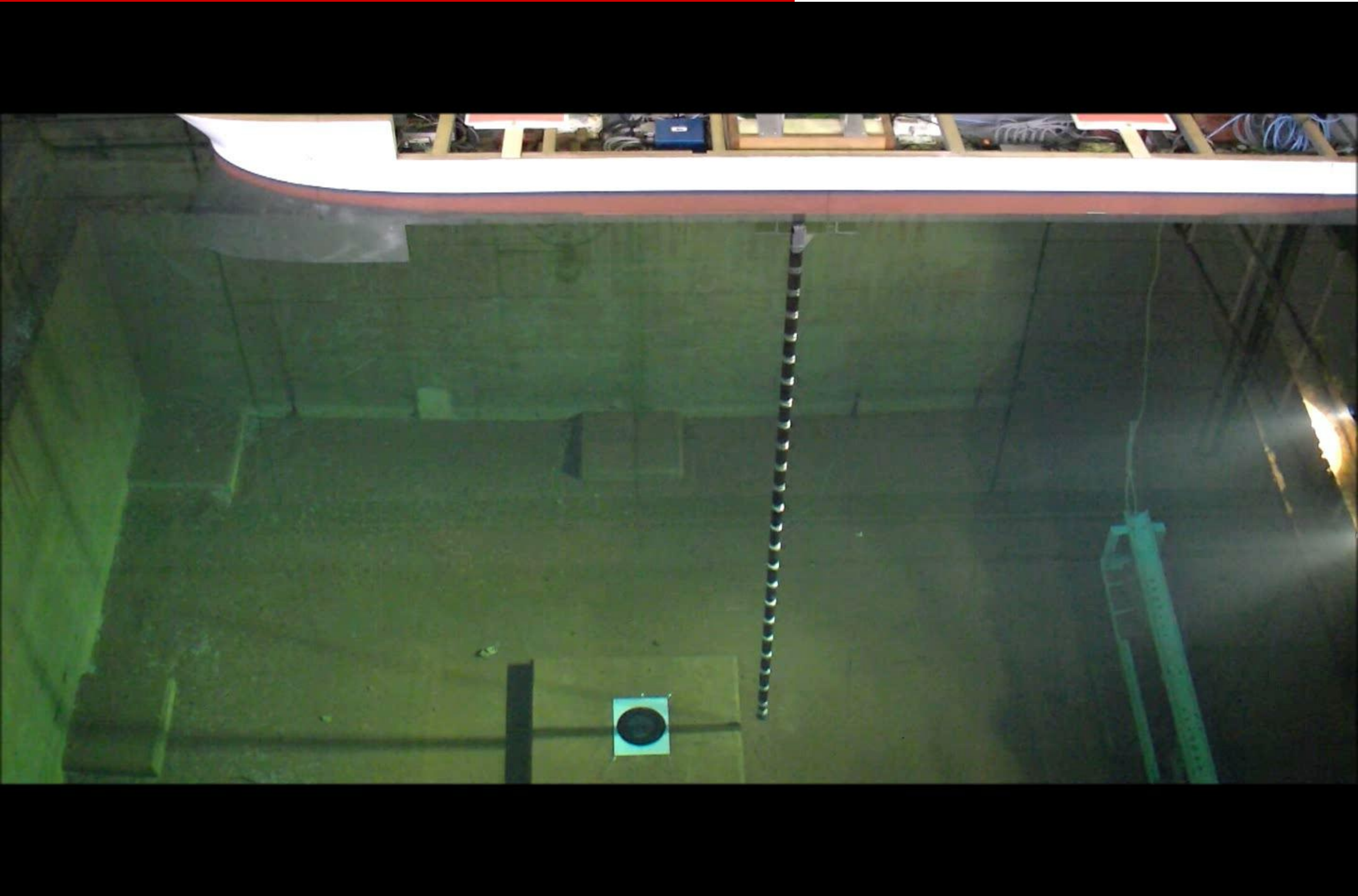
Exp#2 (No Control with Current)



Exp#3 (Unity Feedback)

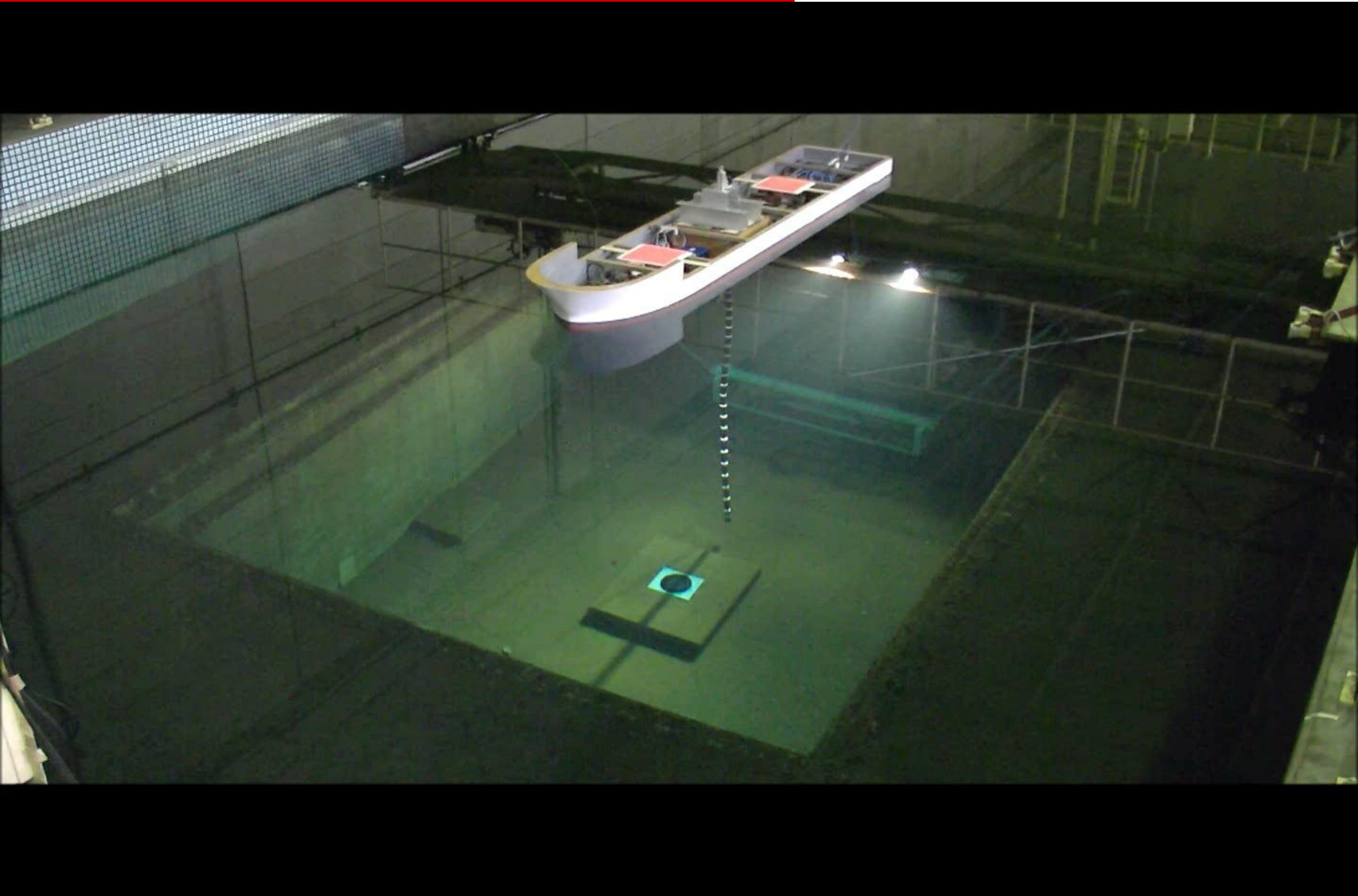


Exp#4 (LQI Control)

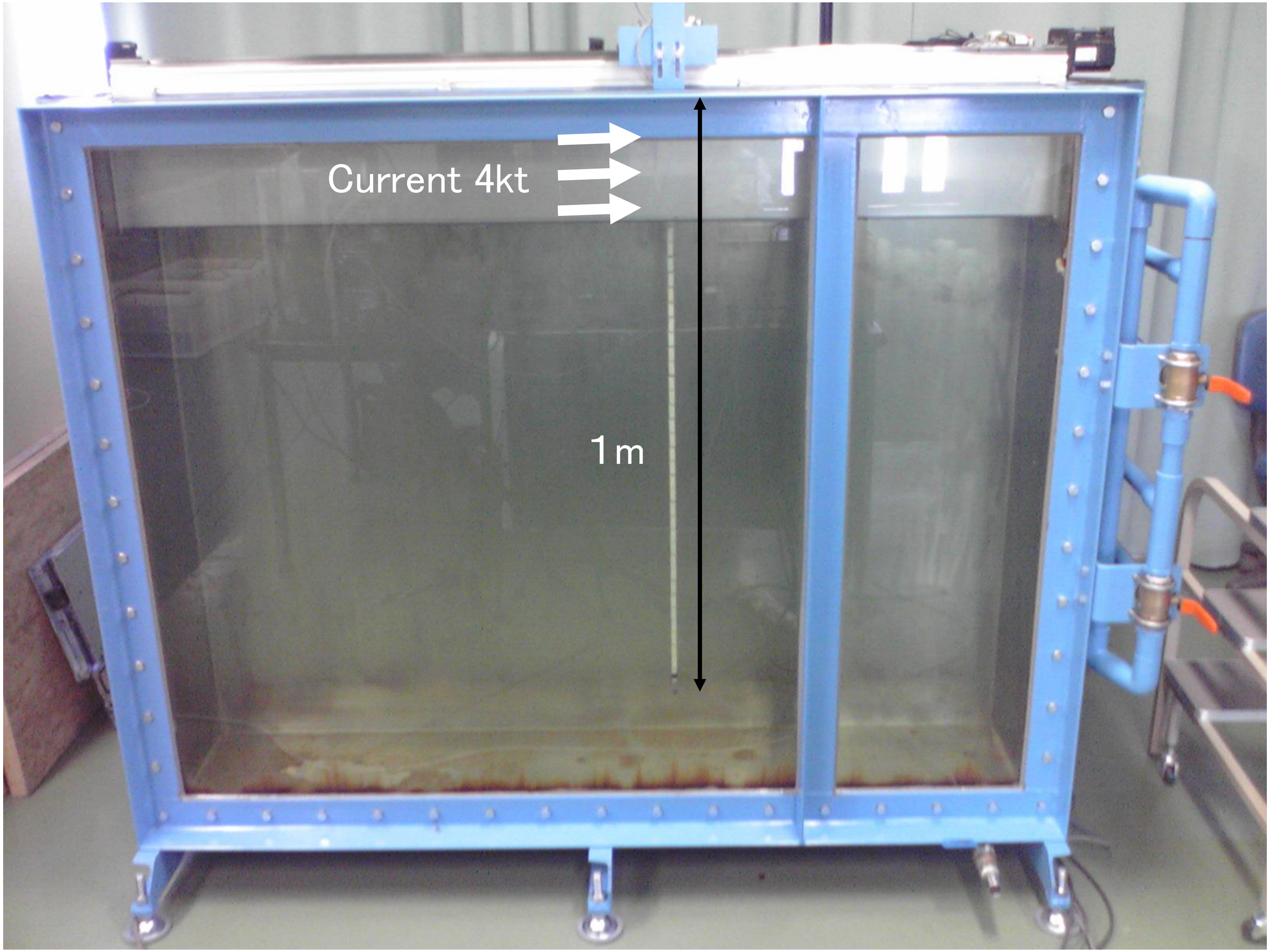


Exp#5 (LQI Control with Current)

Exp#6 (Overview)



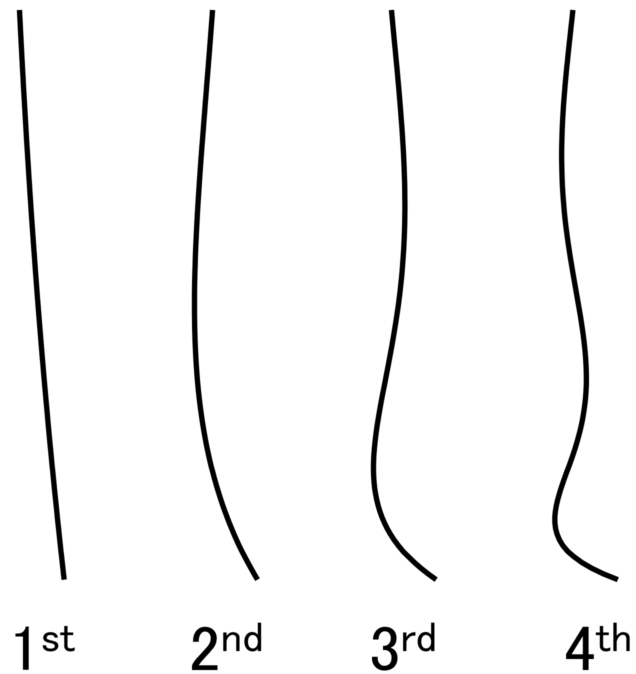
Experimental Set Up (1m)



Riser Pipe Unit Model (1m)

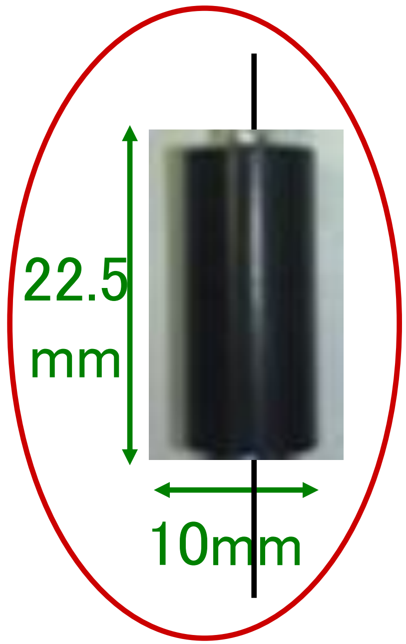
CHIKYU: 210 m length
Assumed Vessel Model: 0.2 m length
Scaling factor : 1/1000

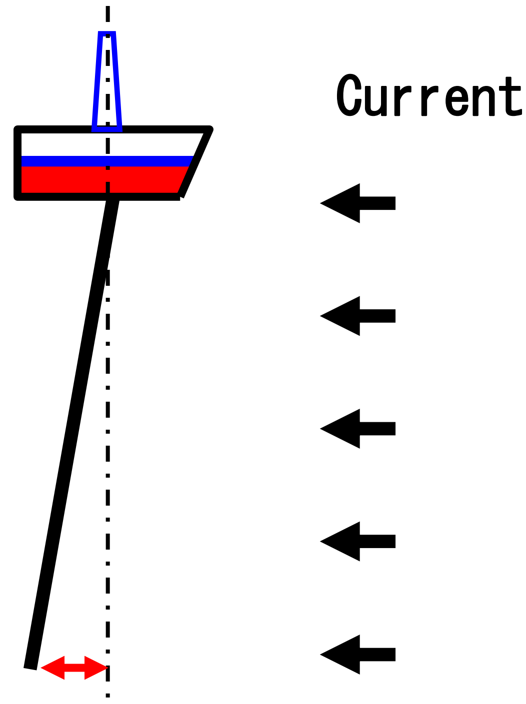
Riser: 2500 m length
Riser Model: 1 m length
Scaling factor : 1/2500



$1/\sqrt{1000}$ period

Dynamic Similarity

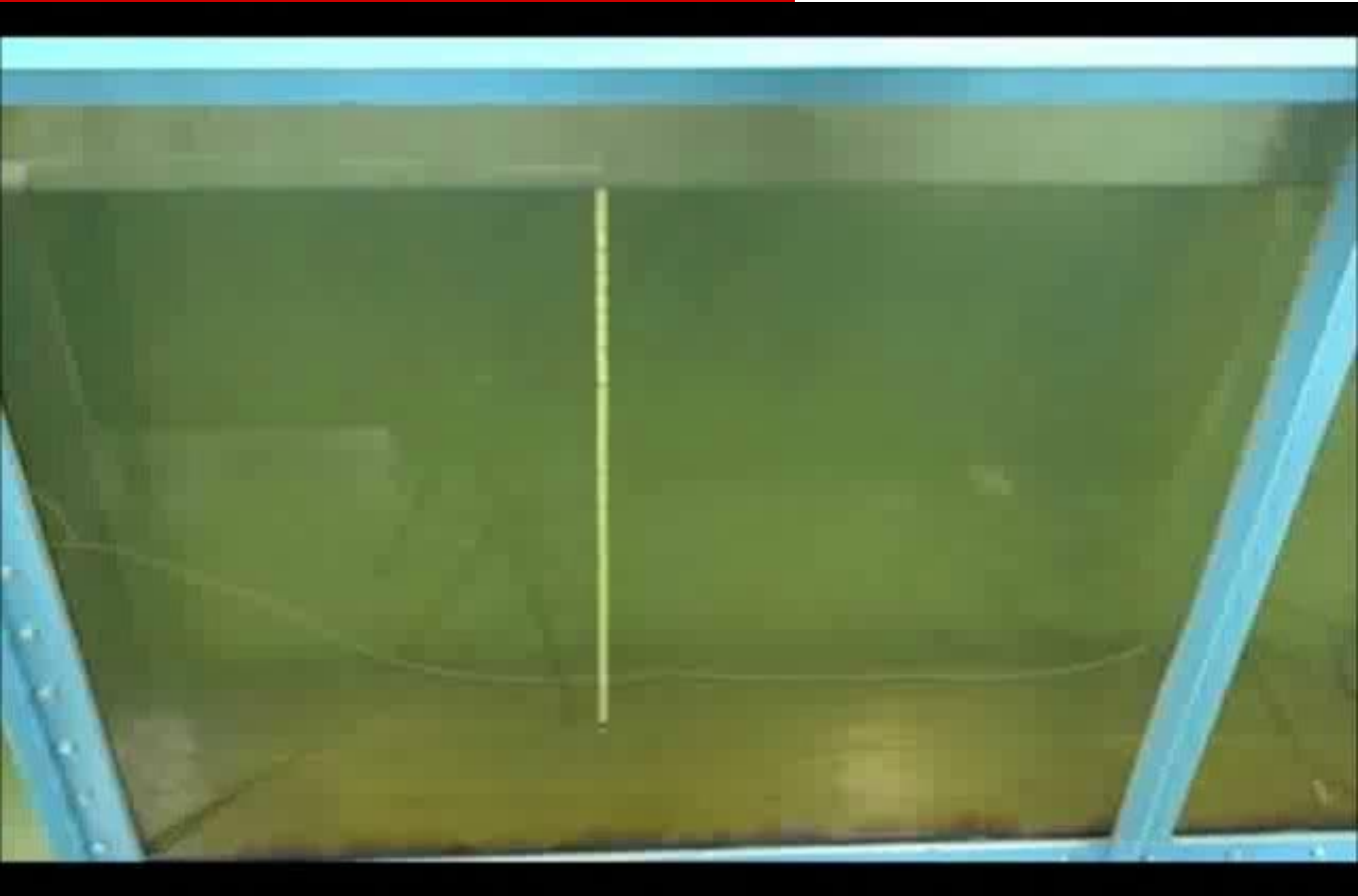




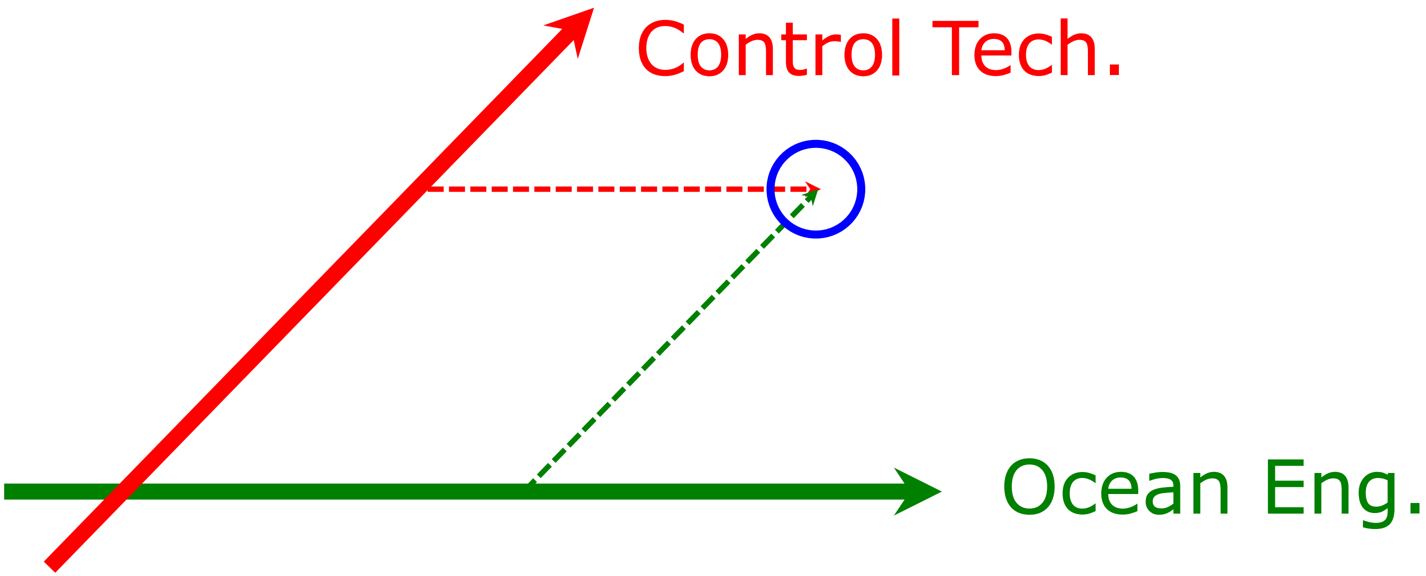
1/1000 deviation

Geometric Similarity

Experiments by 1m riser model



Concluding Remark



Having control technologies as the second axis will expand your engineering ability. Thank you for your attention.

Outline

1 **LQI Control**

Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 **LPV Control**

Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control

Applications

3 Underwater Vehicle

4 Flexible Riser

5 Azimuth thrusters

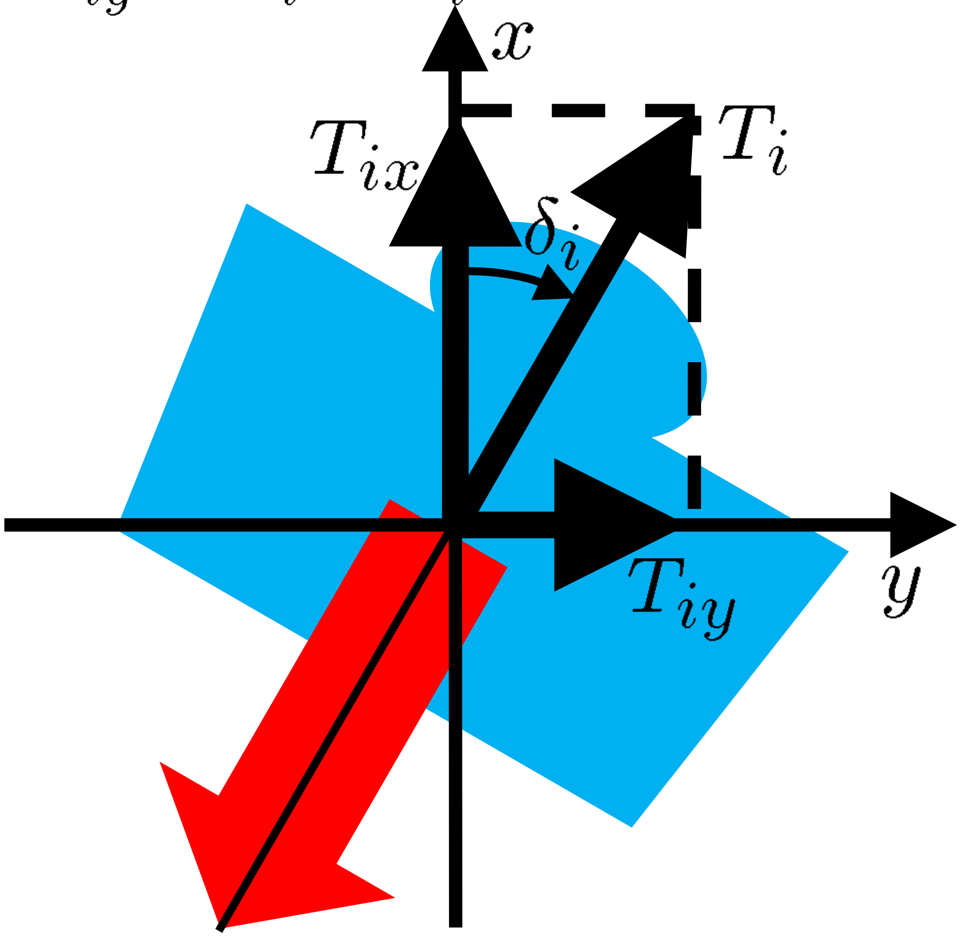
6 Nomoto’s Model

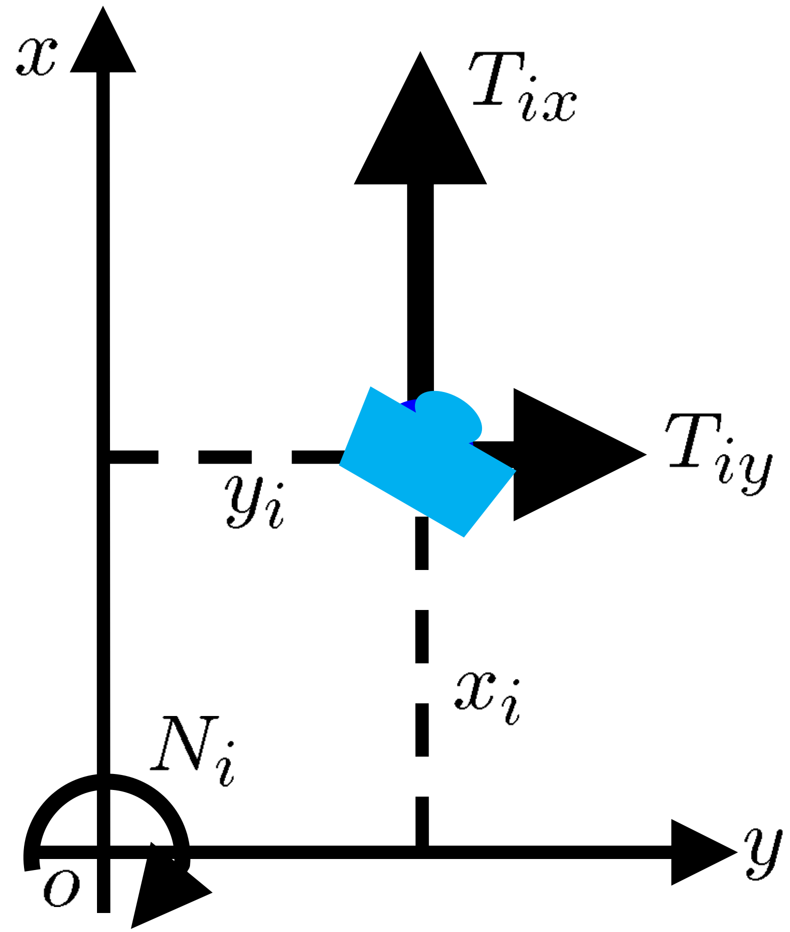
7 Wind Turbine

Thrust Components and Moment

$$T_{ix} = T_i \cos \delta_i$$

$$T_{iy} = T_i \sin \delta_i$$





$$N_{ix} = -\ T_{ix}\ y_i\ +\ T_{iy}\ x_i$$

CA Equation

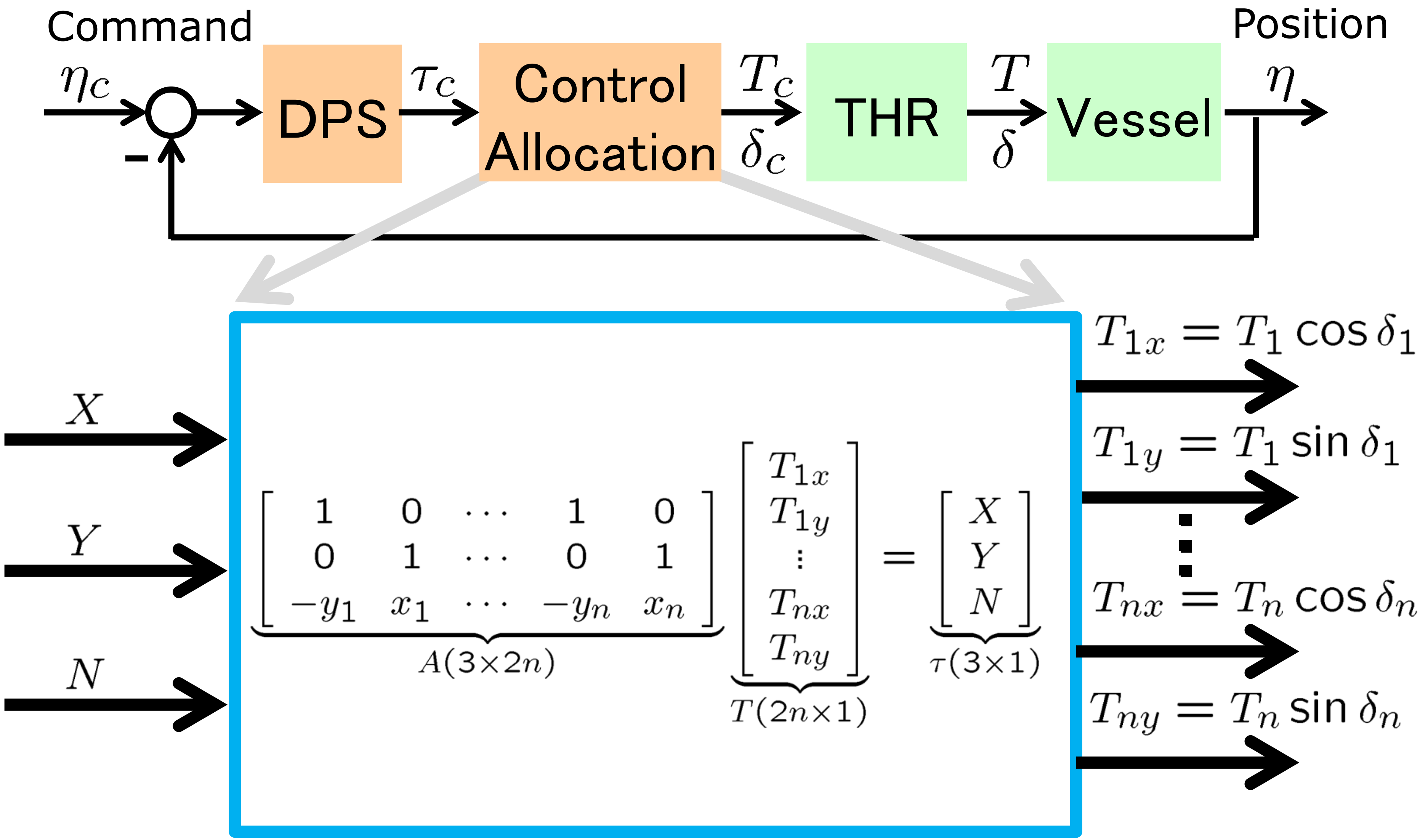
$$T_{1x} + T_{2x} + \cdots + T_{nx} = \sum_{i=1}^{n} T_{ix} = X$$

$$T_{1y} + T_{2y} + \cdots + T_{ny} = \sum_{i=1}^{n} T_{iy} = Y$$

$$N_1 + N_2 + \cdots + N_n = \sum_{i=1}^{n} (-T_{ix}\ y_i + T_{iy}\ x_i) = N$$

$$\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & \cdots & -y_n & x_n \end{bmatrix}}_{A(3\times 2n)} \underbrace{\begin{bmatrix} T_{1x} \\ T_{1y} \\ \vdots \\ T_{nx} \\ T_{ny} \end{bmatrix}}_{T(2n\times 1)} = \underbrace{\begin{bmatrix} X \\ Y \\ N \end{bmatrix}}_{\tau(3\times 1)}$$

CA Problem


Command
η_c
-
DPS
τ_c
Control Allocation
T_c
δ_c
THR
T
δ
Vessel
Position
η
X
Y
N
$\begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & \cdots & -y_n & x_n \end{bmatrix} \begin{bmatrix} T_{1x} \\ T_{1y} \\ \vdots \\ T_{nx} \\ T_{ny} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ N \end{bmatrix}$
$A(3\times 2n)$
$T(2n\times 1)$
$\tau(3\times 1)$
$T_{1x} = T_1 \cos\delta_1$
$T_{1y} = T_1 \sin\delta_1$
$T_{nx} = T_n \cos\delta_n$
$T_{ny} = T_n \sin\delta_n$

General Solution of CA Eq.

Singular Value Decomposition of A

$$A = U \underbrace{\begin{bmatrix} \Sigma_1 & 0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} V_1 & V_2 \end{bmatrix}^T}_{V^T}$$

General Solution of $AT = \tau$

$$T = V_1 \Sigma_1^{-1} U^T \tau + V_2 c$$

c → arbitrary (2*n*-3)-vector

Norm of T

$$\|T\|^2 = \|\Sigma_1^{-1} U^T \tau\|^2 + \|c\|^2$$

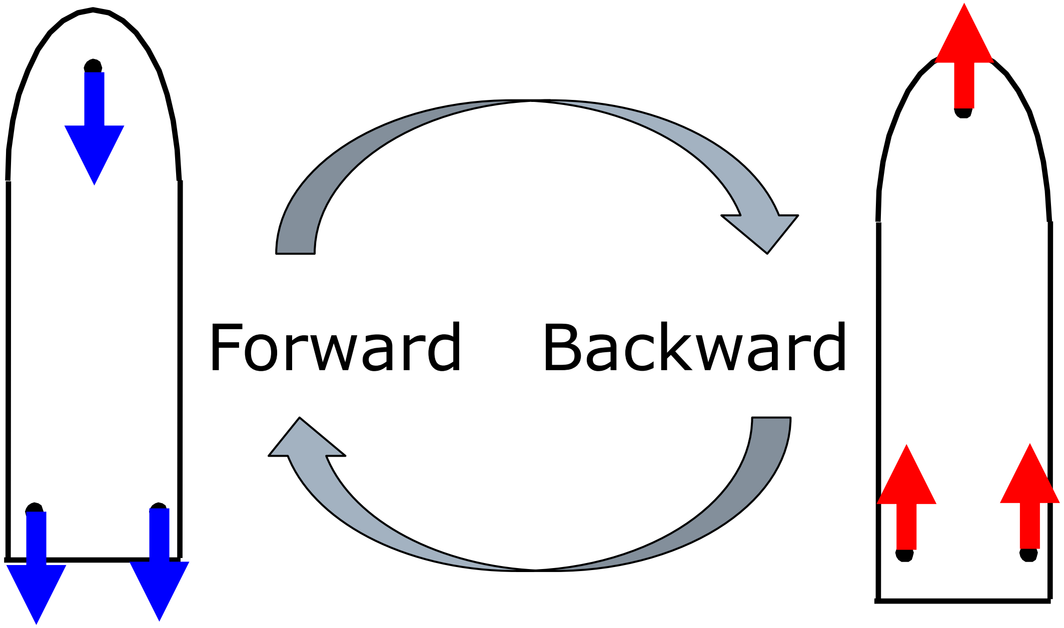
Minimization of $\|T\|$ $(c = 0)$

$$T^* = V_1 \Sigma_1^{-1} U^T \tau$$

Conventional Method

By norm minimization, each THR is apt to play the same role with the same thrust & direction.

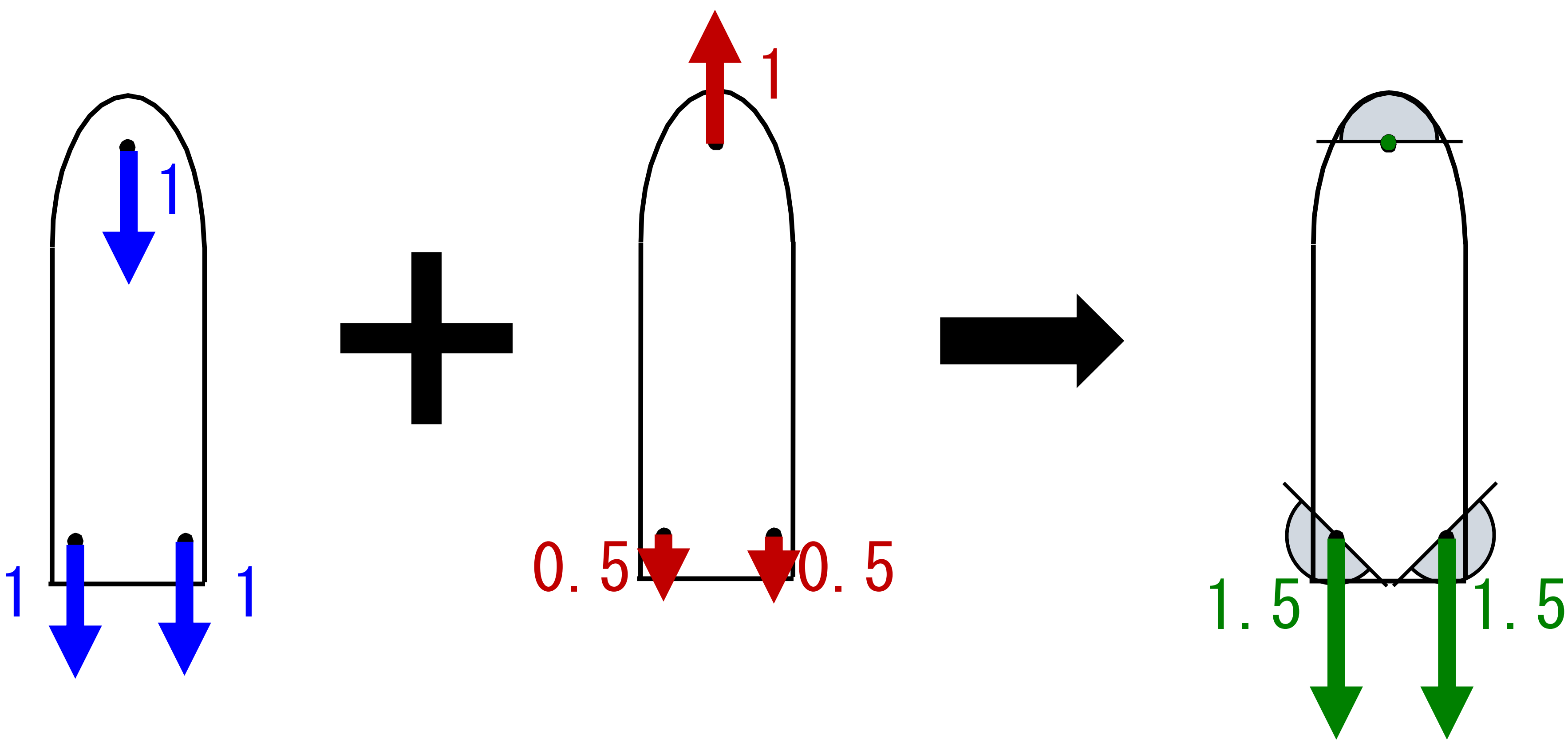
Therefore for the small sign change under the weak disturbance, each THR must always rotate for the forward and backward commands.



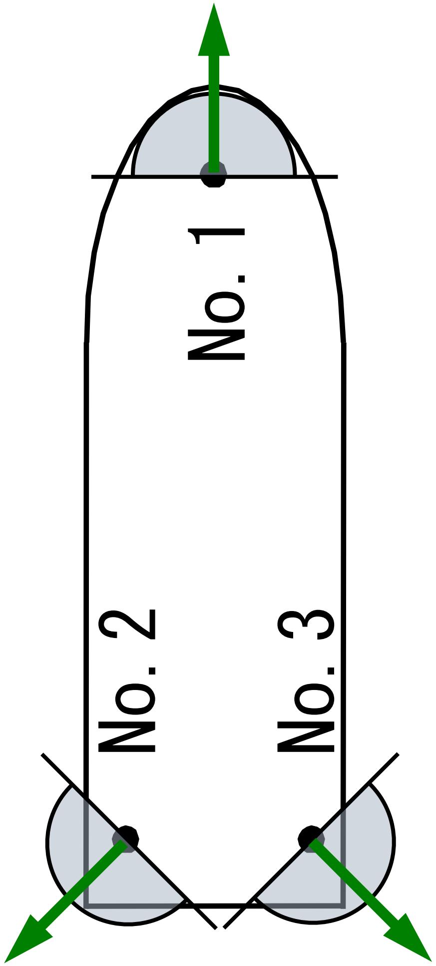
CA with Rotated Angle Constraints 5

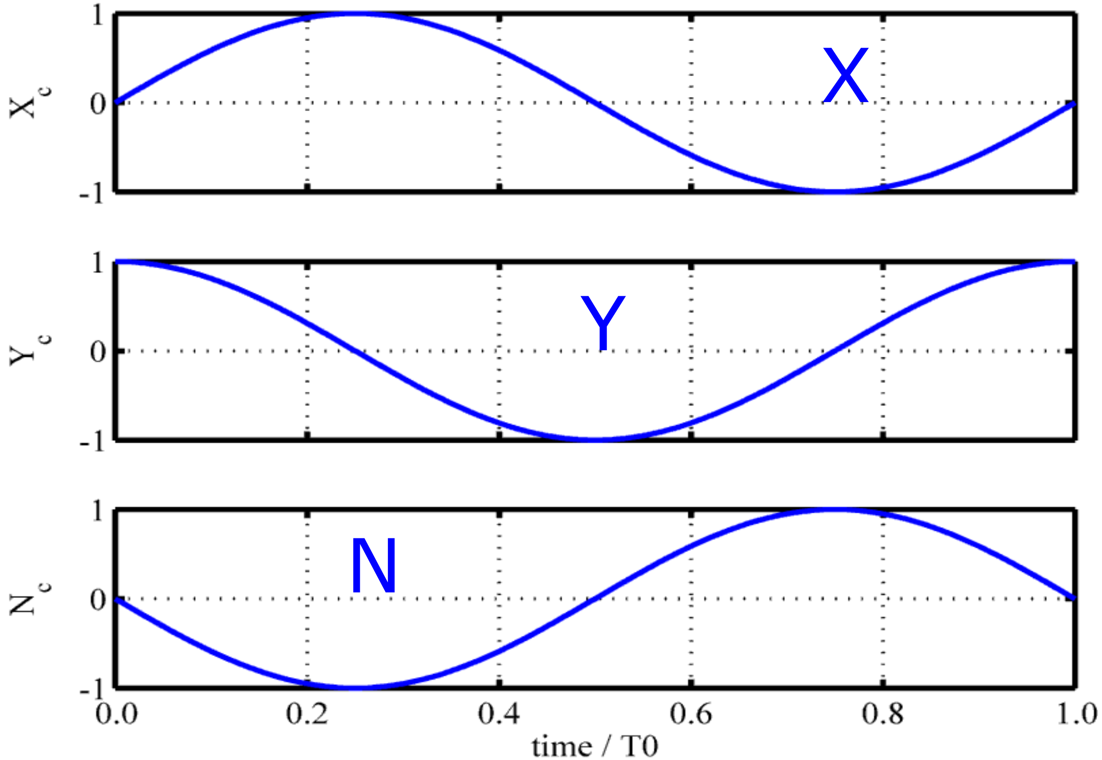
Conventional Compensation Proposal

$$V_1 \Sigma_1^{-1} U^T \tau + V_2 c = T^*$$

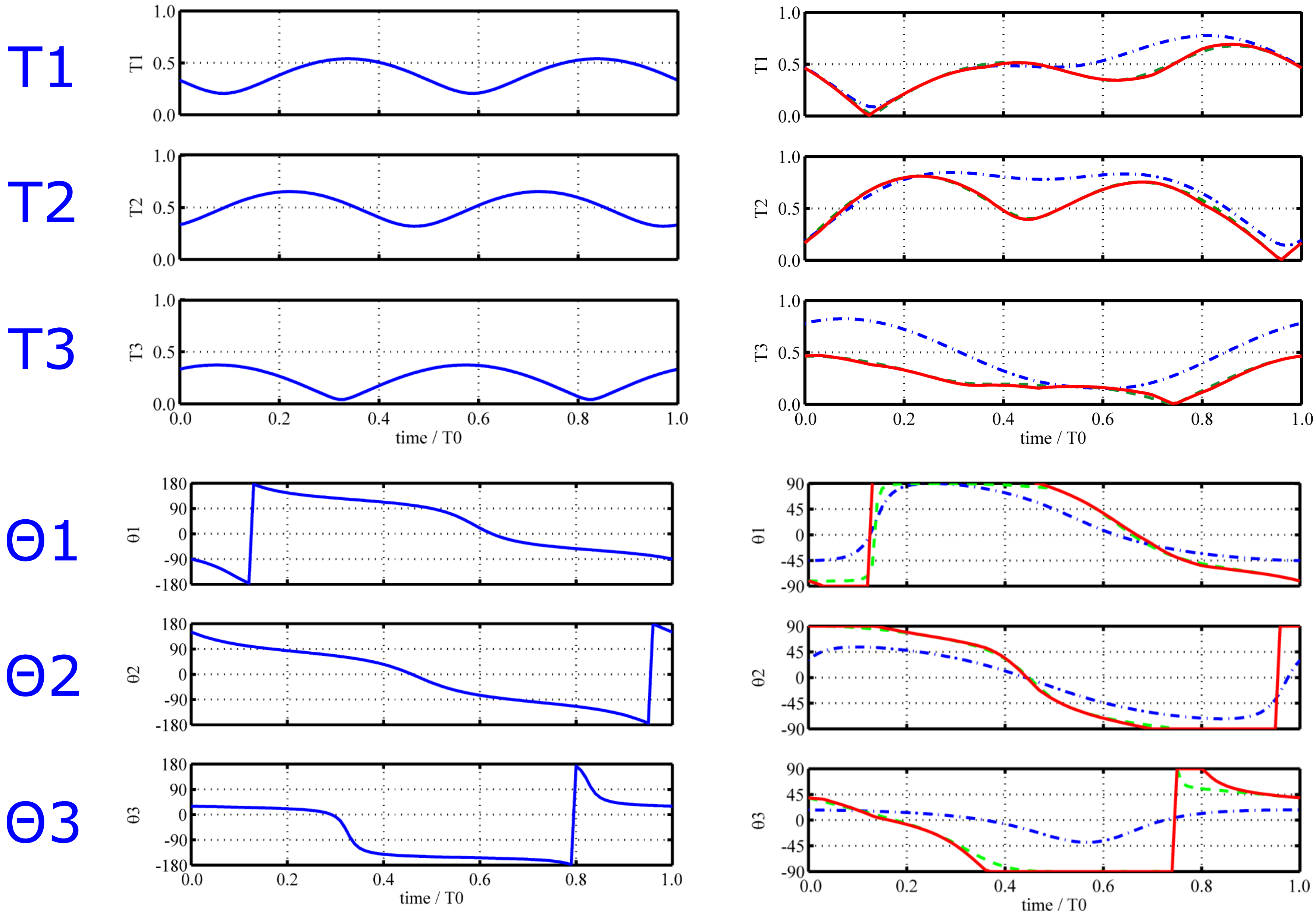


Toy Problem on CA





CA Simulation of Toy Problem



CA Experiment (1)

CA Experiment (2)

Outline

1 LQI Control

Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 LPV Control

Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control

Applications

3 Underwater Vehicle

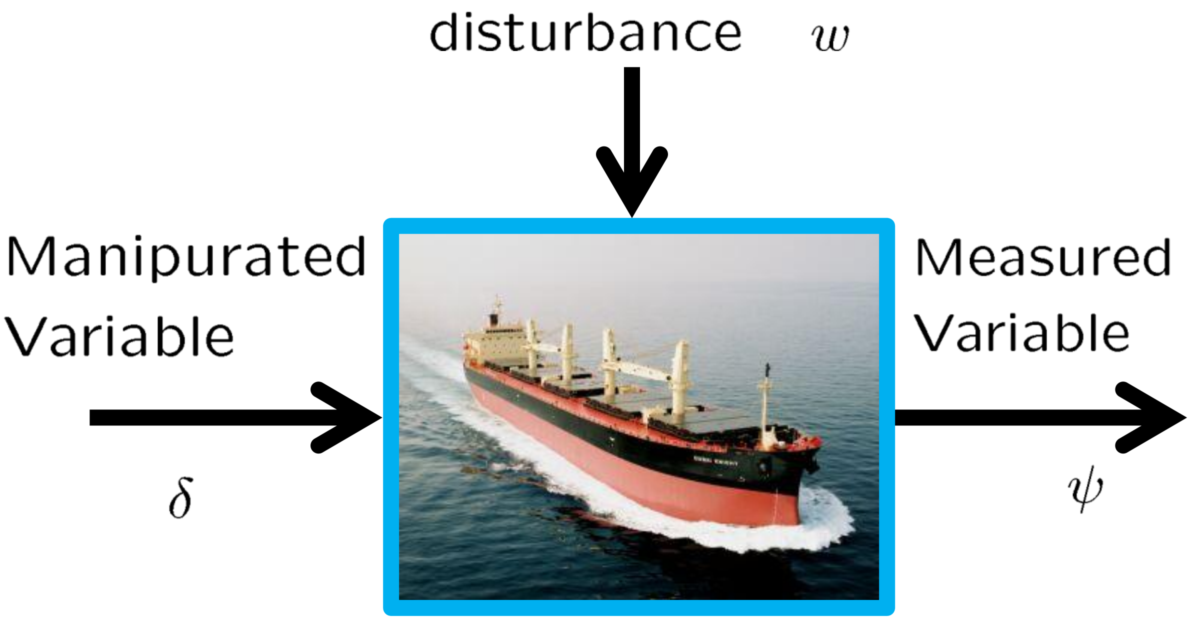
4 Flexible Riser

5 Azimuth thrusters

6 Nomoto’s Model

7 Wind Turbine

MOMOTO Model



Time lag

$$\dot{r}(t) = -\frac{1}{T}r(t) + \frac{K}{T}\delta(t - t_L) + w(t)$$

where

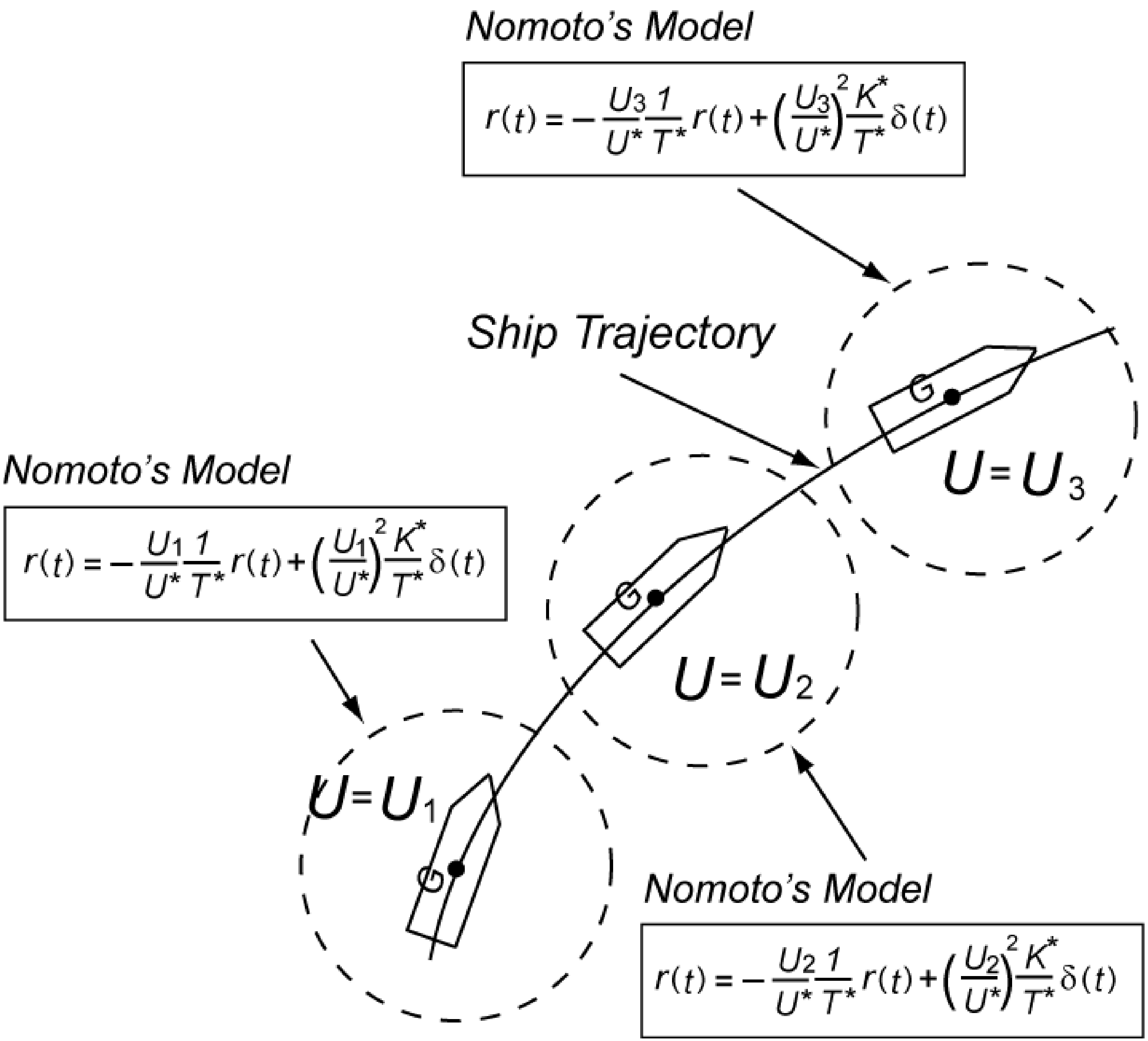
$$T = \frac{L}{U}T', \quad K = \frac{U}{L}K' \quad (U_1 \leq U \leq U_2)$$

Parameter Uncertainty Velocity Variation

Scheduled MOMOTO Model

- Nomoto Model
  $$\dot{r} = -\frac{1}{T}r + \frac{K}{T}\delta$$
  where
  $$T = \frac{L}{U}T', \;\; K = \frac{U}{L}K'$$
- Nominal Speed $U_1 \leq U^* \leq U_2$
  $$T^* = \frac{L}{U^*}T', \;\; K^* = \frac{U^*}{L}K'$$
- Time Constant and Gain Constant
  $$T = \frac{U^*}{U}T^*, \;\; K = \frac{U}{U^*}K^*$$
- Scheduled Nomoto Model
  $$\dot{r} = -\underbrace{\left(\frac{U}{U^*}\right)\frac{1}{T^*}}_{\frac{1}{T(U)}}r + \underbrace{\left(\frac{U}{U^*}\right)^2\frac{K^*}{T^*}}_{\frac{K(U)}{T(U)}}\delta$$

Scheduled MOMOTO Model



Sate Equation

- Motion equation

$$\begin{cases} \dot{\psi} = r \\ \dot{r} = -\left(\frac{U}{U^*}\right)\frac{1}{T^*}r + \left(\frac{U}{U^*}\right)^2\frac{K^*}{T^*}\delta \end{cases}$$

- Rudder Dynamics

$$\dot{\delta} = -\frac{1}{T_a}\delta + \frac{K_a}{T_a}u$$

- State Equation

$$\underbrace{\begin{bmatrix} \dot{\psi} \\ \dot{r} \\ \dot{\delta} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\left(\frac{U}{U^*}\right)\frac{1}{T^*} & \left(\frac{U}{U^*}\right)^2\frac{K^*}{T^*} \\ 0 & 0 & -\frac{1}{T_a} \end{bmatrix}}_{A(U,U^2)} \underbrace{\begin{bmatrix} \psi \\ r \\ \delta \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{K_a}{T_a} \end{bmatrix}}_{B} u$$

- Output Equation

$$\underbrace{\psi}_{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \psi \\ r \\ \delta \end{bmatrix}}_{x}$$

LPV Model with 3 Vertexes

$$\dot{x} = \underbrace{(p_1 A_1 + p_2 A_2 + p_3 A_3)}_{A(U,U^2)} x + Bu$$

where $A_1 = A(U_1, U_1^2), A_2 = A(U_2, U_2^2)$

$A_3 = A(U_3, U_1 U_2)$ with $U_3 = \frac{U_1+U_2}{2}$ and

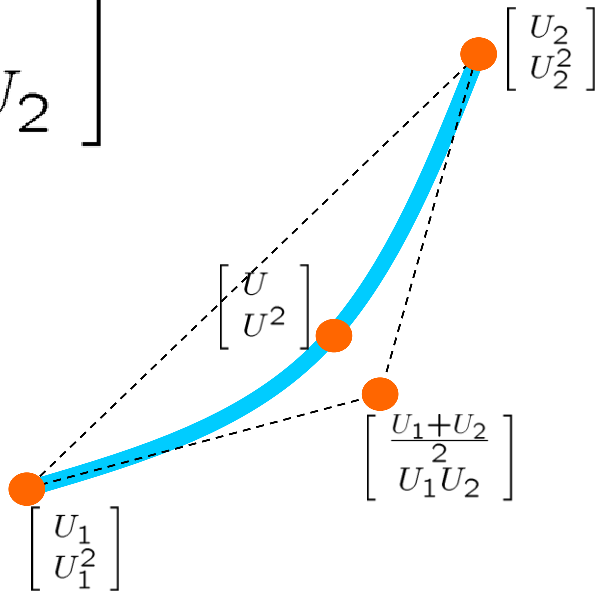
$$p_1 = \frac{1}{p_0} \det \begin{bmatrix} U - U_3 & U_2 - U_3 \\ U^2 - U_1 U_2 & U_2^2 - U_1 U_2 \end{bmatrix}$$

$$p_2 = \frac{1}{p_0} \det \begin{bmatrix} U_1 - U_3 & U - U_3 \\ U_1^2 - U_1 U_2 & U^2 - U_1 U_2 \end{bmatrix}$$

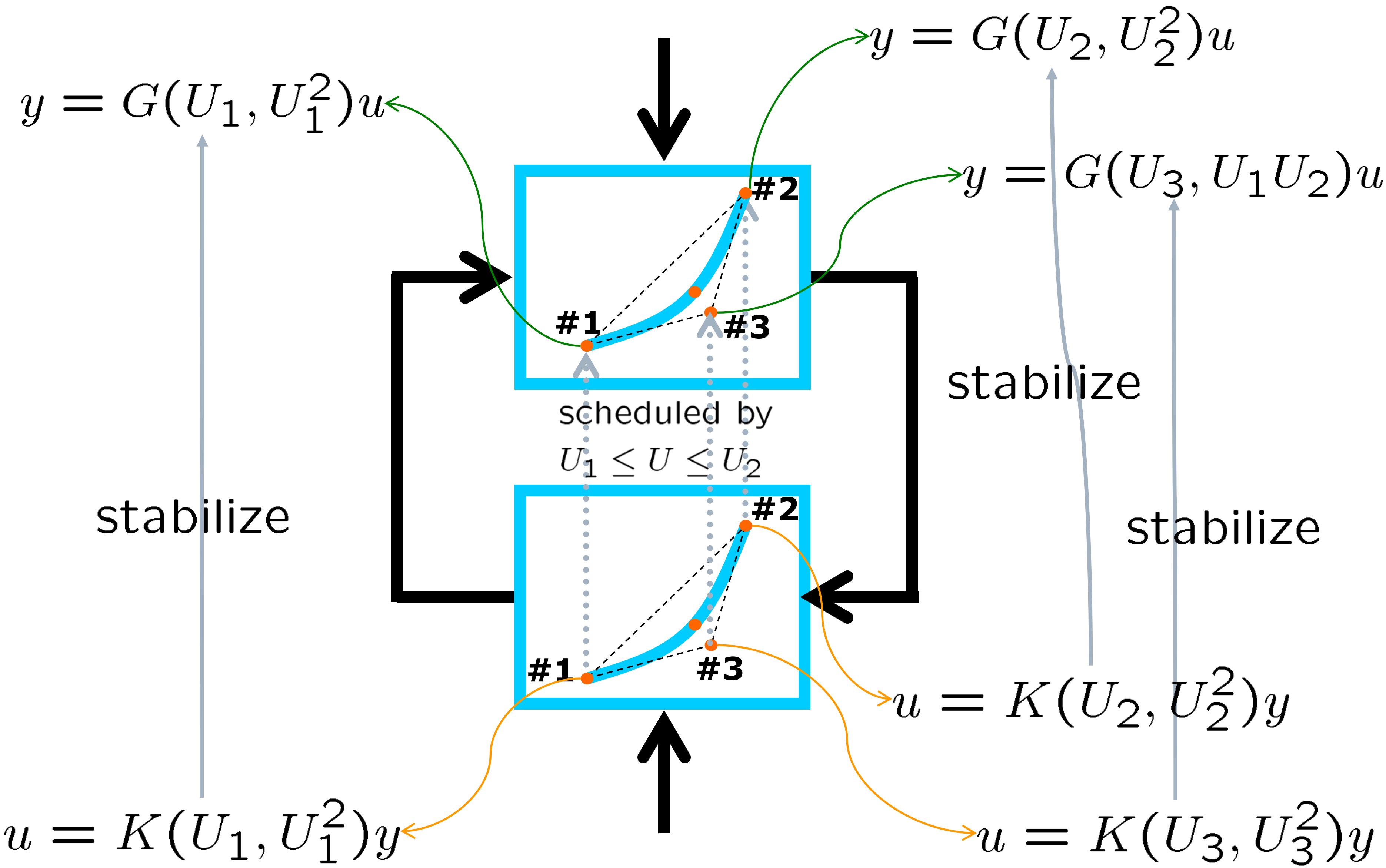
$$p_3 = \frac{1}{p_0} \det \begin{bmatrix} U_1 - U_2 & U_2 - U \\ U_1^2 - U_2^2 & U_2^2 - U^2 \end{bmatrix}$$

$$p_0 = \det \begin{bmatrix} U_1 - U_2 & U_2 - U_3 \\ U_1^2 - U_2^2 & U_2^2 - U_1 U_2 \end{bmatrix}$$

satisfying $p_1 + p_2 + p_3 = 1$



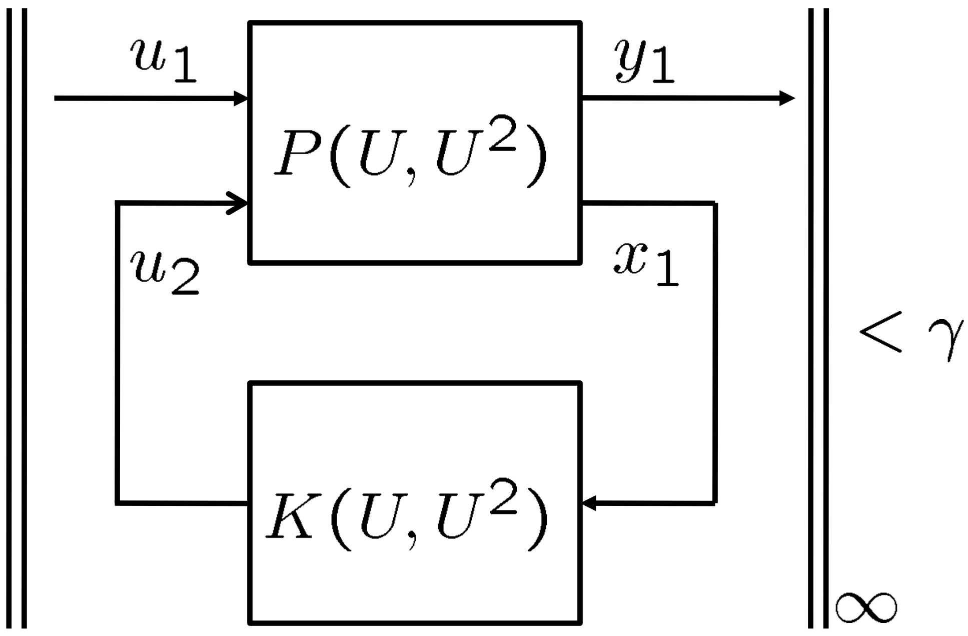
LPV Control System



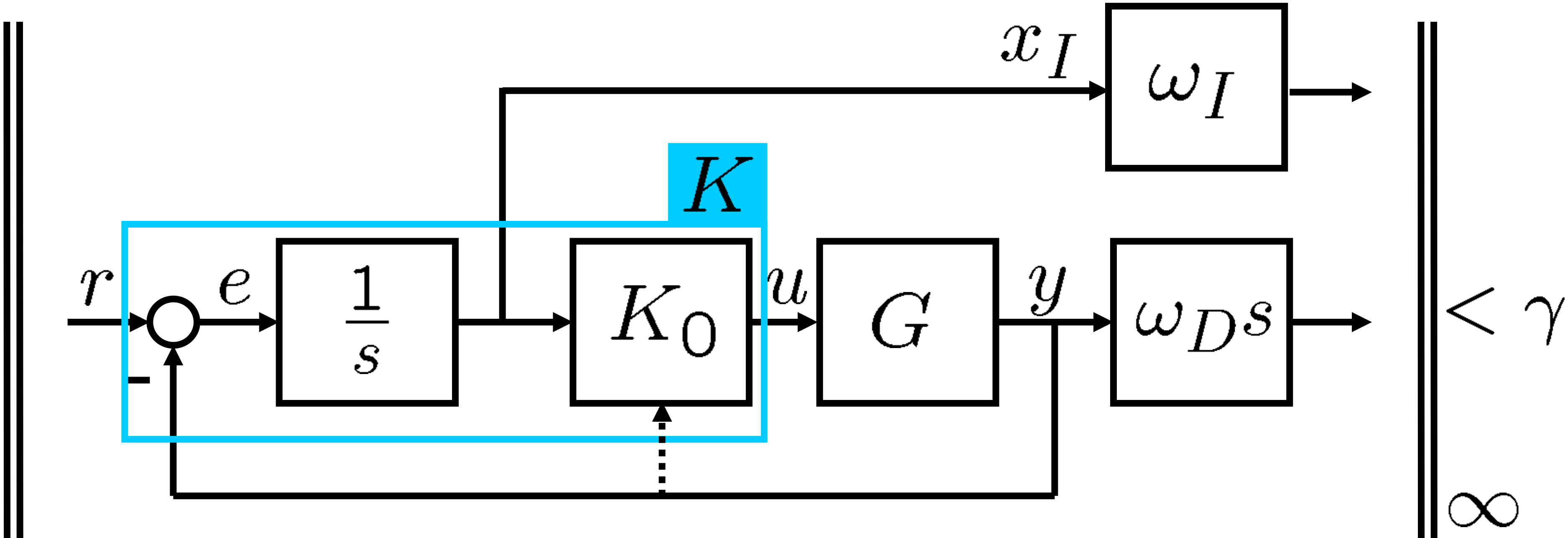
Design Specification

- Spec.#1:
  The closed-loop system is internally stable.

- Spec.#2:
  The L_2-induced gain of the operator is bounded by γ.



Interconnection with Integrator



$$\frac{\omega_I x_I}{r} = \omega_I \frac{\frac{1}{s}}{1+GK_0\frac{1}{s}} = \frac{\overbrace{\omega_I}^{}}{\underbrace{s}_{W_S}} \frac{1}{\underbrace{1+GK}_{S}}$$

$$\frac{\omega_D \dot{y}}{r} = \omega_D s \frac{GK_0\frac{1}{s}}{1+GK_0\frac{1}{s}} = \underbrace{\omega_D s}_{W_T} \frac{GK}{\underbrace{1+GK}_{T}}$$

CLPS by LPV OF

- 2-port representation

$$\left\{\begin{array}{l}
\begin{bmatrix}\dot{x}\\ \dot{x}_I\end{bmatrix} = \underbrace{\begin{bmatrix}A(U,U^2) & 0\\ -C & 0\end{bmatrix}}_{A(U,U^2)}\begin{bmatrix}x\\ x_I\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{B_1} r + \underbrace{\begin{bmatrix}B\\ 0\end{bmatrix}}_{B_2} u\\
\begin{bmatrix}\omega_I x_I\\ \omega_D \dot{y}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & \omega_I\\ \omega_D C A(U,U^2) & 0\end{bmatrix}}_{C_1}\begin{bmatrix}x\\ x_I\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\end{bmatrix}}_{D_{11}} r + \underbrace{\begin{bmatrix}0\\ \omega_D C B\end{bmatrix}}_{D_{12}} u\\
\begin{bmatrix}y\\ x_I\end{bmatrix} = \underbrace{\begin{bmatrix}C & 0\\ 0 & 1\end{bmatrix}}_{C_2}\begin{bmatrix}x\\ x_I\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\end{bmatrix}}_{D_{21}} r
\end{array}\right.$$

$$\begin{aligned}
A(U,U^2) &= p_1(U,U^2)A_1 + p_2(U,U^2)A_2 + p_3(U,U^2)A_3\\
A_K(U,U^2) &= p_1(U,U^2)A_{K1} + p_2(U,U^2)A_{K2} + p_3(U,U^2)A_{K3}\\
B_K(U,U^2) &= p_1(U,U^2)B_{K1} + p_2(U,U^2)B_{K2} + p_3(U,U^2)B_{K3}\\
C_K(U,U^2) &= p_1(U,U^2)C_{K1} + p_2(U,U^2)C_{K2} + p_3(U,U^2)C_{K3}\\
D_K(U,U^2) &= p_1(U,U^2)D_{K1} + p_2(U,U^2)D_{K2} + p_3(U,U^2)D_{K3}
\end{aligned}$$

- output feedback

$$\left\{\begin{array}{l}
\begin{bmatrix}\dot{x}_K\\ \dot{x}_I\end{bmatrix} = \underbrace{\begin{bmatrix}A_K & B_{K2}\\ 0 & 0\end{bmatrix}}_{A_K(U,U^2)}\begin{bmatrix}x_K\\ x_I\end{bmatrix} + \underbrace{\begin{bmatrix}B_{K1} & 0\\ -1 & 1\end{bmatrix}}_{B_K(U,U^2)}\begin{bmatrix}y\\ r\end{bmatrix}\\
u = \underbrace{\begin{bmatrix}C_K & D_{K2}\end{bmatrix}}_{C_K(U,U^2)}\begin{bmatrix}x_K\\ x_I\end{bmatrix} + \underbrace{\begin{bmatrix}D_{K1} & 0\end{bmatrix}}_{D_K(U,U^2)}\begin{bmatrix}y\\ r\end{bmatrix}
\end{array}\right.$$

LMI Based Design of LPV OF

- Minimize γ
  on $R = R^T, S = S^T, \mathcal{A}_{Ki}, \mathcal{B}_{Ki}, \mathcal{C}_{Ki}, D_{Ki}$ $(i = 1, 2, 3)$
  subject to $\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0$ and
  LMI-OF1,2,3,4 for vertex1
  LMI-OF1,2,3,4 for vertex2
  LMI-OF1,2,3,4 for vertex3
- Determine the output feedback controller
  for each vertex A_{Ki}, B_{Ki}, C_{Ki} $(i = 1, 2, 3)$

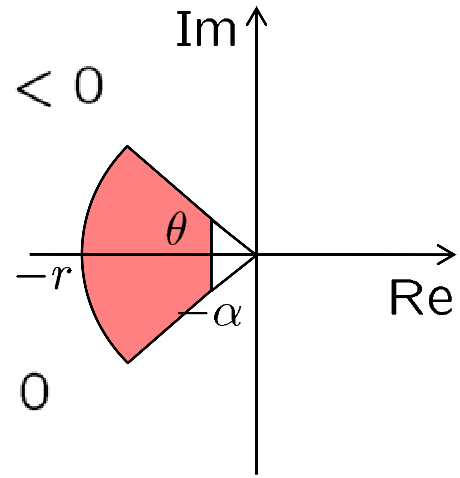
$$
\begin{aligned}
A_{Ki} &= N^{-1}(\mathcal{A}_{Ki} - S(A_i - B_2 D_{Ki} C_2)R - \mathcal{B}_{Ki} C_2 R \\
&\quad - S B_2 \mathcal{C}_{Ki}) M^{-T} \\
B_{Ki} &= N^{-1}(\mathcal{B}_{Ki} - S B_2 D_{Ki}) \\
C_{Ki} &= (\mathcal{C}_{Ki} - D_{Ki} C_2 R) M^{-T}
\end{aligned}
$$

where $I - SR = NM^T$

LMIs for OF Design

- **LMI-OF1**:

$$\begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} + (*)^T + \alpha \begin{bmatrix} R & I \\ I & S \end{bmatrix} < 0$$



- **LMI-OF2**:

$$\begin{bmatrix} -r \begin{bmatrix} R & I \\ I & S \end{bmatrix} & \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} \\ (*)^T & -r \begin{bmatrix} R & I \\ I & S \end{bmatrix} \end{bmatrix} < 0$$

- **LMI-OF3**:

$$\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \otimes \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} + (*)^T < 0$$

- **LMI-OF4**:

$$\begin{bmatrix} \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} + (*)^T & \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ SB_1 + \mathcal{B}_K D_{21} \end{bmatrix} & (*)^T \\ (*)^T & -\gamma^2 I & (*)^T \\ \begin{bmatrix} C_1 R + D_{12}\mathcal{C}_K & C_1 + D_{12} D_K C_2 \end{bmatrix} & D_{11} & -I \end{bmatrix} < 0$$

Scheduled PID Controller

Consider a PID control presented by

$$\delta = K_P(\psi_c - \psi) - K_D r + K_I \int_0^t (\psi_c - \psi(\tau))\, d\tau$$

Assuming $K_i = 0$ and defining $\omega_n = \sqrt{\frac{KK_p}{T}}$, $\zeta = \frac{1+KK_d}{2\sqrt{KK_pT}}$, the following relation should hold.

$$\underbrace{\frac{1}{T}}_{\text{ship motion}} < \underbrace{\omega_n\sqrt{1-2\zeta^2+\sqrt{4\zeta^4-4\zeta^2+2}}}_{\text{controlled motion}} < \underbrace{\frac{1}{T_\delta}}_{\text{steering motion}}$$

Thus the PID gains are calculated as

$$K_P = \frac{T\omega_n^2}{K},\ K_D = \frac{2T\zeta\omega_n - 1}{K},\ K_I = \frac{\omega_n}{10}K_p$$

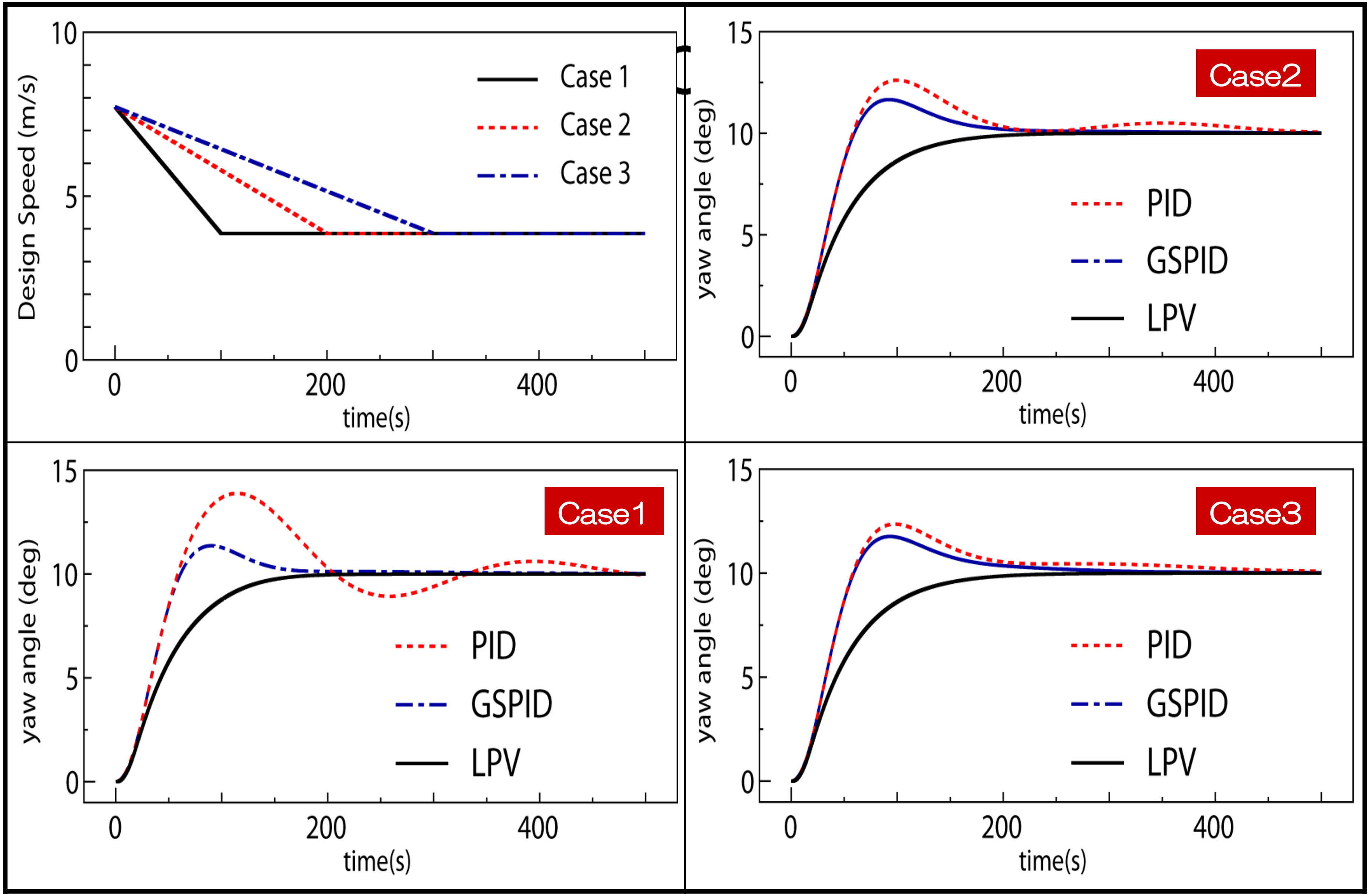
Schedulled PID controller is implemented as

$$K_P(U) = \left(\frac{U^*}{U}\right)^2 \frac{T^*}{K^*}\omega_n^2$$

$$K_D(U) = \left(\frac{U^*}{U}\right)^2 \frac{T^*}{K^*}2\zeta\omega_n - \left(\frac{U^*}{U}\right)\frac{1}{K^*}$$

$$K_I(U) = \left(\frac{U^*}{U}\right)^2 \frac{T^*}{K^*}\frac{\omega_n^3}{10}$$

LPV Control of MONOTO Model



Outline

1 **LQI Control**

Linear-Quadratic-Integral Design of Linear-Time-Invariant Control

2 **LPV Control**

Linear-Matrix-Inequality Based Design of Linear-Parameter-Varying Control

Applications

3 Underwater Vehicle

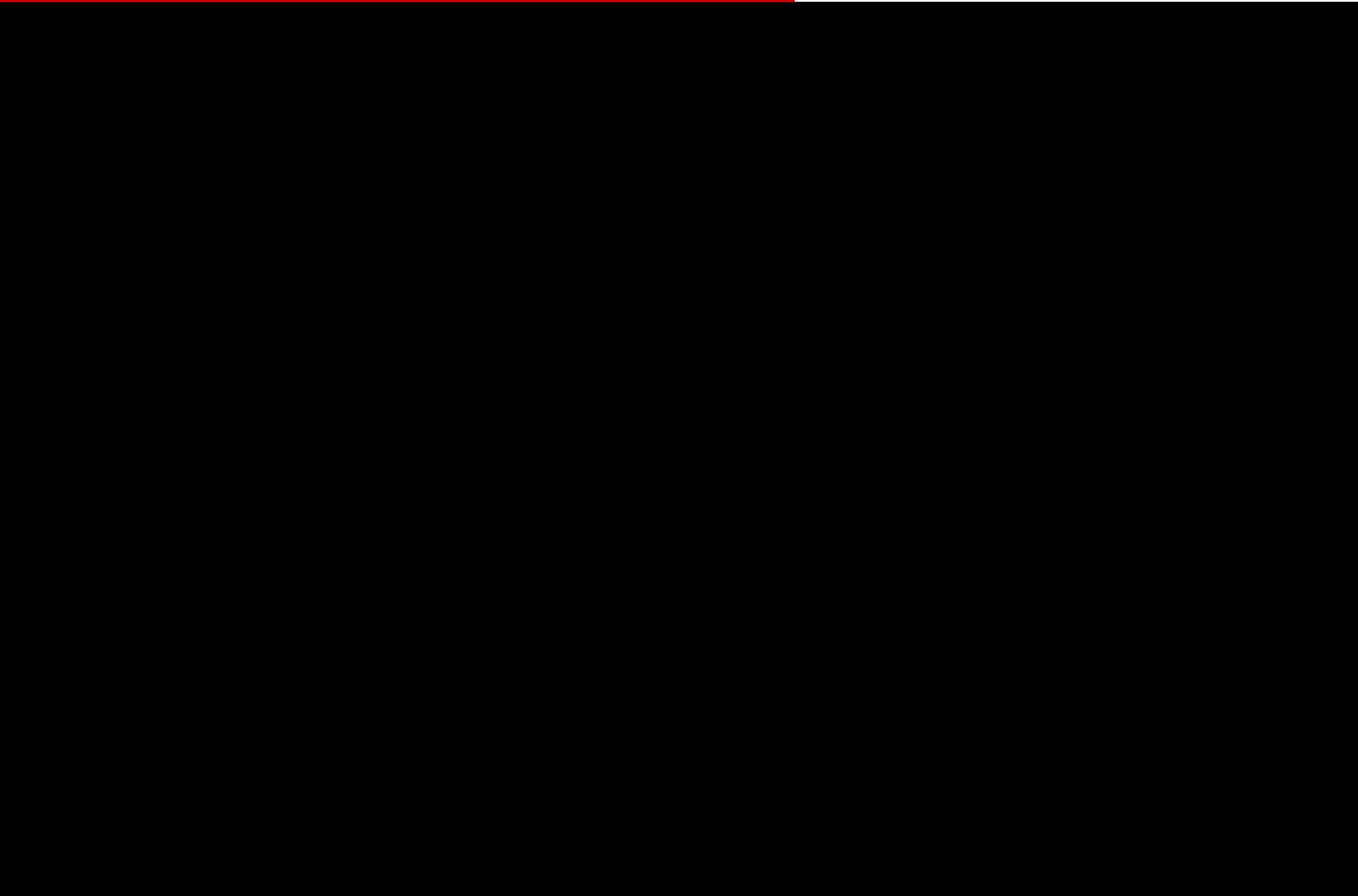
4 Flexible Riser

5 Azimuth thrusters

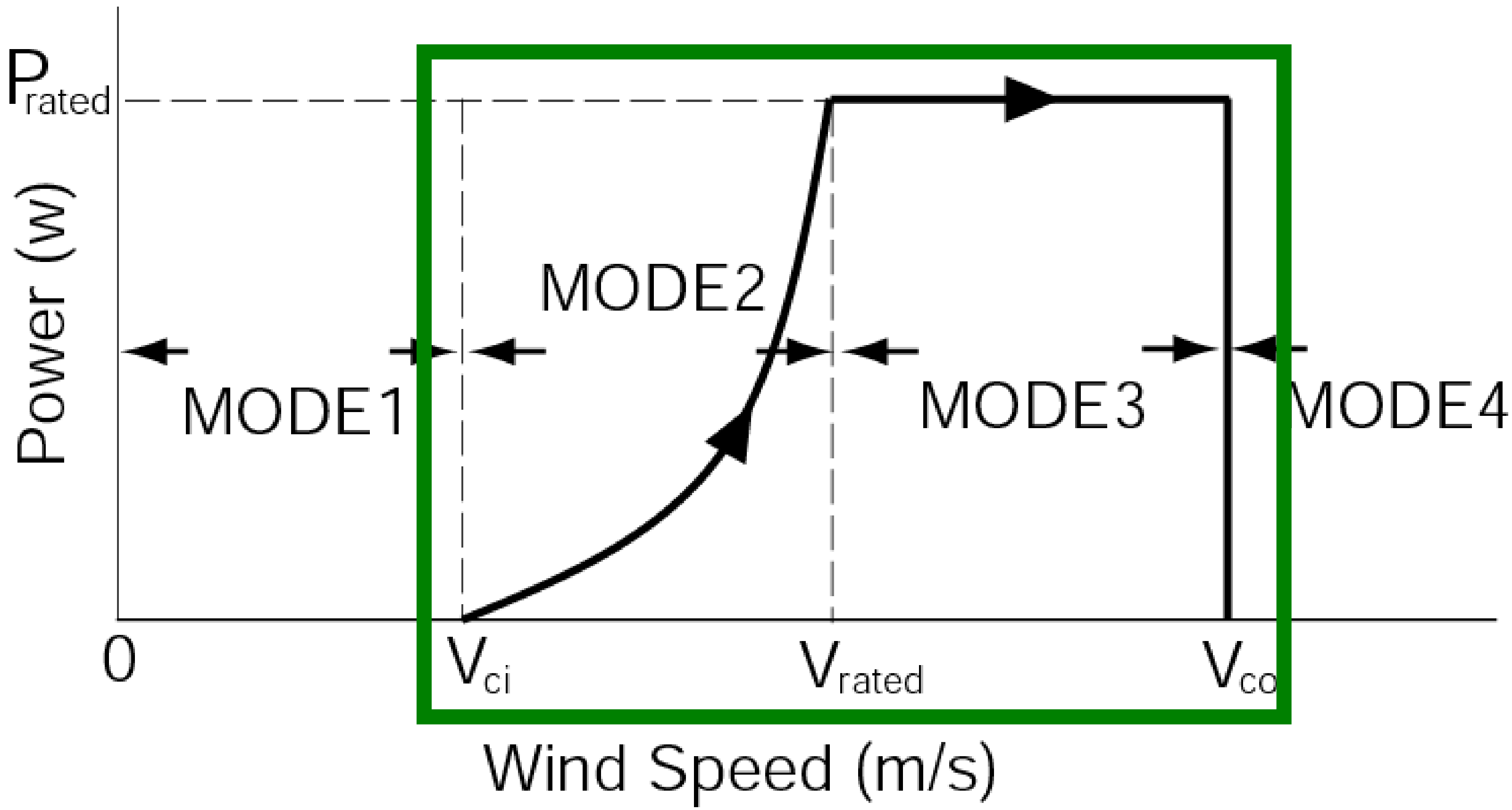
6 Nomoto's Model

7 Wind Turbine

HYWIND



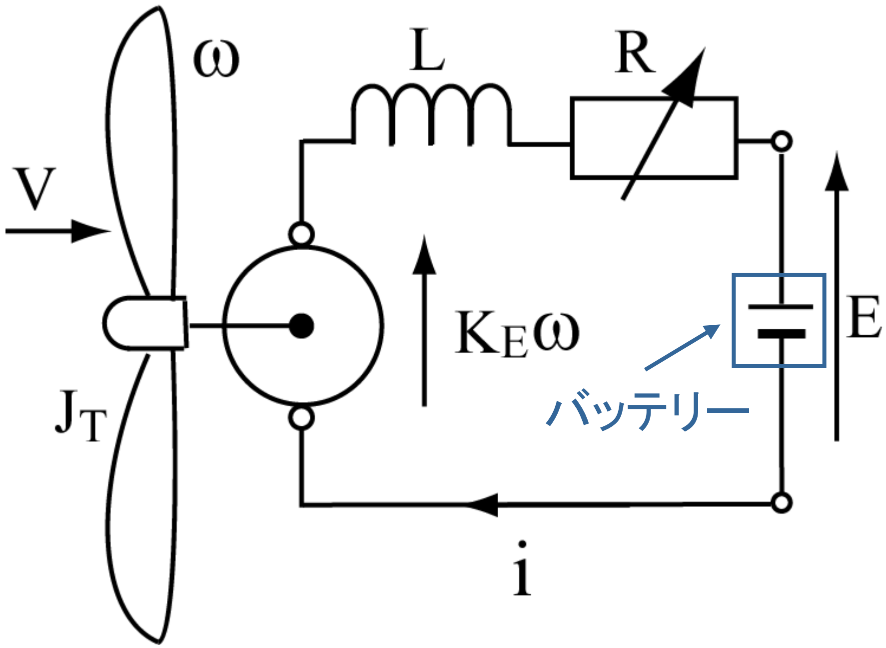
風力発電機の運転モード



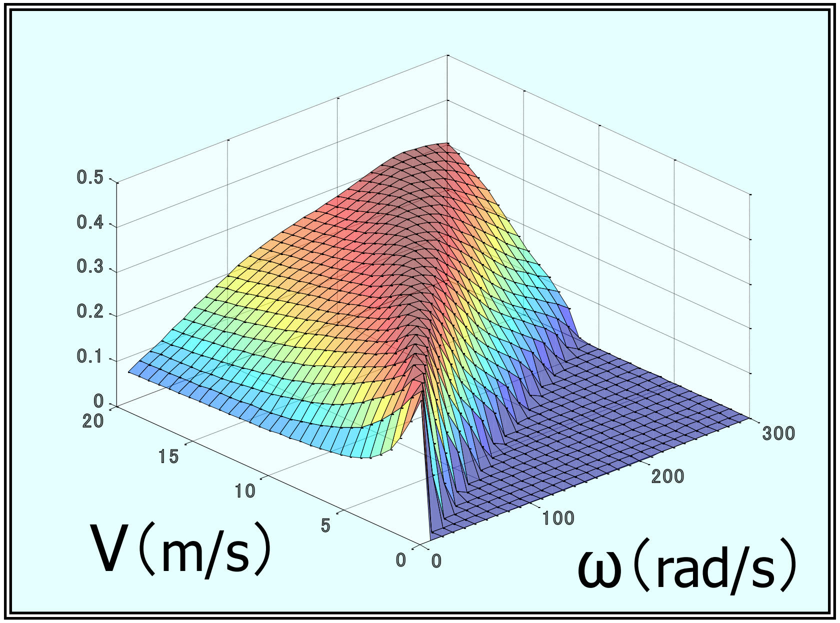
V_{ci} : カットイン風速　　V_{rated}: 定格風速

V_{co} : カットアウト風速　　P_{rated}: 定格出力

風力発電機の数学モデル



風力発電機モデル



風レンズ風車の空力トルク特性

風レンズ風車の数学モデル

$$\begin{cases} J_T\dot{\omega} = \frac{1}{2}\rho A r C_T(\lambda)V^2 - K_T i \\ L\dot{i} + Ri + E = K_E\omega \end{cases} \quad \left(\lambda = \frac{r\omega}{V}\right)$$

可変負荷により任意電流値が実現可能

風力発電機のLPVモデル

$$J_T\dot{\omega} = \boxed{\frac{1}{2}\rho\pi r^3 C_T(\lambda)V^2} - K_T i \qquad (\lambda = \frac{r\omega}{V})$$

$$Q \approx Q^* + \alpha(\omega - \omega^*) + \beta(V - V^*)$$

$$\begin{cases} \alpha = \dfrac{\partial Q}{\partial \omega} = \dfrac{1}{2}\rho r^2 AV\dfrac{\partial C_T}{\partial \lambda} \\ \beta = \dfrac{\partial Q}{\partial V} = \dfrac{1}{2}\rho r AV\left(2C_T - \lambda\dfrac{\partial C_T}{\partial \lambda}\right) \end{cases}$$

$$\frac{d}{dt}(\omega - \omega^*) = \underbrace{\frac{1}{2J_T}\rho r^2 AV\frac{\partial C_T}{\partial \lambda}}_{A(V)}(\omega - \omega^*) - \underbrace{\frac{K_T}{J_T}}_{B}(i - i^*)$$

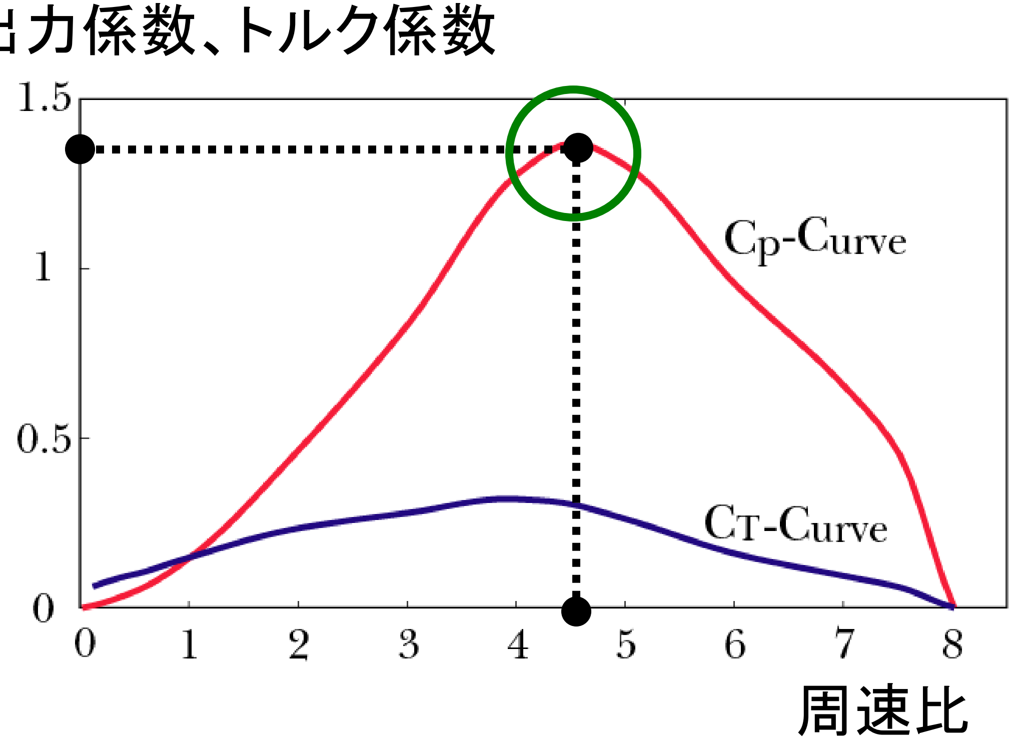
モード２の制御目的

風力エネルギーを最大限、獲得すること

風レンズの周速比が常に**最適周速比**となるように回転数制御

制御目的:

$$\lim_{t \to \infty} \left[\omega - \frac{V}{r} \lambda_{opt} \right] = 0$$



モード２のLPVモデル

$$\frac{d}{dt}(\omega - \omega^*) = \underbrace{\frac{1}{2J_T}\rho r^2 AV \frac{\partial C_T}{\partial \lambda}}_{A(V)}(\omega - \omega^*) - \underbrace{\frac{K_T}{J_T}}_{B}(i - i^*)$$

モード2における風速の変動幅は

$$V_{ci} \leq V \leq V_{rated}$$

次の ポリトピック型LPVモデル を導出することができる

$$\frac{1}{2J_T}\rho r^2 AV \frac{\partial C_T}{\partial \lambda} = p_1\left(\frac{1}{2J_T}\rho r^2 AV_{ci}\frac{\partial C_T}{\partial \lambda}\right) + p_2\left(\frac{1}{2J_T}\rho r^2 AV_{rated}\frac{\partial C_T}{\partial \lambda}\right)$$

ただし、

$$p_1 = \frac{V_{rated} - V}{V_{rated} - V_{ci}}, \ p_2 = \frac{V - V_{ci}}{V_{rated} - V_{ci}} \qquad (p_1 + p_2 = 1)$$

モード３の制御目的

風エネルギーから獲得したパワーを定格出力に抑える

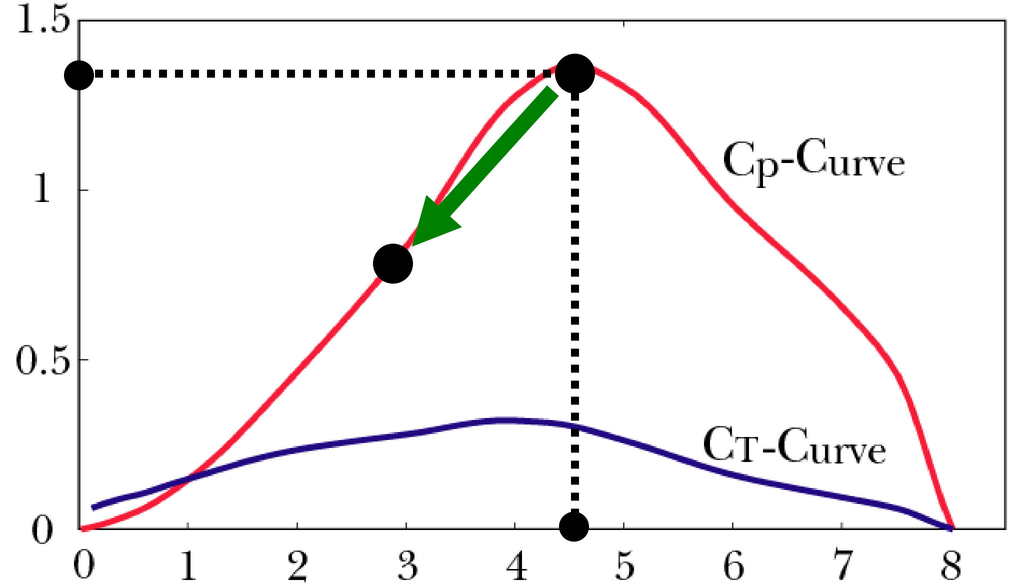
$$P_{rated} = \frac{1}{2}\rho A C_P(\lambda) V^3$$

$$\lambda(V) = C_P^{-1}\left(\frac{2P_{rated}}{\rho A V^3}\right)$$

制御目的:

$$\lim_{t\to\infty}\left[\omega - \frac{V}{r}C_P^{-1}\left(\frac{2P_{rated}}{\rho A V^3}\right)\right] = 0$$

出力係数、トルク係数



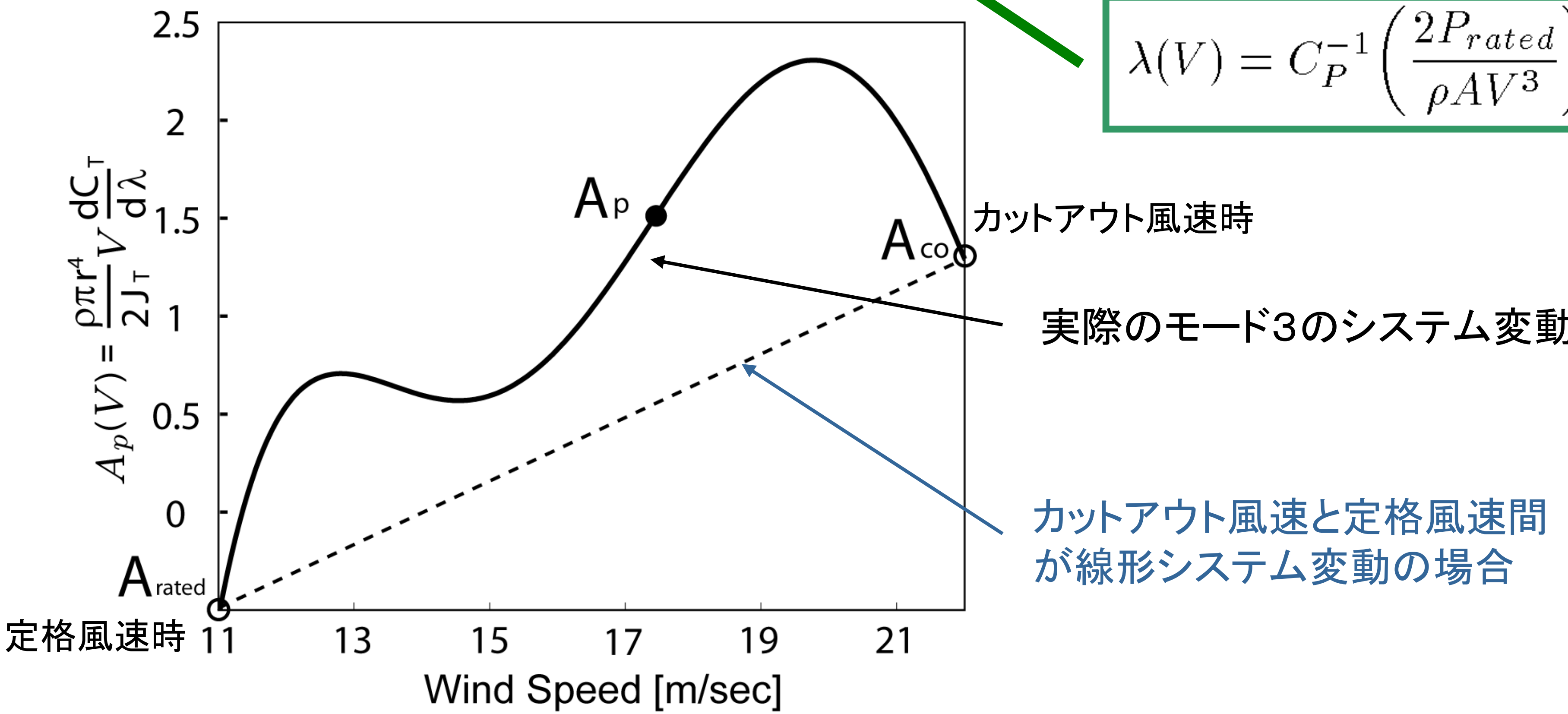
周速比

モード３のLPVモデル

$$\frac{d}{dt}(\omega-\omega^*)=\frac{1}{2J_T}\rho r^2 AV\frac{\partial C_T}{\partial \lambda}(\omega-\omega^*)-\frac{K_T}{J_T}(i-i^*)$$

$$\lambda(V)=C_P^{-1}\left(\frac{2P_{rated}}{\rho AV^3}\right)$$

図6. 風速と係数Aの関係



ポリトピック型LPVモデルを導出することができない

モード３のLPVモデル

LPVモデリング問題を解決する1つのアプローチ

トルク係数曲線を2次関数近似

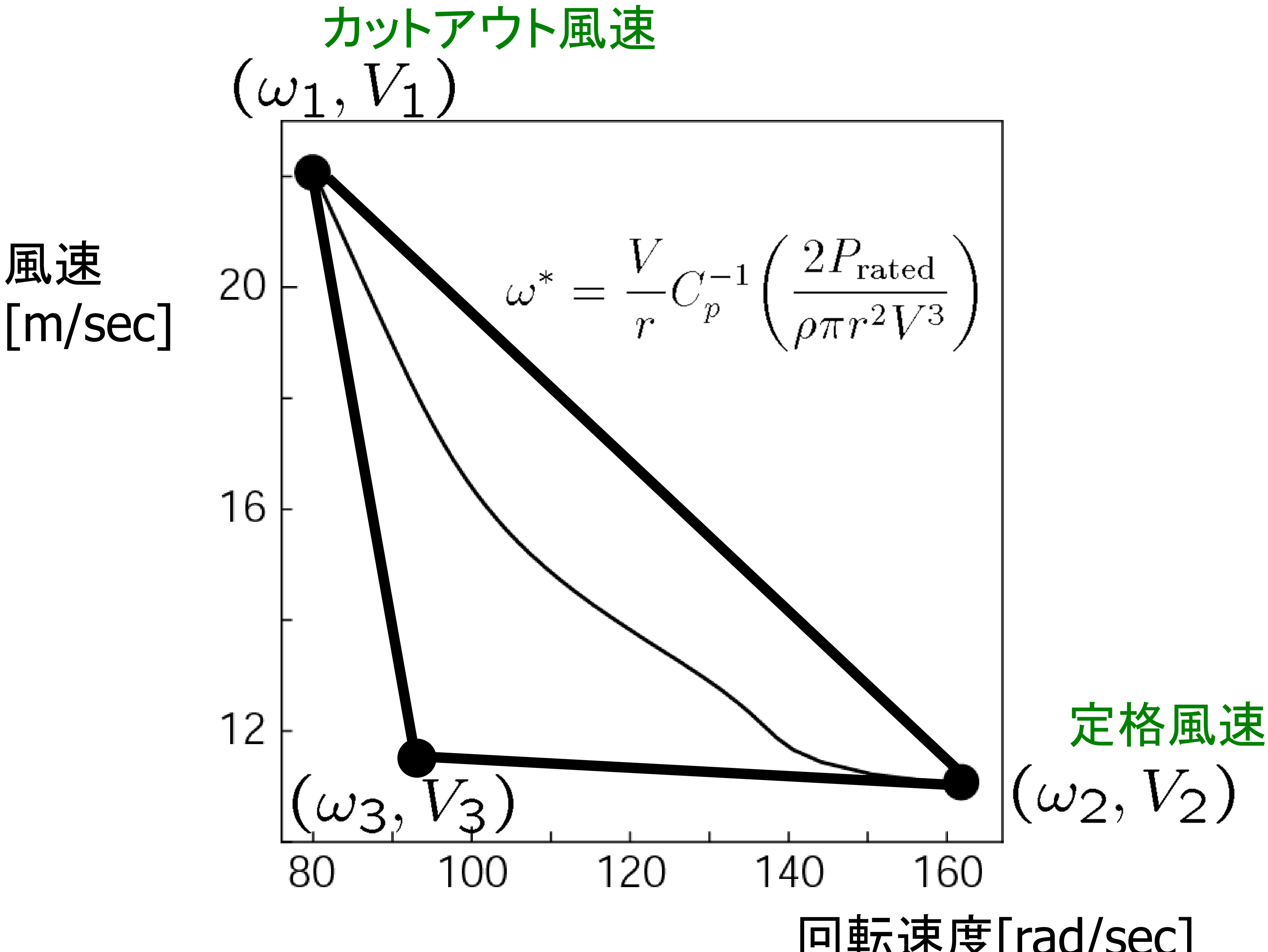
$$C_T(\lambda) \simeq c_2\lambda^2 + c_1\lambda + c_0$$

↓

風速と回転数の2つのパラメータに線形依存した
風力発電機の状態方程式が導出される

$$\frac{d}{dt}(\omega - \omega^*) = \left(\rho\pi r^5 c_2 \omega + \frac{1}{2}\rho\pi r^4 c_1 V\right)(\omega - \omega^*) - \frac{K_T}{J_T}(i - i^*)$$

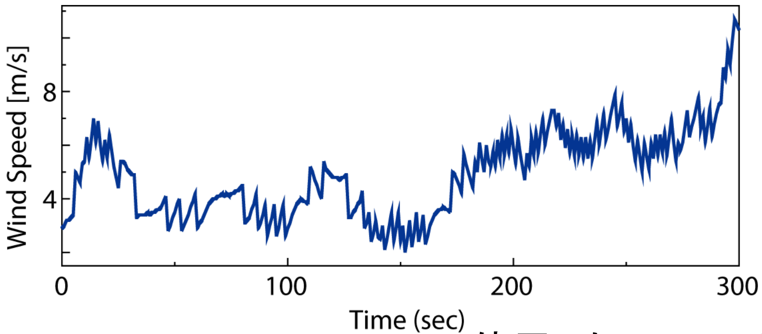
モード３のLPVモデル

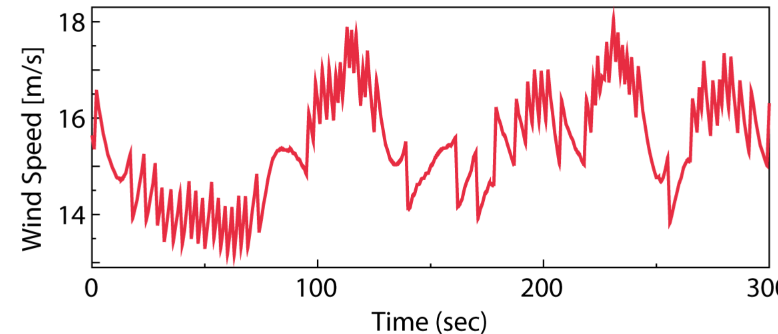


数値シミュレーションによる検討

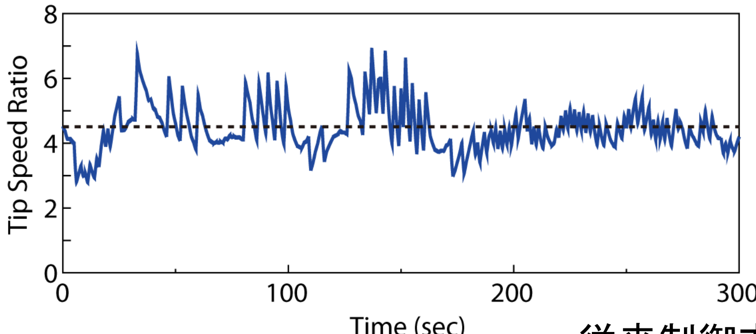
モード2の計算結果

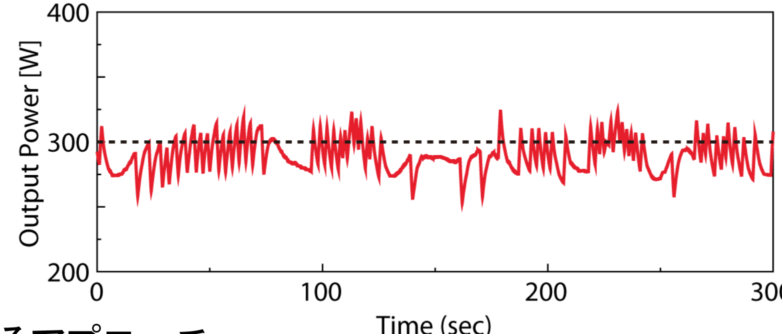
モード3の計算結果



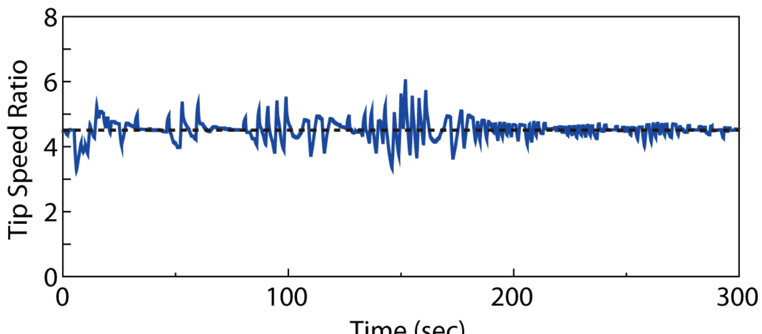


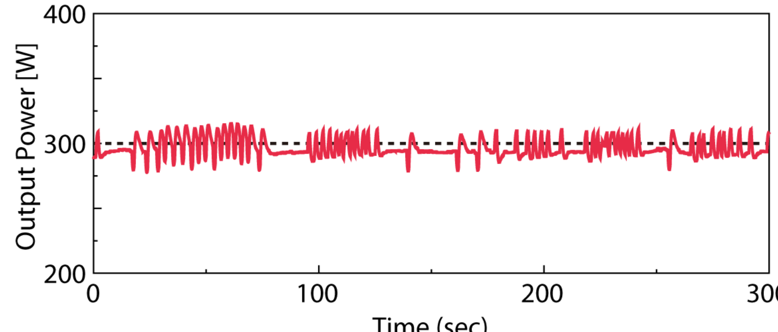
使用したフィールド計測風速データの時系列





従来制御方式によるアプローチ





線形パラメータ変動制御方式