制御屋からみた船舶制御について

第357回　KFRセミナー

日時：2024年3月13日（水）12：30～17：00
場所：大阪公立大学　I-siteなんば　講義室C2＋C3

九州大学名誉教授　梶原 宏之
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自己紹介（制御系CAD、RCPSP法）

７年　九大箱崎時代

1999	2000	2001	2002	2003	2004	2005
九大教授	線形システム制御入門		高氏DR		木村先生赴任	大坪氏DR

１２年　九大伊都時代

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
中尾氏DR	五百木氏DR			人位氏DR		ハッサン氏DR	孟氏DR	段氏DR システム制御工学演習		岩下氏DR	九大退職 木村先生寄付講座教授

５年　長総大時代

2018	2019	2020	2021	2022
長総大特命教授 九大名誉教授				

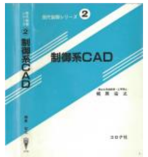
2023	2024

（DPACS）

1979	1980	1981
DPACS開発		

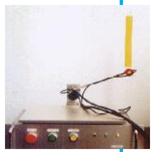
８年　岡大時代（制御系CAD）

1982	1983	1984	1985	1986	1987	1998	1989
岡大講師		岡大助教授	工学博士			制御系CAD	



９年　九工大時代（LPV制御）

1990	1991	1992	1993	1994	1995	1996	1997	1998
九工大助教授		DELTA実験			INRIA訪問（LPV制御）	INRIA訪問（ADIP実験）	ONERA-CERT訪問	



７年　東工大時代

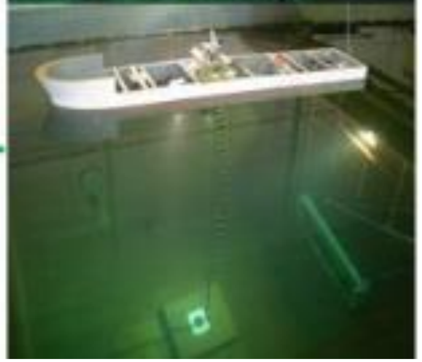
1952		1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974
佐賀生れ									福岡高校入学			九工大入学			

1975	1976	1977	1978
東工大修士		東工大助手	2重倒立振子実験

システム制御技術の研鑽と造船工程計画法の実用化

制御実験

Control Technologies	System Characteristics	Applications
1.1 PID Control	SISO	Nomoto Ship Model (only simulation)
1.2 Sliding Mode Control	Nonlinear	SWATH (experiment)
2.1 LQG Control	MIMO	Underwater Vehicle (experiment)
2.2 Robust Control	Uncertainties	Underwater Vehicle (experiment)
2.3 LPV Control	Gain Scheduling LMI	Underwater Vehicle (experiment)
3.1 Control Allocation	Over-actuated	DPS (experiment)
3.2 Flexible Structure Control	Mode Expansion	Flexible Riser (experiment)



概要

・制御屋からみた船舶制御について

（梶原 宏之 氏：九州大学名誉教授）１６：００～１６：５０

システム制御技術を概観し、これらを船舶制御に応用した事例について紹介する。
①野本モデルの同定（無定位系）②多変数制御への対応（LQI制御）③速度変動への対応（LPV制御）④DPS関連の手法（CA手法、HILS手法）⑤ロバスト性への対応（SM制御）

【1】野本モデルの同定（無定位系）
【2】多変数制御への対応（LQI制御）
【3】速度変動への対応（LPV制御）
【4】DPS関連の手法（CA手法、HILS手法）
【5】ロバスト性への対応（SM制御）

LQI: Linear Quadratic with Integral

LPV: Linear Parameter Varying

CA: Control Allocation

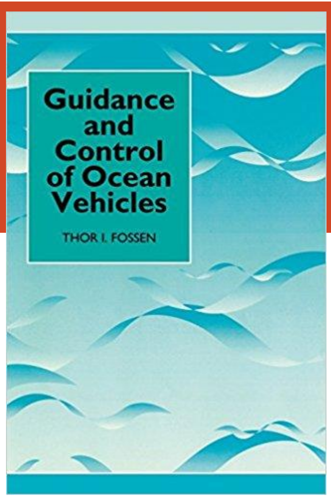
HILS: Hardware In the Loop Simulation

SM: Sliding Mode

【1】野本モデルの同定(無定位系)

- 船舶海洋分野では、1次遅れがなぜ野本モデル？
- Z試験は不要ではないか？
- 無定位系の同定のためには単位FBを適用！

野本謙作先生



Nomoto's 1st-Order Model

A 1st-order approximation is obtained by letting the effective time constant be equal to: $T = T_1 + T_2 - T_3$.

- Time-domain:

$$T\ddot{\psi} + \dot{\psi} = K\delta$$

- Transfer function:

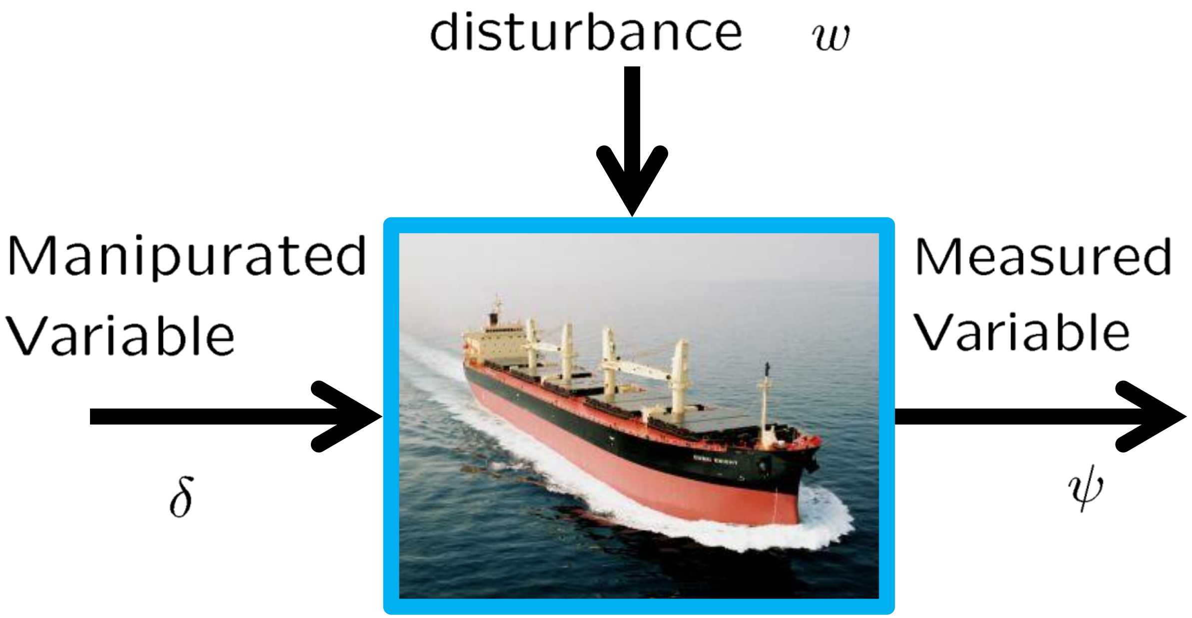
$$\frac{\psi}{\delta}(s) = \frac{K}{s\,(1 + Ts)}$$

The 1st-order Nomoto model should only be used for low frequencies. This is illustrated in the following example where the frequency response of Nomoto's 1st- and 2nd-order models is compared in an amplitude–phase diagram.

Example 5.1 (Nomoto's 1st- and 2nd-Order Models)
In this example we will consider a stable cargo ship and an unstable oil tanker.

	Cargo ship (Mariner class) Chislett and Strøm-Tejsen (1965a)	Oil tanker (full loaded) Dyne and Trägårdh (1975)
L (m)	161	350
u_0 (m/s)	7.7	8.1
∇ (dwt)	16622	389100
K (1/s)	0.185	−0.019
T_1 (s)	118.0	−124.1
T_2 (s)	7.8	16.4
T_3 (s)	18.5	46.0

Nomoto’s Model



Time lag

$$\dot{r}(t) = -\frac{1}{T}r(t) + \frac{K}{T}\delta(t - t_L) + w(t)$$

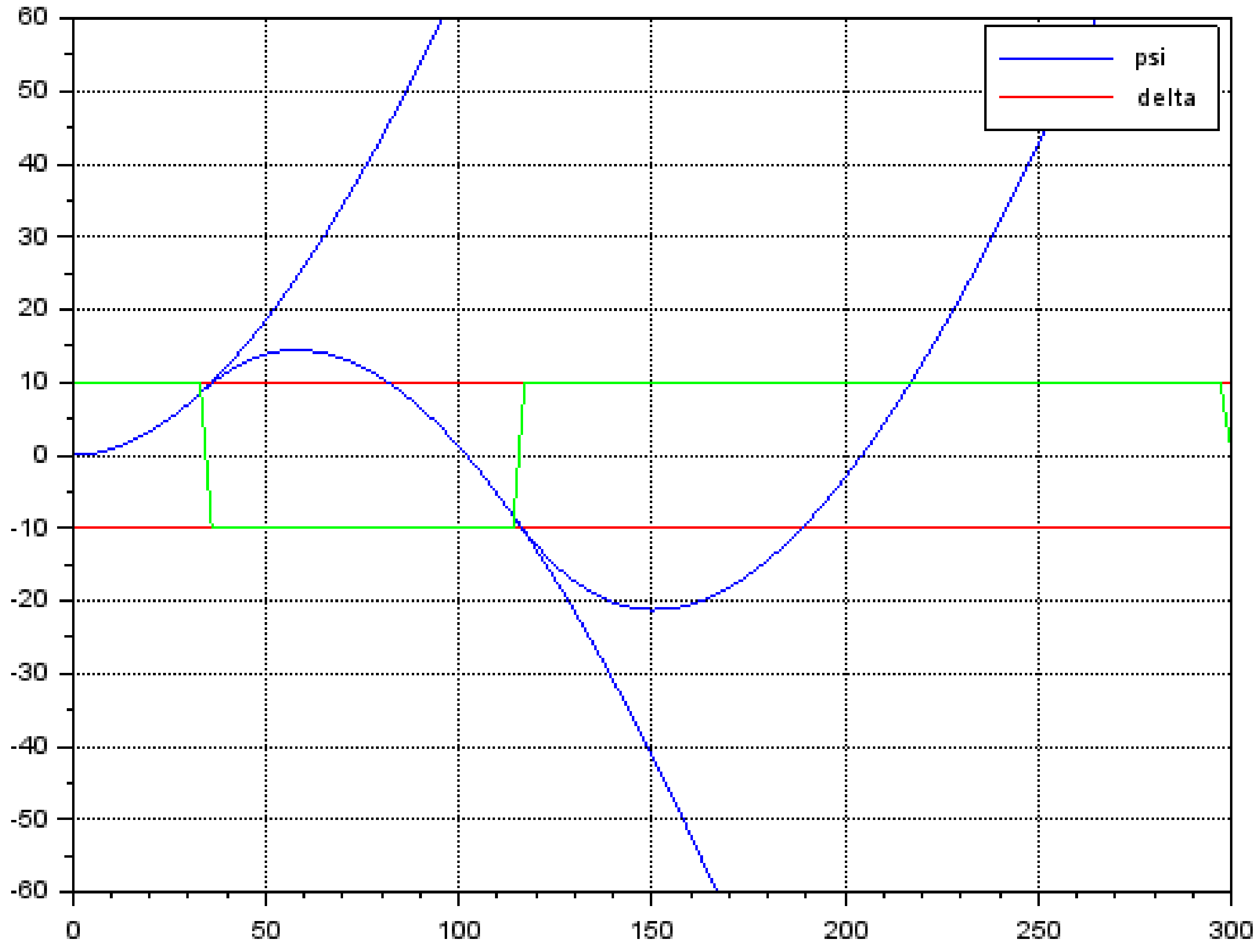
where $\dot{\psi}(t) = r(t)$

$$T = \frac{L}{U}T', \;\; K = \frac{U}{L}K' \qquad (U_1 \leq U \leq U_2)$$

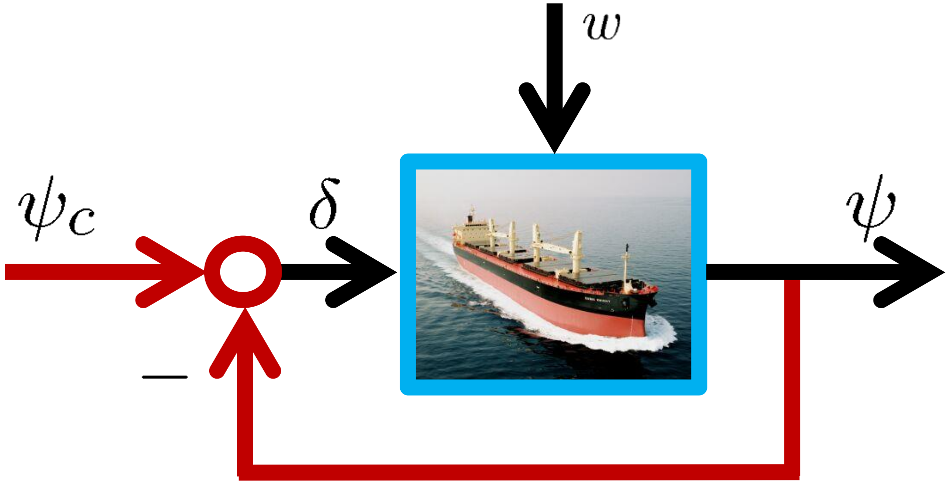
Parameter Uncertainty

Velocity Variation

Z試験


psi
delta
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
0
50
100
150
200
250
300

単位フィードバック



- $\delta(t) = \psi_c - \psi(t)$
- motion equation
$$\begin{cases} \dot{\psi}(t) = r(t) \\ \dot{r}(t) = -\frac{1}{T} r(t) + \frac{K}{T}(\psi_c - \psi(t - t_L)) + w(t) \end{cases}$$
- state equation
$$\underbrace{\begin{bmatrix} \dot{\psi}(t) \\ \dot{r}(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{K}{T}\frac{\psi(t-t_L)}{\psi(t)} & -\frac{1}{T} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \psi(t) \\ r(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 \\ \frac{K}{T} \end{bmatrix}}_{B} \psi_c + \begin{bmatrix} 0 \\ w(t) \end{bmatrix}$$

t_L=0 ⇒ −K/T

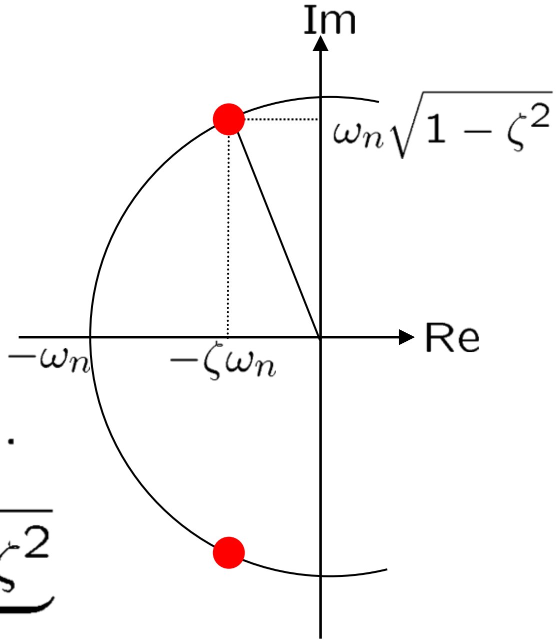
２次振動系のインパルス/ステップ応答

- 2nd-order system

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}}_{B} u \ (\zeta < 1)$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x$$

ζ:damping coef., ω_n:natural angular freq.



- eigenvalues of A: $\lambda = \underbrace{-\zeta\omega_n}_{\lambda_R} \pm j \underbrace{\omega_n\sqrt{1-\zeta^2}}_{\lambda_I}$
- impluse resp.: $G(t) = \frac{\omega_n^2}{\lambda_I} e^{\lambda_R t} \sin \lambda_I t$
- step resp.: $S(t) = 1 - \frac{\omega_n}{\lambda_I} e^{\lambda_R t} \sin(\lambda_I t - \tan^{-1} \frac{\lambda_I}{\lambda_R})$
- $(T_p, 1 + p_0) = \left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, 1 + \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) \right)$

２次振動系のインパルス/ステップ応答

- impluse response:
$$G(t) = C\exp(At)B = CV\exp(\Lambda t)V^{-1}B$$
$$= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\sqrt{2}\begin{bmatrix} 1 & 0 \\ \lambda_R & \lambda_I \end{bmatrix}}_{VJ} \underbrace{e^{\lambda_R t}\begin{bmatrix} \cos\lambda_I t & \sin\lambda_I t \\ \sin\lambda_I t & \cos\lambda_I t \end{bmatrix}}_{\exp(J^{-1}\Lambda J)} \underbrace{\frac{1}{\sqrt{2}\lambda_I}\begin{bmatrix} \lambda_I & 0 \\ -\lambda_R & 1 \end{bmatrix}}_{(VJ)^{-1}} \underbrace{\begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}}_{B}$$
$$= \frac{\omega_n^2}{\lambda_I}e^{\lambda_R t}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \cos\lambda_I t & \sin\lambda_I t \\ \sin\lambda_I t & \cos\lambda_I t \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\omega_n^2}{\lambda_I}e^{\lambda_R t}\sin\lambda_I t$$
- step response: Using $\int e^{ax}\sin bx dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$,
$$S(t) = \int_0^t G(\tau)d\tau = \int_0^t \frac{\omega_n^2}{\lambda_I}e^{\lambda_R\tau}\sin\lambda_I\tau d\tau = \frac{\omega_n^2}{\lambda_I}\left[e^{\lambda_R\tau}\sin\lambda_I\tau\right]_0^t$$
$$= \frac{\omega_n^2}{\lambda_I}\frac{1}{\lambda_R^2+\lambda_I^2}(e^{\lambda_R t}(\lambda_R\sin\lambda_I t - \lambda_I\cos\lambda_I t) + \lambda_I)$$
$$= 1 + \frac{\omega_n}{\lambda_I}e^{\lambda_R t}(\sin\lambda_I t \times \frac{\lambda_R}{\omega_n} - \cos\lambda_I t \times \frac{\lambda_I}{\omega_n})$$
$$= 1 + \frac{\omega_n}{\lambda_I}e^{\lambda_R t}(\sin\lambda_I t\cos\theta - \cos\lambda_I t\sin\theta)$$
$$= 1 + \frac{\omega_n}{\lambda_I}e^{\lambda_R t}\sin(\lambda_I t - \theta) \quad (\theta = \tan^{-1}\frac{\lambda_I}{\lambda_R})$$
$$= 1 + \frac{\omega_n}{\lambda_I}e^{\lambda_R t}\sin(\lambda_I t + \phi - \pi) \quad (\phi = \pi - \theta = \tan^{-1}\frac{\lambda_I}{-\lambda_R})$$
$$= \boxed{1 - \frac{\omega_n}{\lambda_I}e^{\lambda_R t}\sin(\lambda_I t + \phi) \quad (\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta})}$$

２次振動系のインパルス/ステップ応答

- $(T_p, 1+p_0) = \left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, 1+\exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}})\right)$

$$\dot{S}(t) = \frac{\omega_n}{\lambda_I}(\lambda_R e^{\lambda_R t}\sin(\lambda_I t+\phi) + e^{\lambda_R t}\cos(\lambda_I t+\phi)\times\lambda_I)$$

$$0 = \frac{\omega_n}{\lambda_I}(\lambda_R e^{\lambda_R t}\sin(\lambda_I t+\phi) + e^{\lambda_R t}\cos(\lambda_I t+\phi)\times\lambda_I)$$

$$0 = \sin(\lambda_I t+\phi)\times\lambda_R + \cos(\lambda_I t+\phi)\times\lambda_I \quad (\phi = \tan^{-1}\frac{\lambda_I}{-\lambda_R})$$

$$0 = \sin(\lambda_I t+\phi)\cos\phi - \cos(\lambda_I t+\phi)\sin\phi = \sin(\lambda_I t+\phi-\phi) = \sin\lambda_I t$$

$$\lambda_I T_p = \pi \quad\Rightarrow\quad T_p = \frac{\pi}{\lambda_I}$$

$$S(T_p) = 1 - \frac{\omega_n}{\lambda_I}e^{\lambda_R\frac{\pi}{\lambda_I}}\sin(\lambda_I\frac{\pi}{\lambda_I}+\phi) = 1 - \frac{\omega_n}{\lambda_I}e^{\frac{\lambda_R}{\lambda_I}\pi}\sin(\pi+\phi) = 1 + e^{\frac{\lambda_R}{\lambda_I}\pi}$$

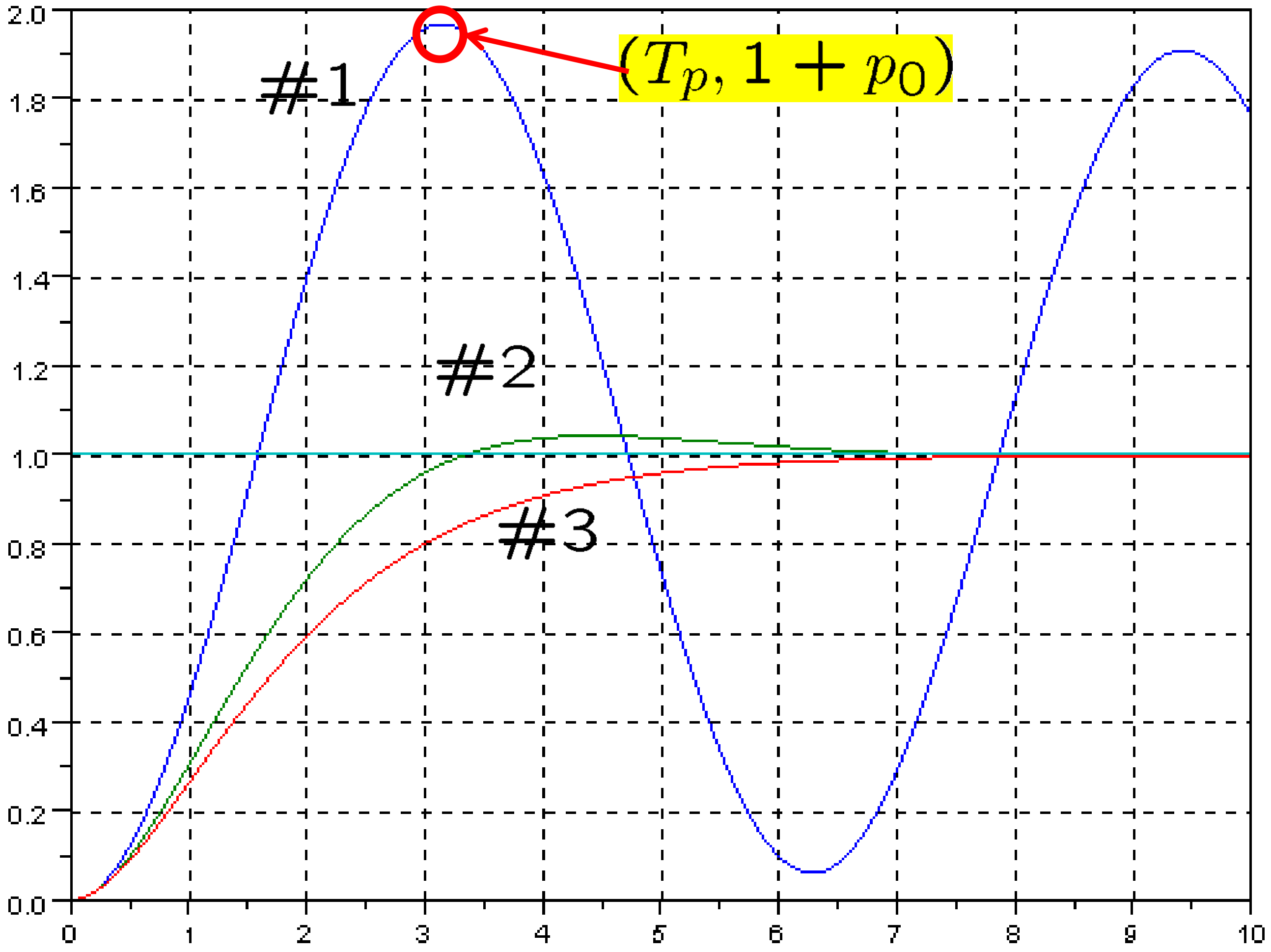
- $(\zeta, \omega_n) = \left(\sqrt{\frac{(\log p_0)^2}{(\log p_0)^2+\pi^2}}, \frac{\sqrt{(\log p_0)^2+\pi^2}}{T_p}\right)$

$$\begin{cases} T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \\ p_0 = \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) \end{cases} \Rightarrow \begin{cases} T_p^2 = \frac{\pi^2}{\omega_n^2(1-\zeta^2)} \\ \log p_0 = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}} \end{cases} \Rightarrow \begin{cases} \omega_n^2 = \frac{\pi^2}{T_p^2(1-\zeta^2)} \\ (\log p_0)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2} \end{cases}$$

$$\Rightarrow \begin{cases} \zeta^2 = \frac{(\log p_0)^2}{(\log p_0)^2+\pi^2} \\ \omega_n^2 = \frac{\pi^2}{T_p^2}\frac{(\log p_0)^2+\pi^2}{\pi^2} \end{cases} \Rightarrow \begin{cases} \zeta = \sqrt{\frac{(\log p_0)^2}{(\log p_0)^2+\pi^2}} \\ \omega_n = \frac{\sqrt{(\log p_0)^2+\pi^2}}{T_p} \end{cases}$$

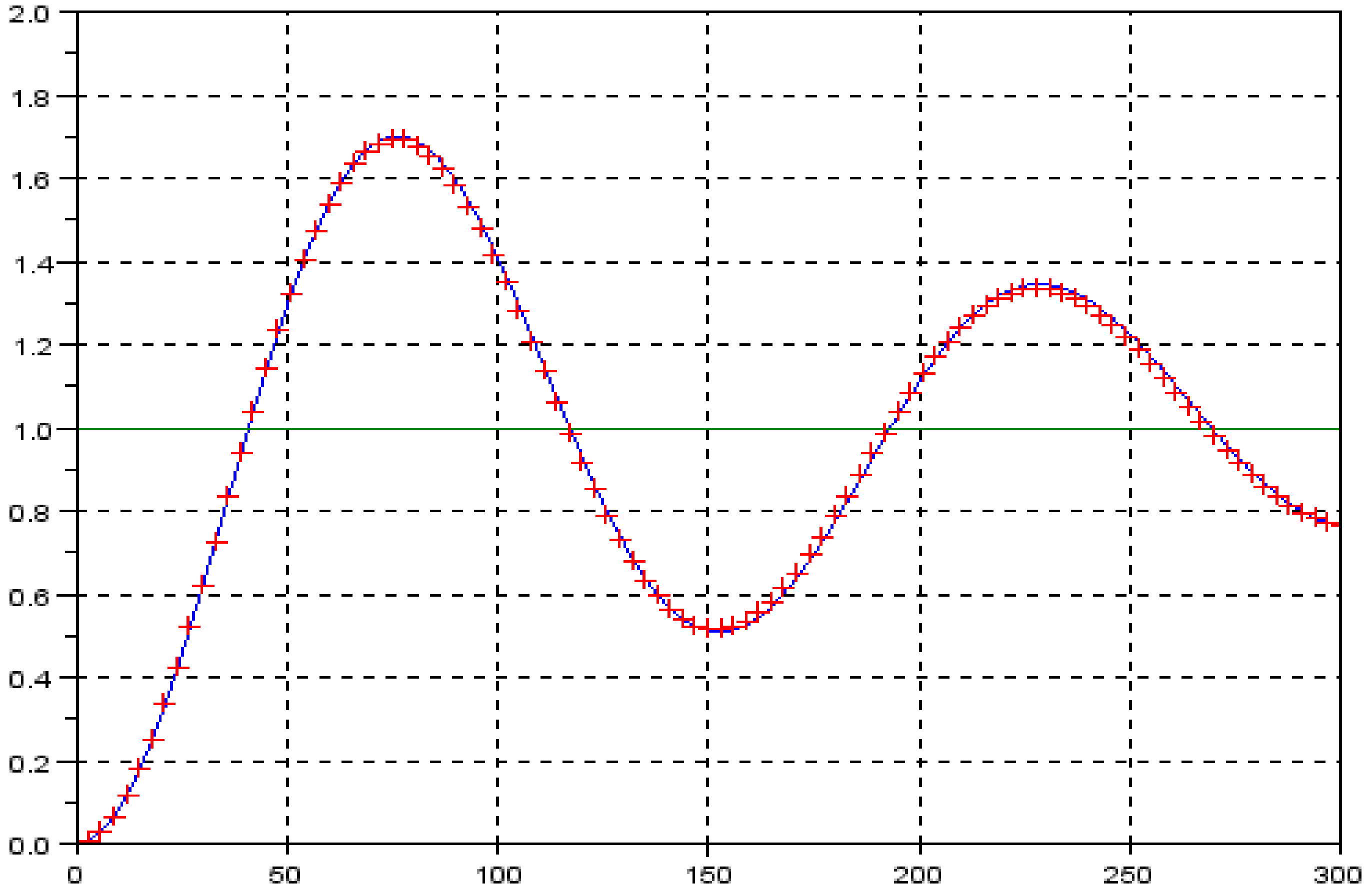
2次系のステップ応答の例

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -2\zeta \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x, \quad \zeta = 0.01, \frac{1}{\sqrt{2}}, 1$$



２次振動系の同定

$$(T_p, 1+p_0) \ \Rightarrow \ (\zeta, \omega_n) = \left(\sqrt{\frac{(\log p_0)^2}{(\log p_0)^2 + \pi^2}}, \frac{\sqrt{(\log p_0)^2 + \pi^2}}{T_p} \right)$$

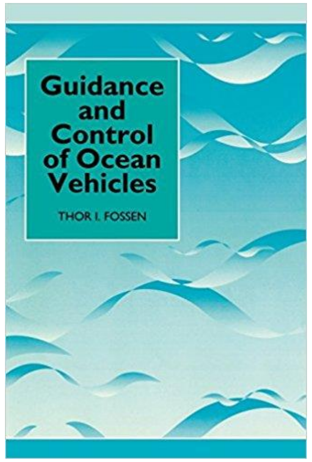


【3】速度変動への対応（LPV制御）

- 目の前に障害物が現れたらどうする
- 速度が変わると流体力微係数が大きく変わる
- 速度を変動パラメータとしてスケジューリングを行う！

33. Hiroyuki KAJIWARA A, Pierre APKARIAN, Pascal GAHINET: "LPV Techniques for Control of an Inverted Pendulum", IEEE Control Systems Magazine, vol.19, no.1, pp.44-54, 1999

Scheduled NOMOTO Model



- Nomoto Model

  $\dot{r} = -\frac{1}{T}r + \frac{K}{T}\delta$

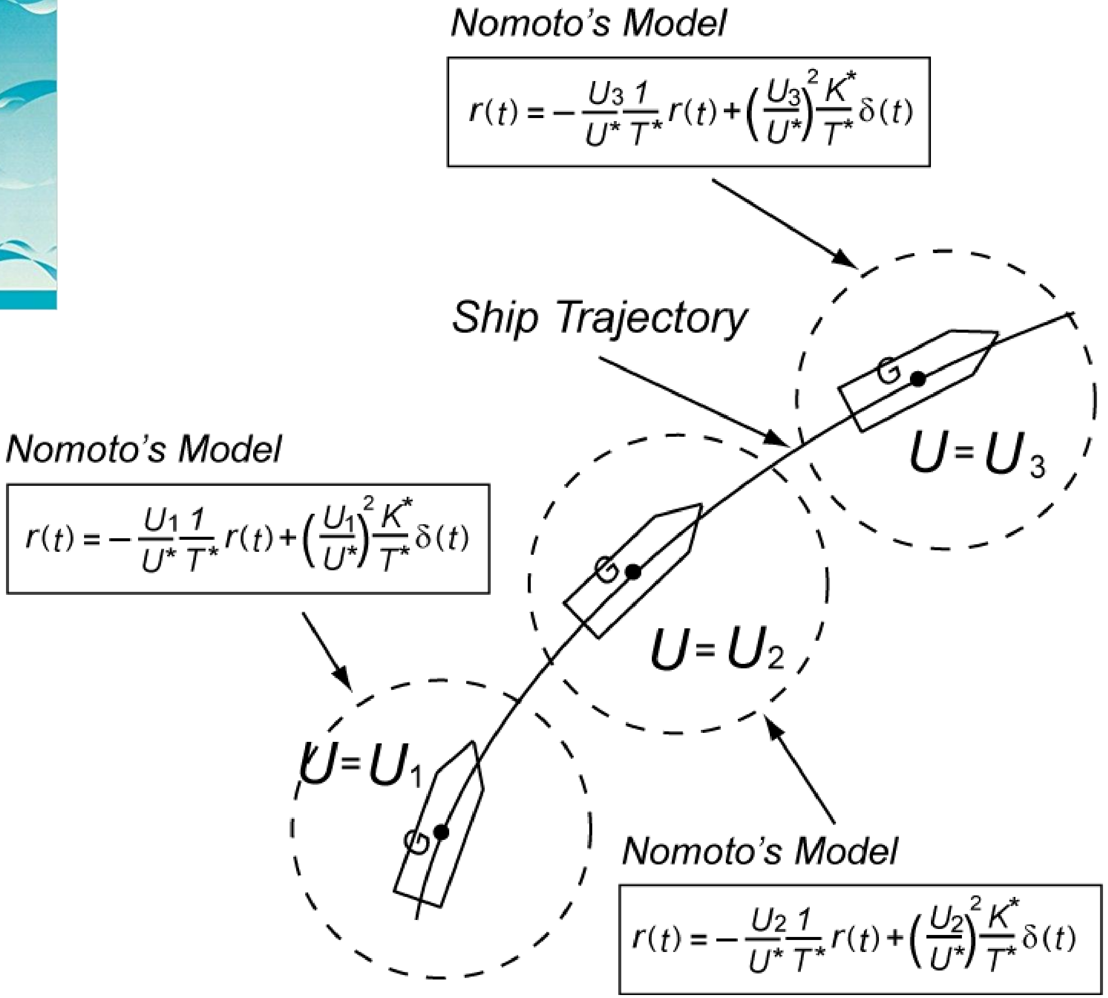
  where

  $T = \frac{L}{U}T', \ K = \frac{U}{L}K'$
- Nominal Speed $U_1 \leq U^* \leq U_2$

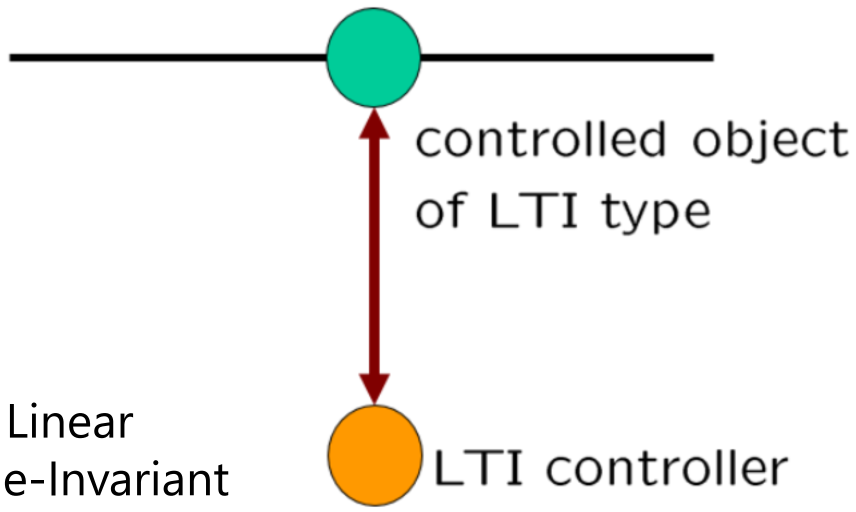
  $T^* = \frac{L}{U^*}T', \ K^* = \frac{U^*}{L}K'$
- Time Constant and Gain Constant

  $T = \frac{U^*}{U}T^*, \ K = \frac{U}{U^*}K^*$
- Scheduled Nomoto Model

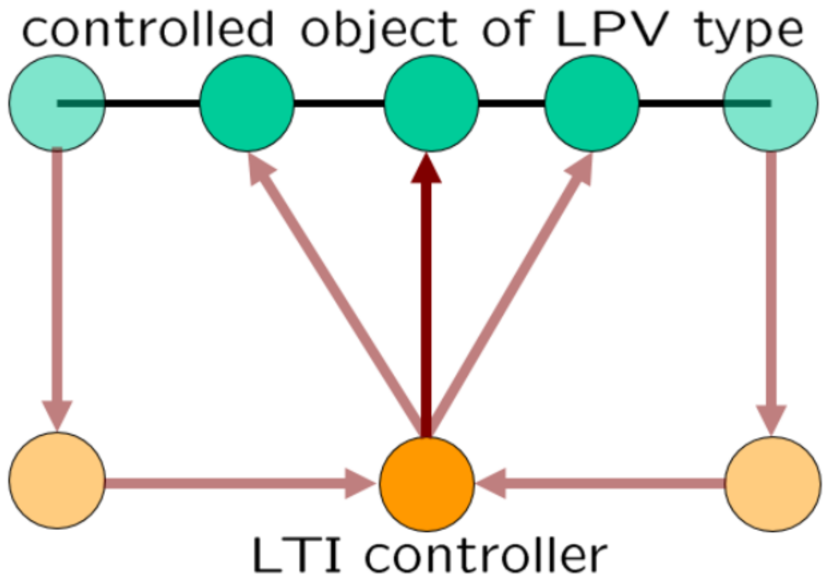
$$\dot{r} = -\underbrace{\left(\frac{U}{U^*}\right)\frac{1}{T^*}}_{\frac{1}{T(U)}} r + \underbrace{\left(\frac{U}{U^*}\right)^2 \frac{K^*}{T^*}}_{\frac{K(U)}{T(U)}} \delta$$

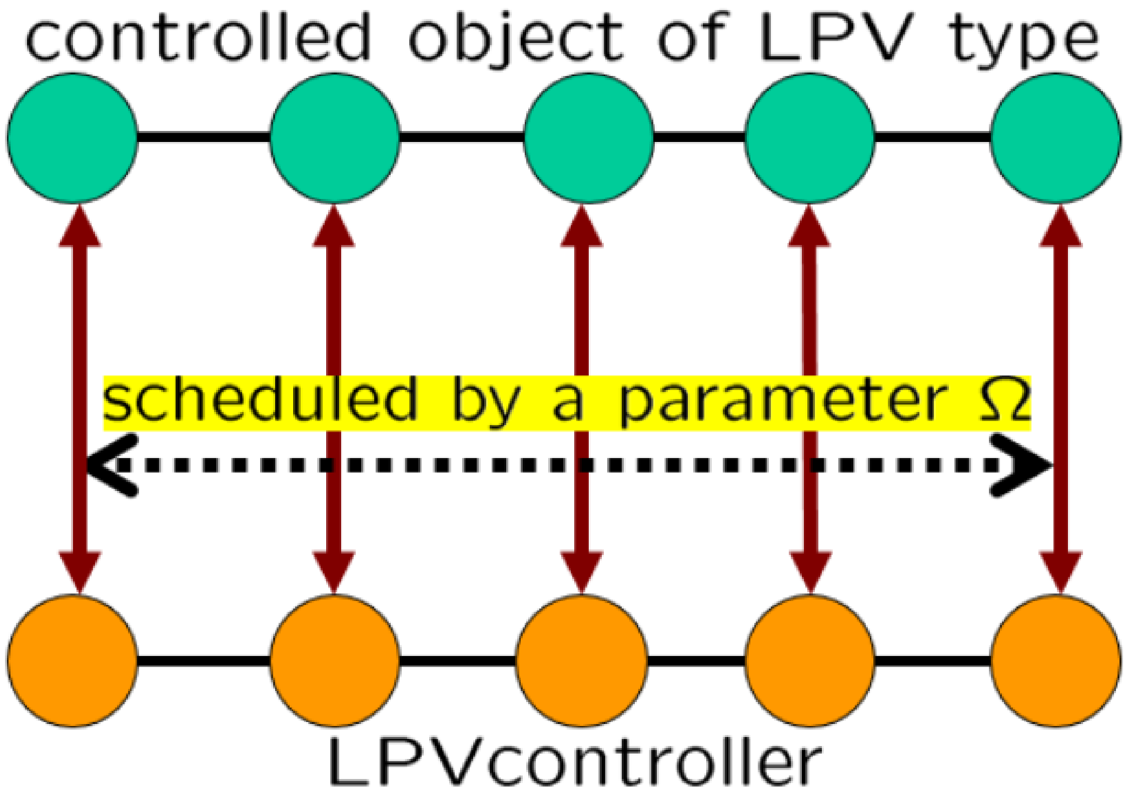


LTI Control and LPV Control



LTI: Linear
Time-Invariant

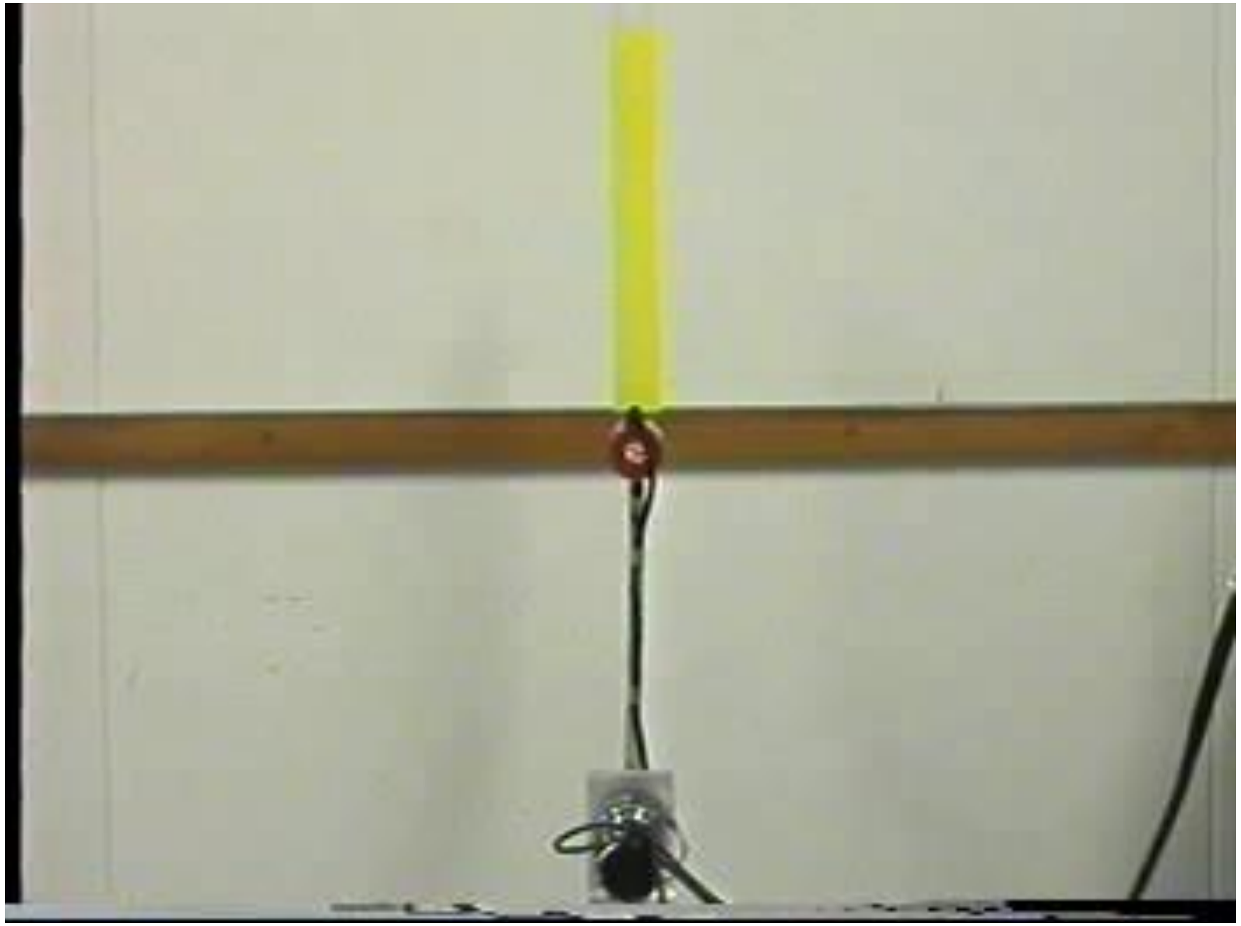




LPV: Linear Parameter Varying

Arm-Driven Inverted Pendulum

LTI Controller



LPV Controller

State Equation

- Motion equation

$$\begin{cases} \dot{\psi} = r \\ \dot{r} = -\left(\frac{U}{U^*}\right)\frac{1}{T^*}r + \left(\frac{U}{U^*}\right)^2\frac{K^*}{T^*}\delta \end{cases}$$

- Rudder Dynamics

$$\dot{\delta} = -\frac{1}{T_a}\delta + \frac{K_a}{T_a}u$$

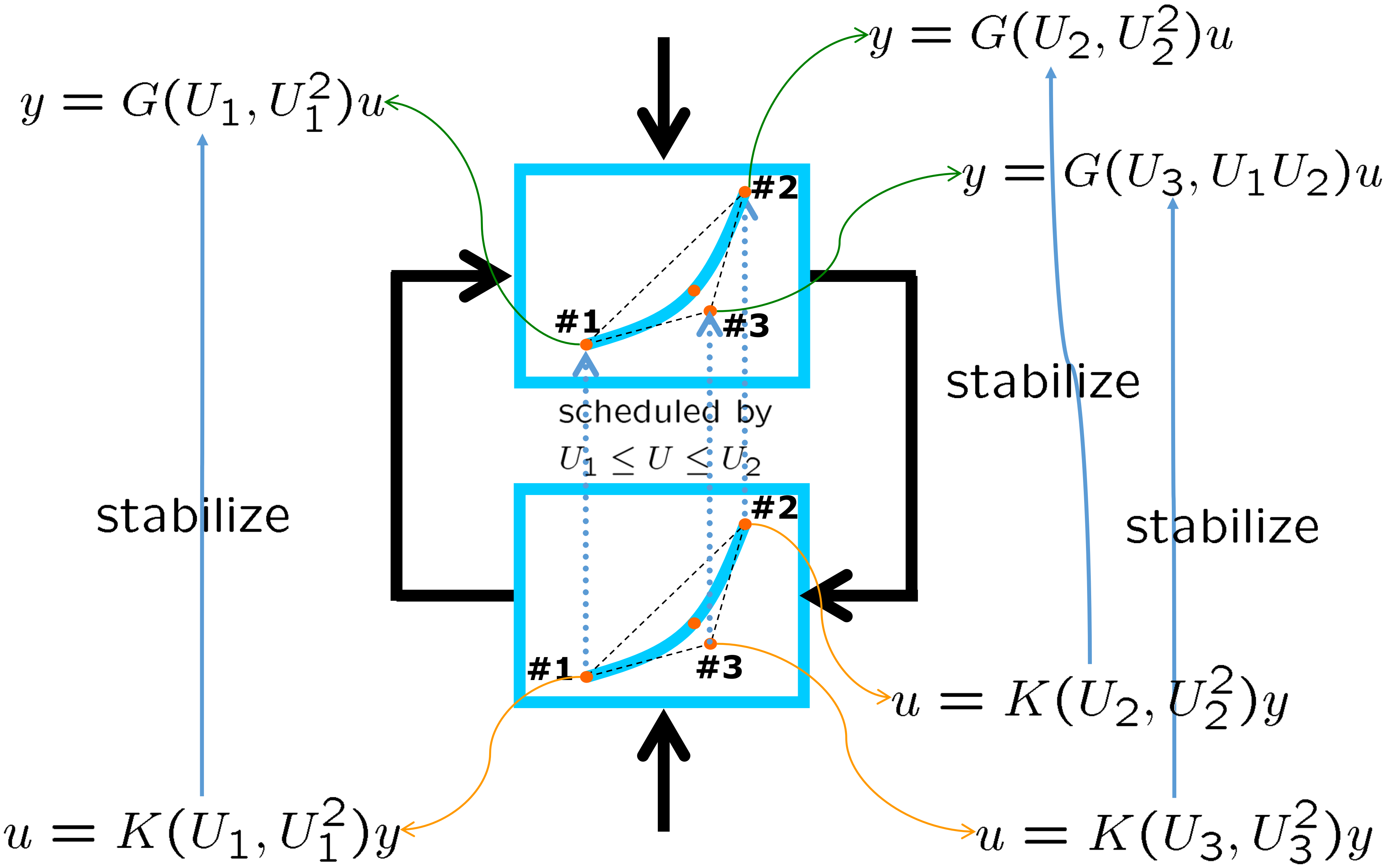
- State Equation

$$\underbrace{\begin{bmatrix} \dot{\psi} \\ \dot{r} \\ \dot{\delta} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\left(\frac{U}{U^*}\right)\frac{1}{T^*} & \left(\frac{U}{U^*}\right)^2\frac{K^*}{T^*} \\ 0 & 0 & -\frac{1}{T_a} \end{bmatrix}}_{A(U,U^2)} \underbrace{\begin{bmatrix} \psi \\ r \\ \delta \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{K_a}{T_a} \end{bmatrix}}_{B} u$$

- Output Equation

$$\underbrace{\psi}_{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \psi \\ r \\ \delta \end{bmatrix}}_{x}$$

LPV Control System



LPV Model with 3 Vertexes

$$\dot{x} = \underbrace{(p_1 A_1 + p_2 A_2 + p_3 A_3)}_{A(U,U^2)} x + Bu$$

where $A_1 = A(U_1, U_1^2), A_2 = A(U_2, U_2^2)$
$A_3 = A(U_3, U_1 U_2)$ with $U_3 = \frac{U_1+U_2}{2}$ and

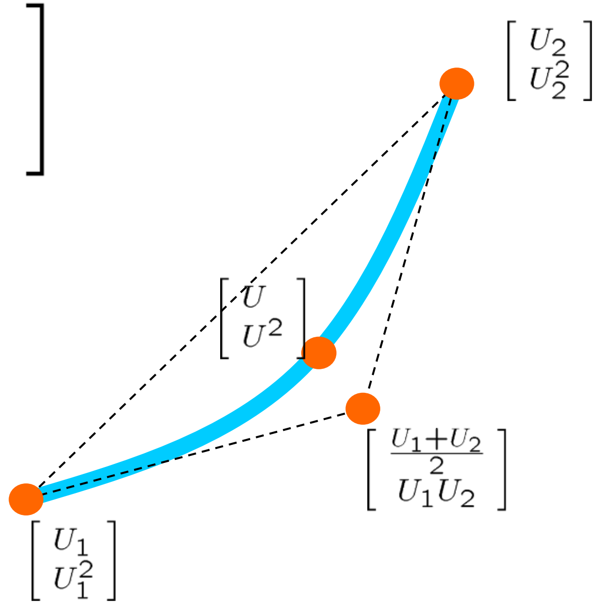
$$p_1 = \frac{1}{p_0} \det \begin{bmatrix} U - U_3 & U_2 - U_3 \\ U^2 - U_1 U_2 & U_2^2 - U_1 U_2 \end{bmatrix}$$

$$p_2 = \frac{1}{p_0} \det \begin{bmatrix} U_1 - U_3 & U - U_3 \\ U_1^2 - U_1 U_2 & U^2 - U_1 U_2 \end{bmatrix}$$

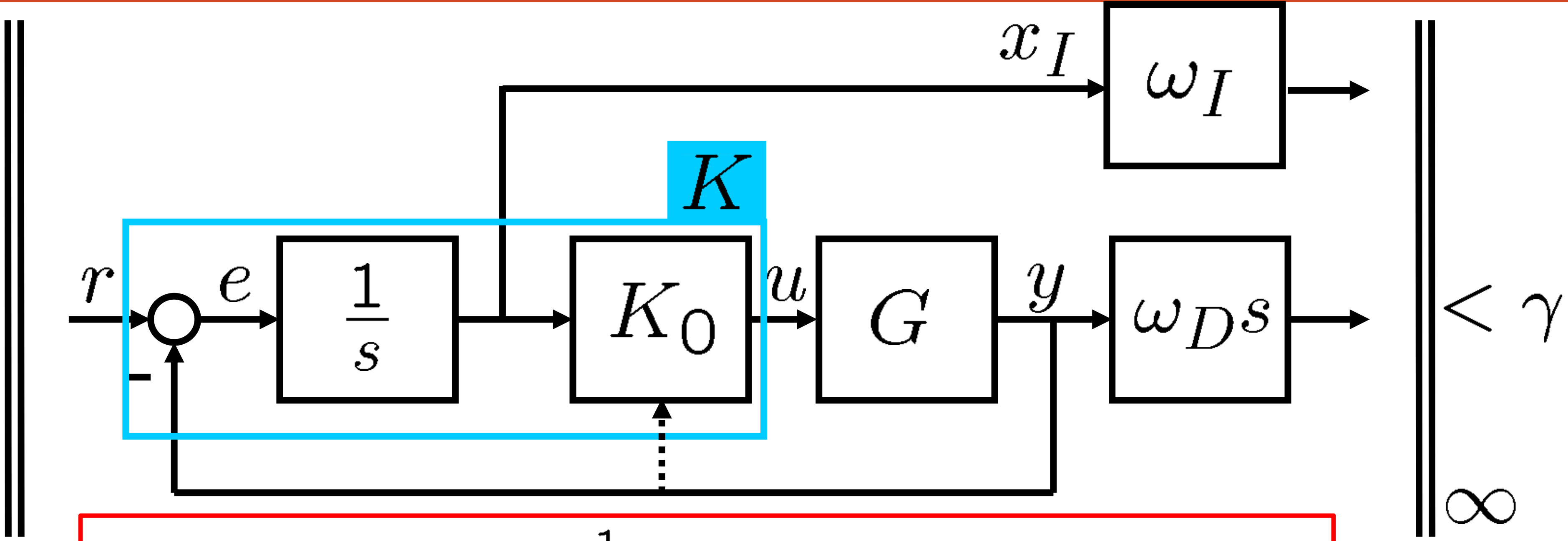
$$p_3 = \frac{1}{p_0} \det \begin{bmatrix} U_1 - U_2 & U_2 - U \\ U_1^2 - U_2^2 & U_2^2 - U^2 \end{bmatrix}$$

$$p_0 = \det \begin{bmatrix} U_1 - U_2 & U_2 - U_3 \\ U_1^2 - U_2^2 & U_2^2 - U_1 U_2 \end{bmatrix}$$

satisfying $p_1 + p_2 + p_3 = 1$



Interconnection with Integrator



$$\frac{\omega_I x_I}{r} = \omega_I \frac{\frac{1}{s}}{1+GK_0\frac{1}{s}} = \underbrace{\frac{\omega_I}{s}}_{W_S}\underbrace{\frac{1}{1+GK}}_{S}$$

$$\frac{\omega_D \dot{y}}{r} = \omega_D s \frac{GK_0\frac{1}{s}}{1+GK_0\frac{1}{s}} = \underbrace{\omega_D s}_{W_T}\underbrace{\frac{GK}{1+GK}}_{T}$$

CLPS by LPV Output FB

- 2-port representation

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{x}_I \end{bmatrix} = \underbrace{\begin{bmatrix} A(U,U^2) & 0 \\ -C & 0 \end{bmatrix}}_{A(U,U^2)} \begin{bmatrix} x \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_1} r + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_2} u \\ \begin{bmatrix} \omega_I x_I \\ \omega_D \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \omega_I \\ \omega_D C A(U,U^2) & 0 \end{bmatrix}}_{C_1} \begin{bmatrix} x \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D_{11}} r + \underbrace{\begin{bmatrix} 0 \\ \omega_D C B \end{bmatrix}}_{D_{12}} u \\ \begin{bmatrix} y \\ x_I \end{bmatrix} = \underbrace{\begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix}}_{C_2} \begin{bmatrix} x \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D_{21}} r \end{cases}$$

$$\begin{aligned} A(U,U^2) &= p_1(U,U^2)A_1 + p_2(U,U^2)A_2 + p_3(U,U^2)A_3 \\ A_K(U,U^2) &= p_1(U,U^2)A_{K1} + p_2(U,U^2)A_{K2} + p_3(U,U^2)A_{K3} \\ B_K(U,U^2) &= p_1(U,U^2)B_{K1} + p_2(U,U^2)B_{K2} + p_3(U,U^2)B_{K3} \\ C_K(U,U^2) &= p_1(U,U^2)C_{K1} + p_2(U,U^2)C_{K2} + p_3(U,U^2)C_{K3} \\ D_K(U,U^2) &= p_1(U,U^2)D_{K1} + p_2(U,U^2)D_{K2} + p_3(U,U^2)D_{K3} \end{aligned}$$

- output feedback

$$\begin{cases} \begin{bmatrix} \dot{x}_K \\ \dot{x}_I \end{bmatrix} = \underbrace{\begin{bmatrix} A_K & B_{K2} \\ 0 & 0 \end{bmatrix}}_{A_K(U,U^2)} \begin{bmatrix} x_K \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} B_{K1} & 0 \\ -1 & 1 \end{bmatrix}}_{B_K(U,U^2)} \begin{bmatrix} y \\ r \end{bmatrix} \\ u = \underbrace{\begin{bmatrix} C_K & D_{K2} \end{bmatrix}}_{C_K(U,U^2)} \begin{bmatrix} x_K \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} D_{K1} & 0 \end{bmatrix}}_{D_K(U,U^2)} \begin{bmatrix} y \\ r \end{bmatrix} \end{cases}$$

LMI Based Design of LPV Output FB

- Minimize γ
  on $R = R^T, S = S^T, \mathcal{A}_{Ki}, \mathcal{B}_{Ki}, \mathcal{C}_{Ki}, D_{Ki}$ $(i = 1, 2, 3)$
  subject to $\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0$ and
  LMI-OF1,2,3,4 for vertex1
  LMI-OF1,2,3,4 for vertex2
  LMI-OF1,2,3,4 for vertex3
- Determine the output feedback controller
  for each vertex A_{Ki}, B_{Ki}, C_{Ki} $(i = 1, 2, 3)$

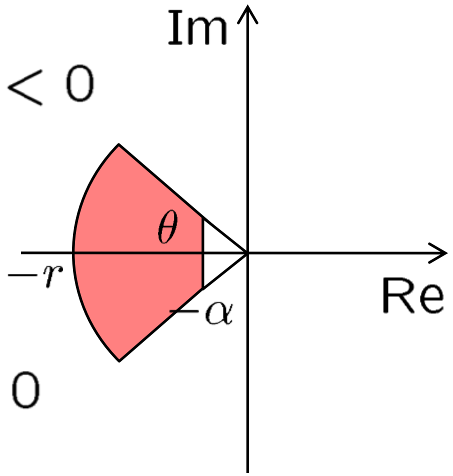
$$
\begin{aligned}
A_{Ki} &= N^{-1}(\mathcal{A}_{Ki} - S(A_i - B_2 D_{Ki} C_2)R - \mathcal{B}_{Ki} C_2 R \\
&\quad - S B_2 \mathcal{C}_{Ki}) M^{-T} \\
B_{Ki} &= N^{-1}(\mathcal{B}_{Ki} - S B_2 D_{Ki}) \\
C_{Ki} &= (\mathcal{C}_{Ki} - D_{Ki} C_2 R) M^{-T}
\end{aligned}
$$

where $I - SR = NM^T$

LMIs for Output FB Design

- **LMI-OF1**:

$$\begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} + (*)^T + \alpha \begin{bmatrix} R & I \\ I & S \end{bmatrix} < 0$$



- **LMI-OF2**:

$$\begin{bmatrix} -r\begin{bmatrix} R & I \\ I & S \end{bmatrix} & \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} \\ (*)^T & -r\begin{bmatrix} R & I \\ I & S \end{bmatrix} \end{bmatrix} < 0$$

- **LMI-OF3**:

$$\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \otimes \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} + (*)^T < 0$$

- **LMI-OF4**:

$$\begin{bmatrix} \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_2 \\ \mathcal{A}_K & SA + \mathcal{B}_K C_2 \end{bmatrix} + (*)^T & \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ SB_1 + \mathcal{B}_K D_{21} \end{bmatrix} & (*)^T \\ (*)^T & -\gamma^2 I & (*)^T \\ \begin{bmatrix} C_1 R + D_{12}\mathcal{C}_K & C_1 + D_{12} D_K C_2 \end{bmatrix} & D_{11} & -I \end{bmatrix} < 0$$

Scheduled PID Controller

Consider a PID control presented by

$$\delta = K_P(\psi_c - \psi) - K_D r + K_I \int_0^t (\psi_c - \psi(\tau))\, d\tau$$

Assuming $K_i = 0$ and defining $\omega_n = \sqrt{\frac{KK_p}{T}}$, $\zeta = \frac{1+KK_d}{2\sqrt{KK_pT}}$, the following relation should hold.

$$\underbrace{\frac{1}{T}}_{\text{ship motion}} < \underbrace{\omega_n\sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}}_{\text{controlled motion}} < \underbrace{\frac{1}{T_\delta}}_{\text{steering motion}}$$

Thus the PID gains are calculated as

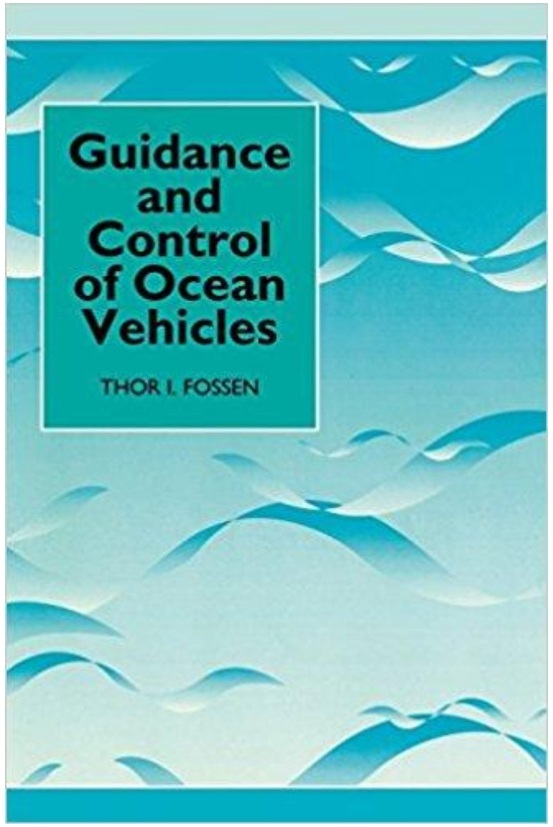
$$K_P = \frac{T\omega_n^2}{K}, \quad K_D = \frac{2T\zeta\omega_n - 1}{K}, \quad K_I = \frac{\omega_n}{10}K_p$$

Schedulled PID controller is implemented as

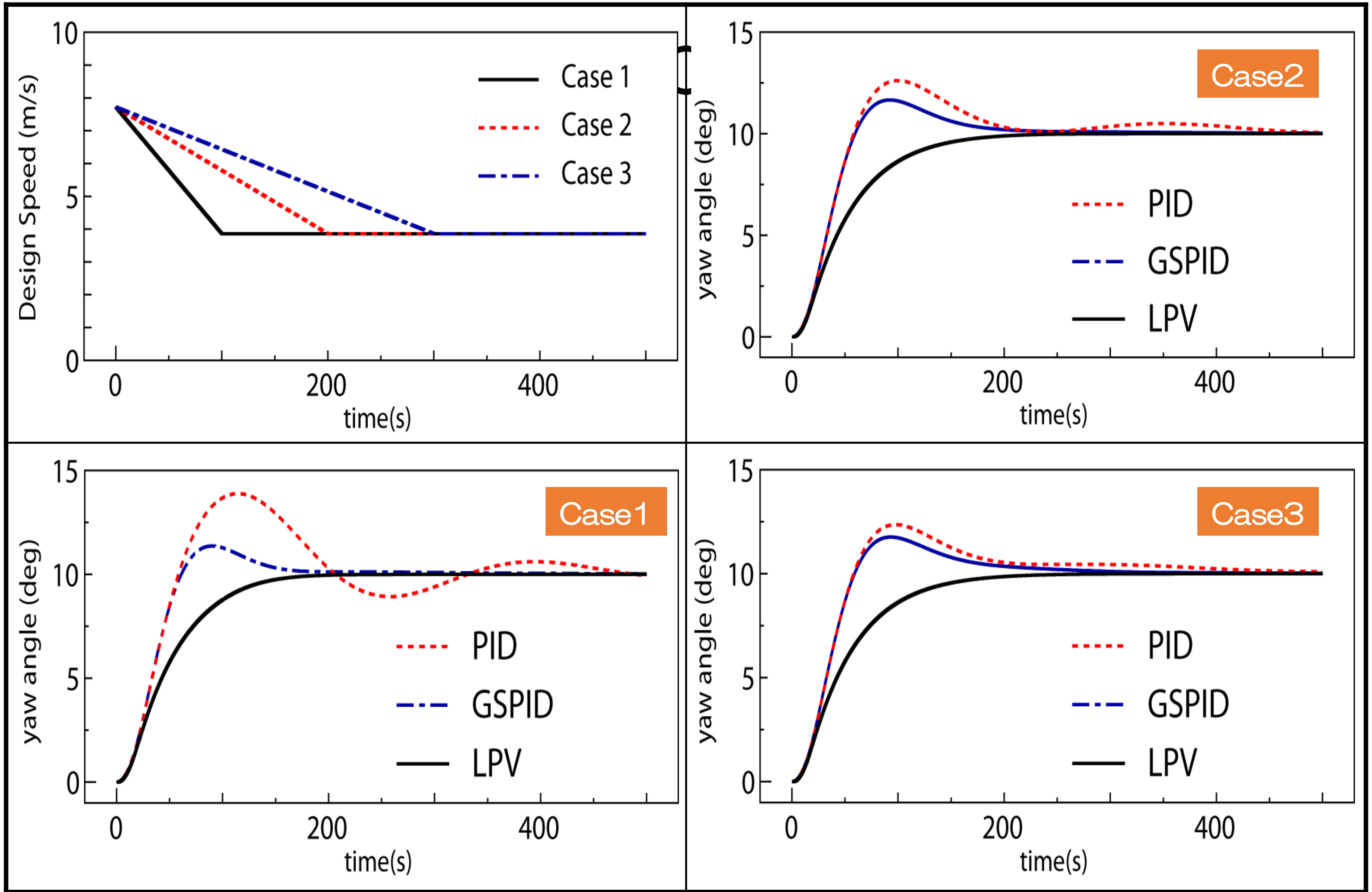
$$K_P(U) = \left(\frac{U^*}{U}\right)^2 \frac{T^*}{K^*}\omega_n^2$$

$$K_D(U) = \left(\frac{U^*}{U}\right)^2 \frac{T^*}{K^*}2\zeta\omega_n - \left(\frac{U^*}{U}\right)\frac{1}{K^*}$$

$$K_I(U) = \left(\frac{U^*}{U}\right)^2 \frac{T^*}{K^*}\frac{\omega_n^3}{10}$$



LPV Control of NOMOTO Model

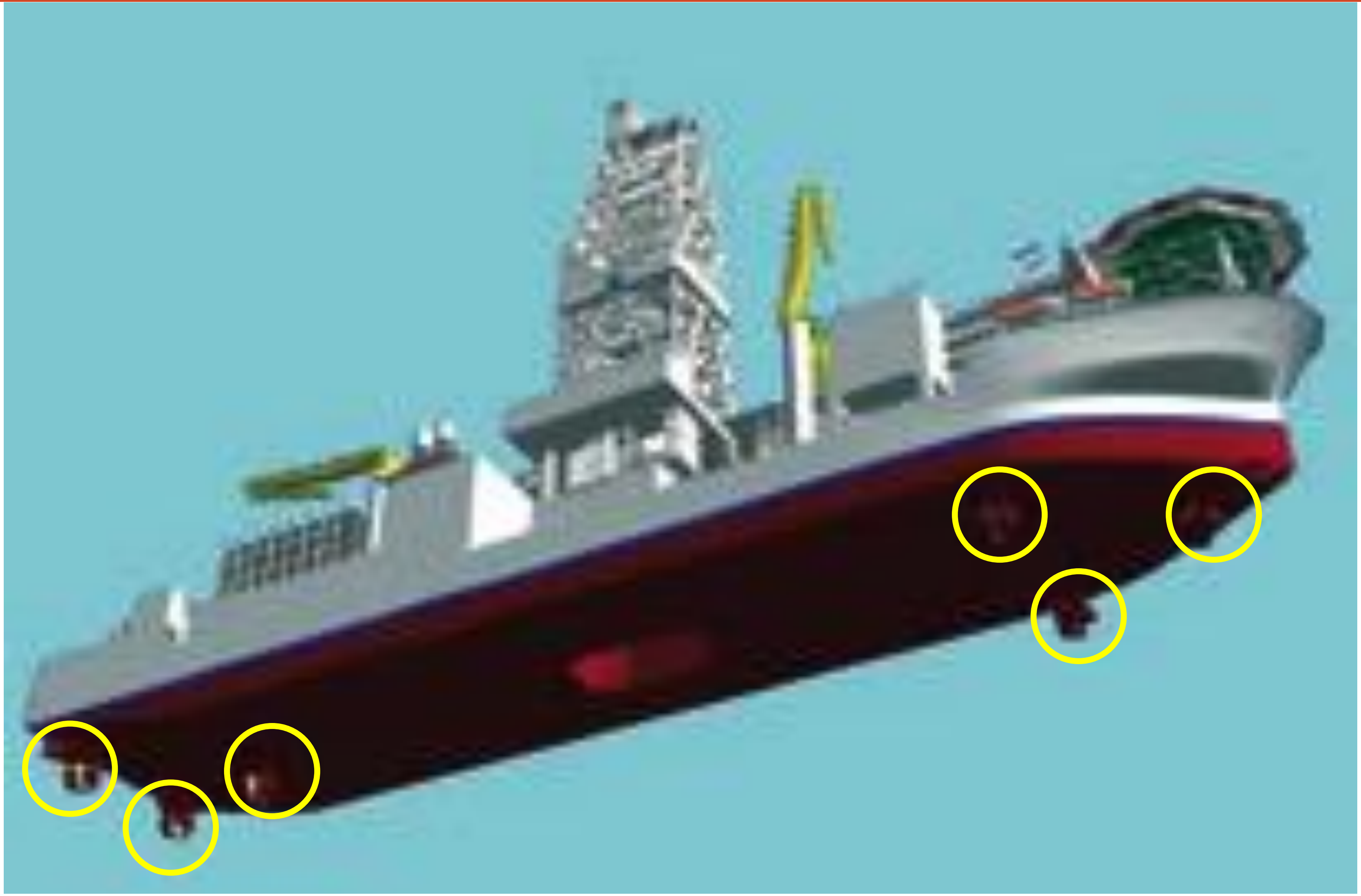


【4】DPS関連の手法（CA手法）

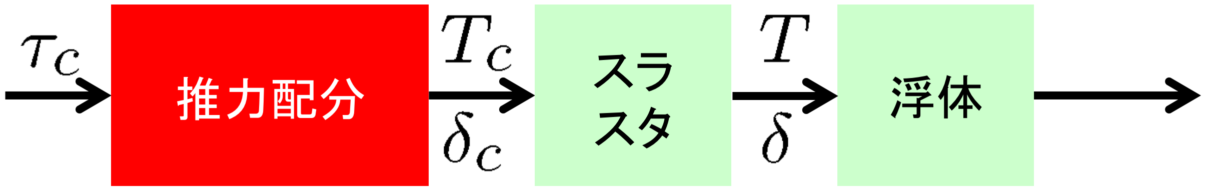
- CA(Control Allocation)問題とは
- 連立方程式求解問題（方程式の数より未知変数の数が多い）
- 特異値分解の利用！

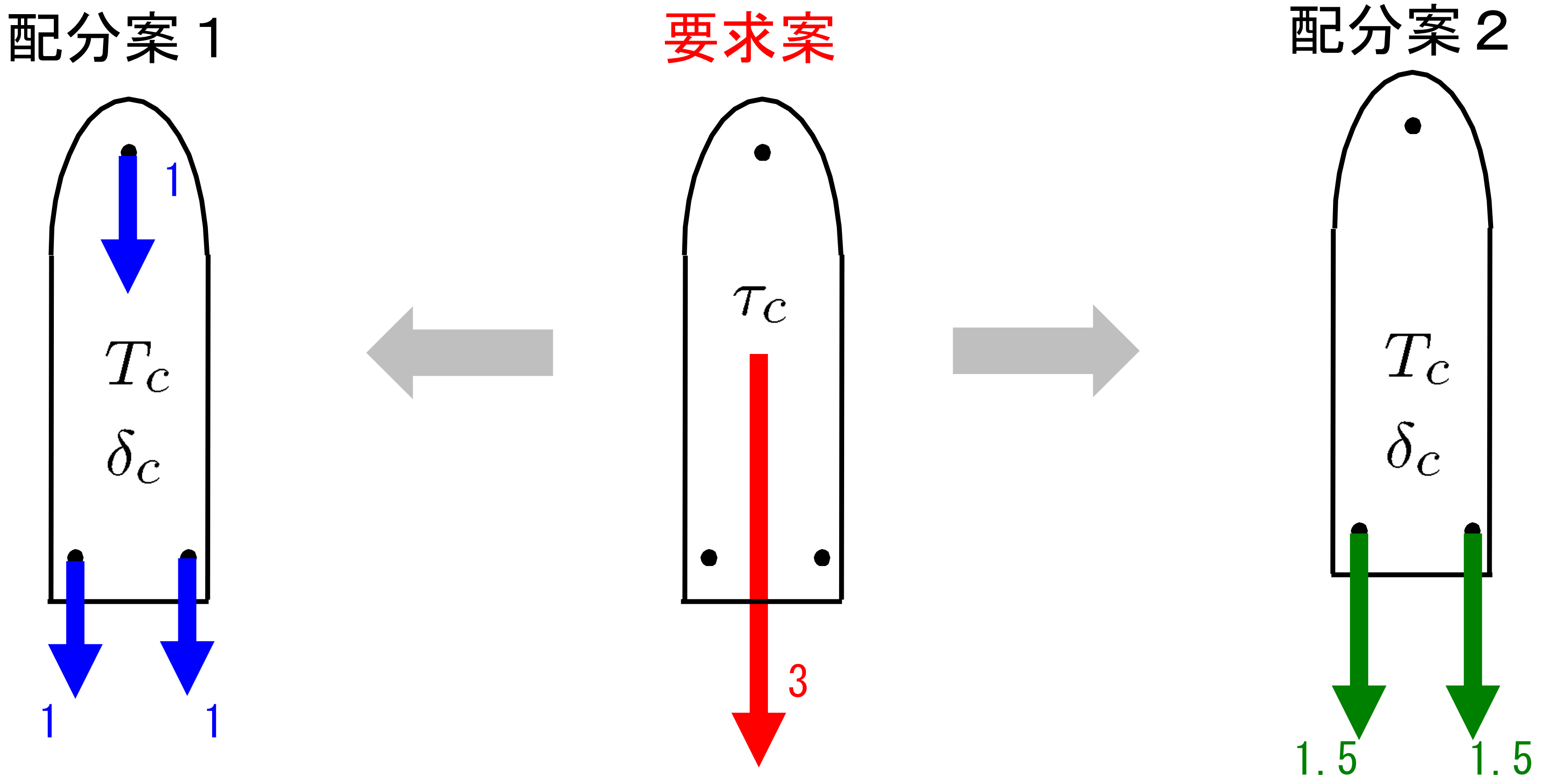
73. 大坪和久, 梶原宏之: "アジマススラスタ首振角に制約がある場合の推力配分法について", 日本船舶海洋工学会論文集, no.6, pp.177-182, 2007
74. 五百木陵行, 梶原宏之: "区分的線形補間による首振角制限付きアジマススラスタの最適推力配分法", 日本船舶海洋工学会論文集, no.6, pp.183-190, 2007

CHIKYU: Azimuth Thrusters

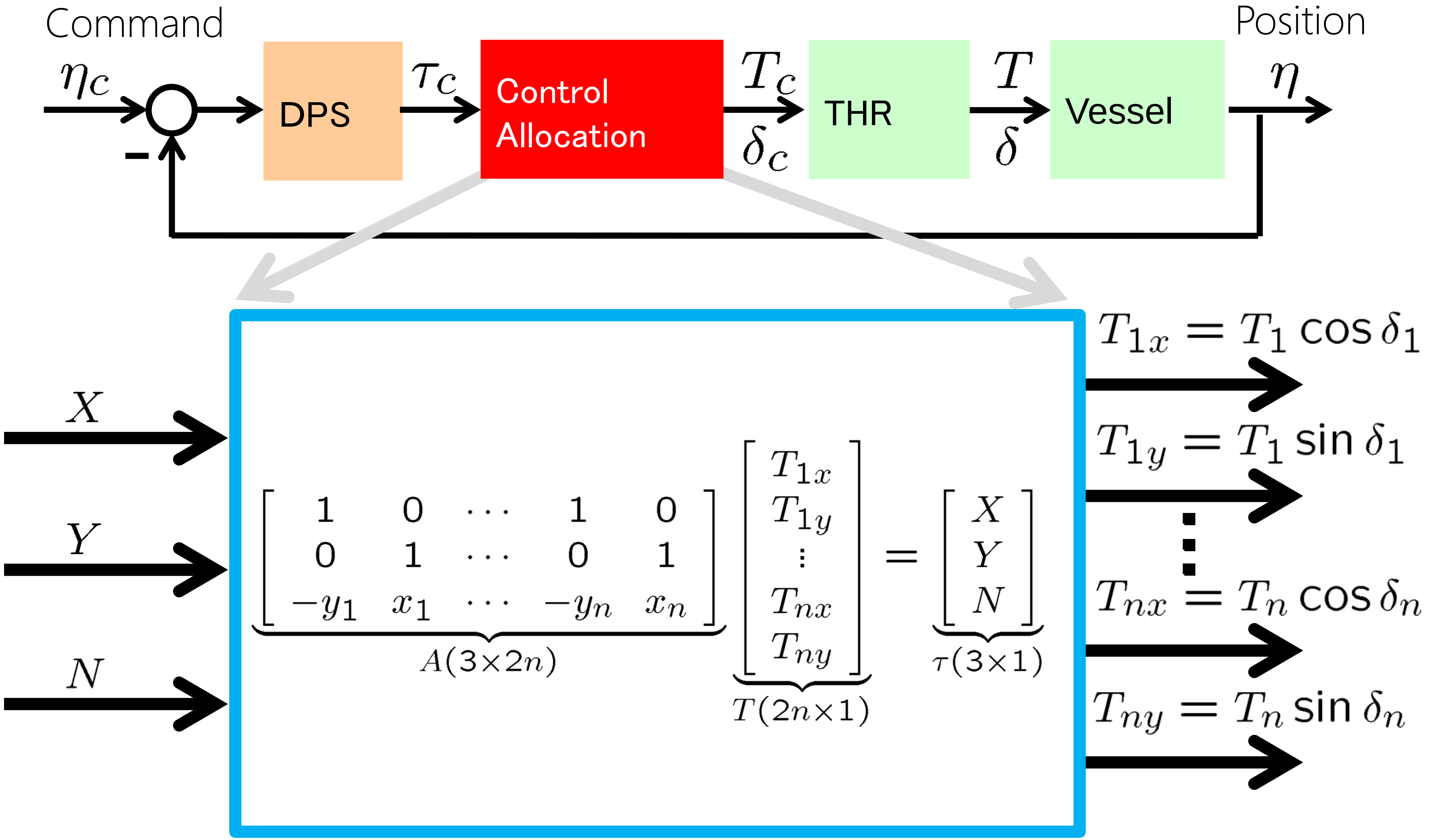


推力配分とは


τ_c
推力配分
T_c
δ_c
スラスタ
T
δ
浮体

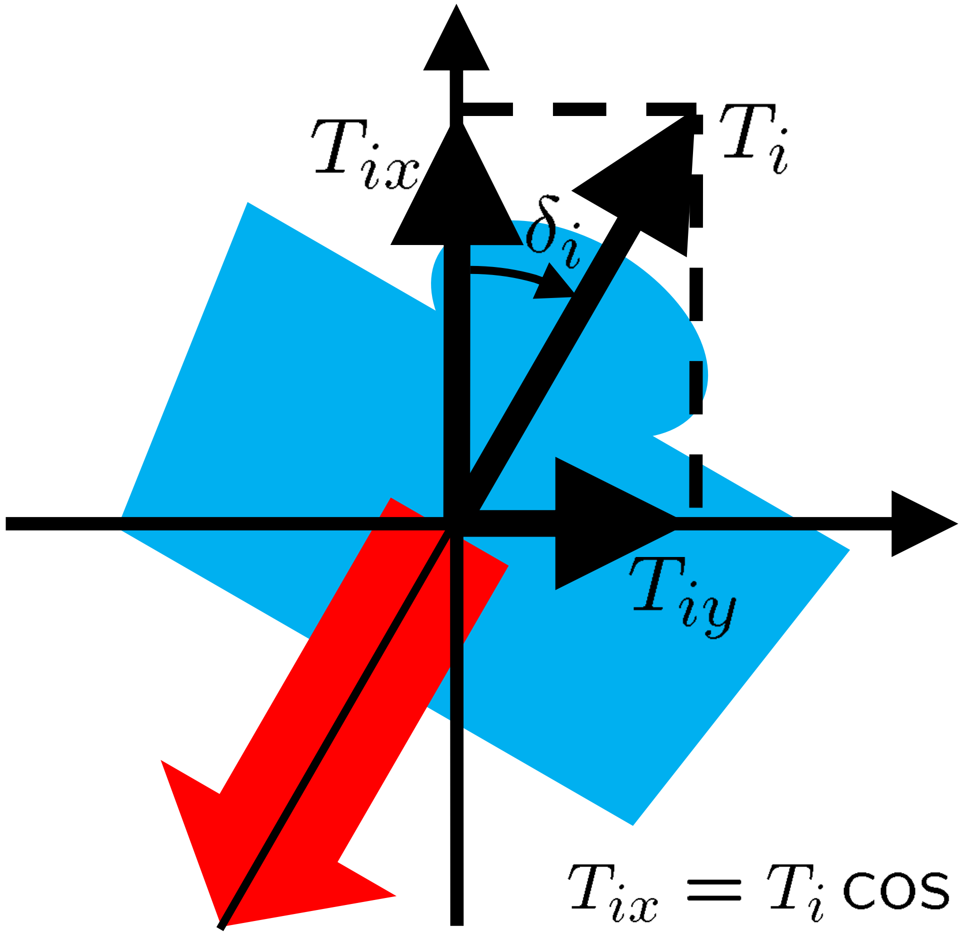

配分案１
1
T_c
δ_c
1
1
要求案
τ_c
3
配分案２
T_c
δ_c
1.5
1.5

CA (Control Allocation) Problem

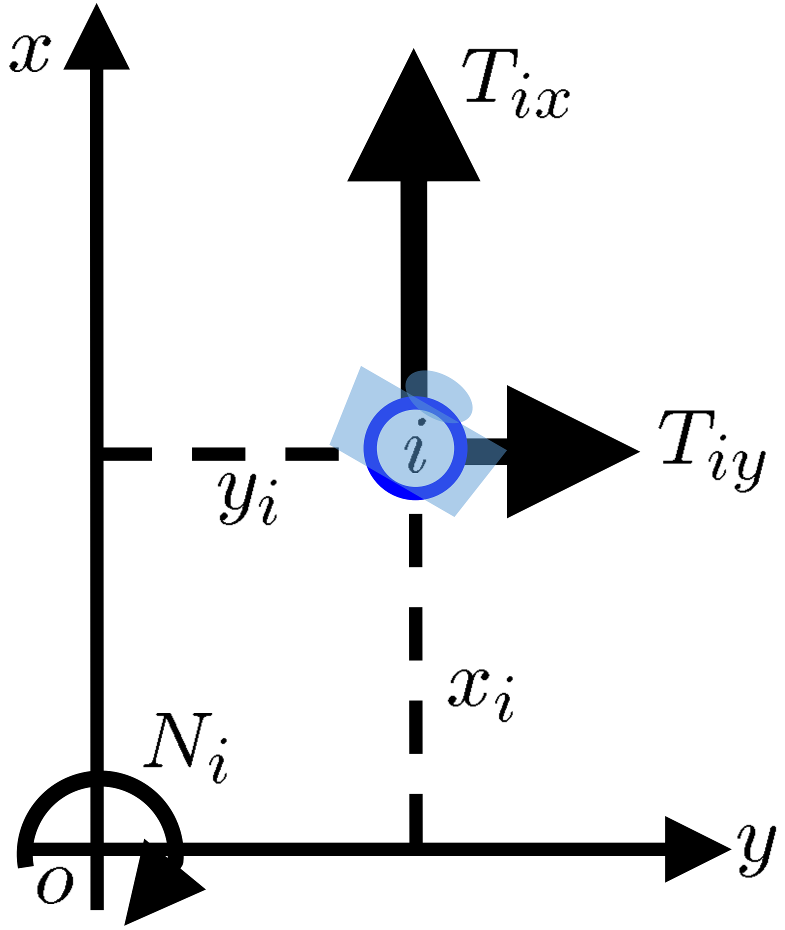


$$\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & \cdots & -y_n & x_n \end{bmatrix}}_{A(3\times 2n)} \underbrace{\begin{bmatrix} T_{1x} \\ T_{1y} \\ \vdots \\ T_{nx} \\ T_{ny} \end{bmatrix}}_{T(2n\times 1)} = \underbrace{\begin{bmatrix} X \\ Y \\ N \end{bmatrix}}_{\tau(3\times 1)}$$

スラストス成分とモーメント



$$T_{ix} = T_i \cos\delta_i$$
$$T_{iy} = T_i \sin\delta_i$$



$$N_{ix} = -\ T_{ix}\ y_i\ +\ T_{iy}\ x_i$$

推力配分方程式

$$T_{1x} + T_{2x} + \cdots + T_{nx} = \sum_{i=1}^{n} T_{ix} = X$$

$$T_{1y} + T_{2y} + \cdots + T_{ny} = \sum_{i=1}^{n} T_{iy} = Y$$

$$N_1 + N_2 + \cdots + N_n = \sum_{i=1}^{n} (-T_{ix}\ y_i + T_{iy}\ x_i) = N$$

$$T_{ix} = T_i \cos \delta_i$$

$$T_{iy} = T_i \sin \delta_i$$

$$\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & \cdots & -y_n & x_n \end{bmatrix}}_{A(3\times 2n)} \underbrace{\begin{bmatrix} T_{1x} \\ T_{1y} \\ \vdots \\ T_{nx} \\ T_{ny} \end{bmatrix}}_{T(2n\times 1)} = \underbrace{\begin{bmatrix} X \\ Y \\ N \end{bmatrix}}_{\tau(3\times 1)}$$

従来法の導出(推力最小化)

A の特異値分解

$$A = U \underbrace{\begin{bmatrix} \Sigma_1 & 0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} V_1 & V_2 \end{bmatrix}^T}_{V^T}$$

$AT = \tau$ の一般解

$$T = V_1 \Sigma_1^{-1} U^T \tau + V_2 c$$

任意のベクトル
(2*n*-3次元)

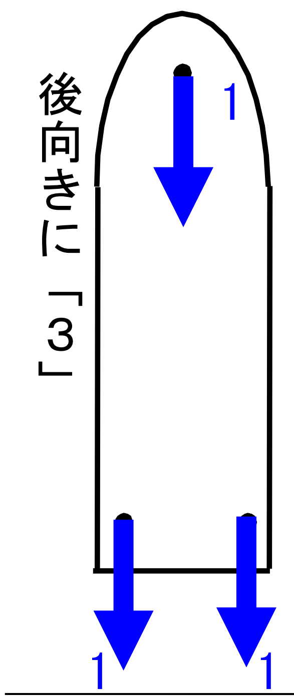
T のノルム

$$\|T\|^2 = \|\Sigma_1^{-1} U^T \tau\|^2 + \|c\|^2$$

推力の最小化($c = 0$ を代入)

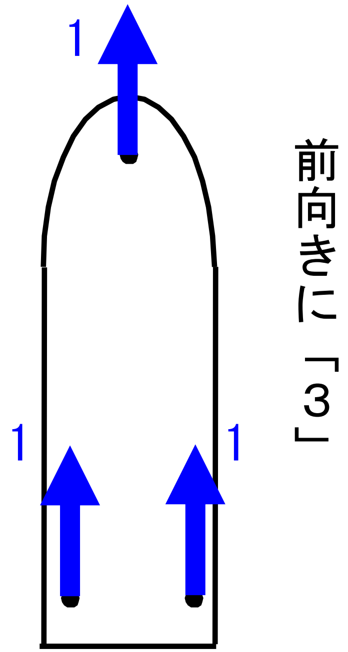
$$T^* = V_1 \Sigma_1^{-1} U^T \tau$$

従来法の特徴



推力最小化　により
各スラスタが均等に役割分担

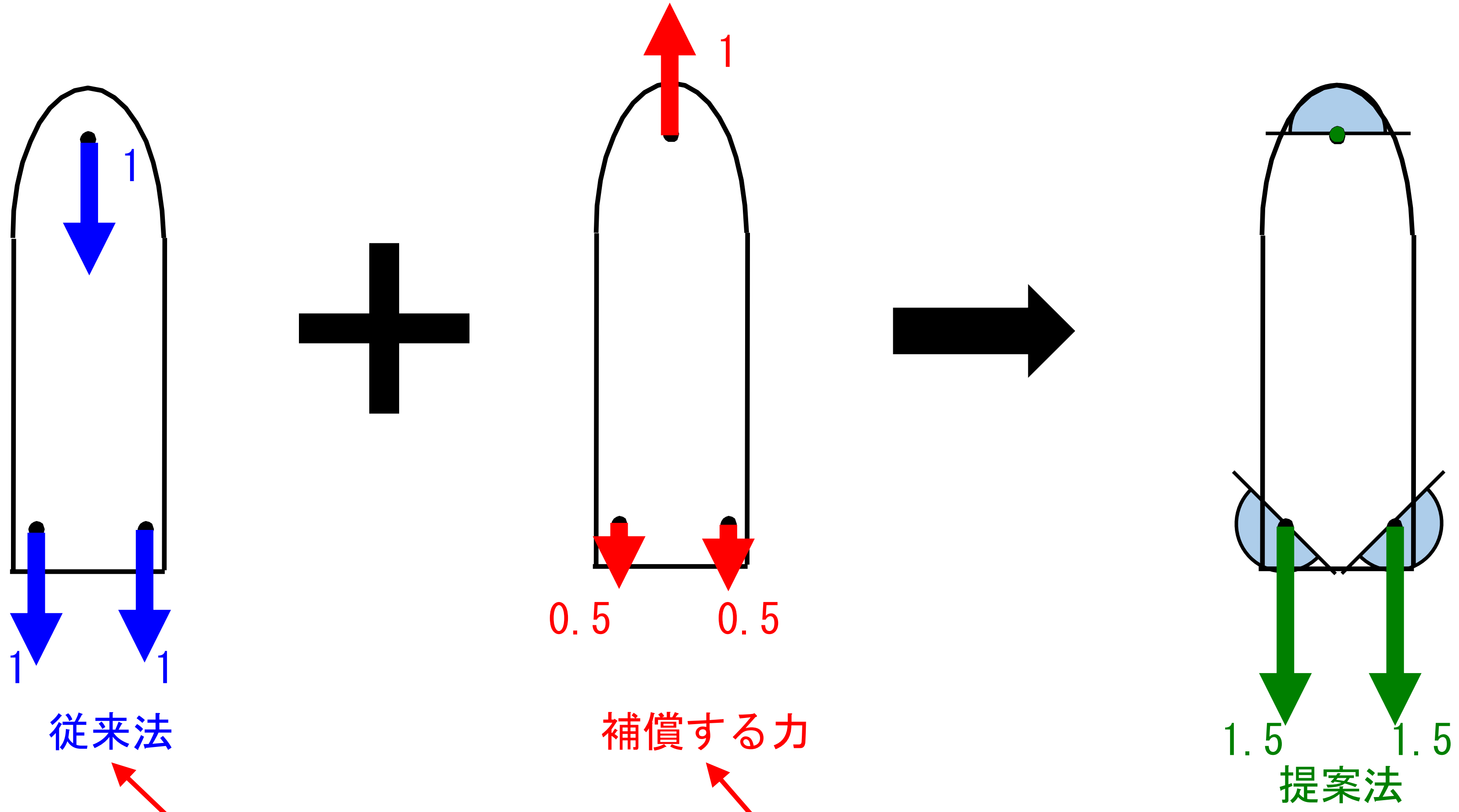
スラスタの向きが揃いやすい



推力指令の符号変化
に対しては
首振り角を変えて対応

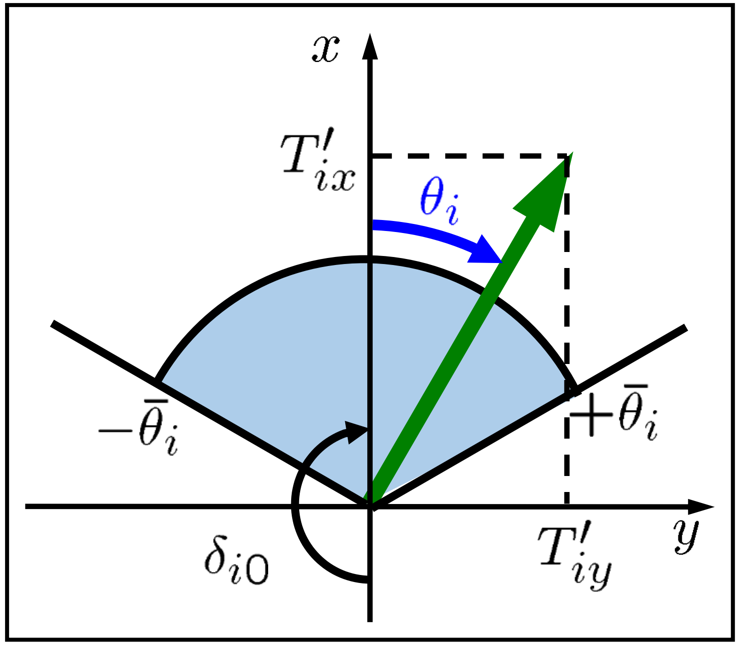
弱外乱中では
指令の符号変化が
多いため
首振り量が増大

首振角制限付き推力配分問題



$$V_1 \Sigma_1^{-1} U^T \tau + V_2 c = T^*$$

推力配分方程式の修正



$$T'_{ix} > 0,\ \ |T'_{iy}| \le T'_{ix} \tan\bar{\theta}_i$$

$$T'_{ix} = T_i \cos\theta_i,\ \ T'_{iy} = T_i \sin\theta_i$$

$$\theta_i = \delta_i - \delta_{i0}\ \ (|\theta_i| \le \bar{\theta}_i)$$

$$(i = 1, \cdots, n)$$

$$\underbrace{\left[\begin{array}{cccc} \cdots & \cos\delta_{i0} & -\sin\delta_{i0} & \cdots \\ \cdots & \sin\delta_{i0} & \cos\delta_{i0} & \cdots \\ \hline \cdots & \begin{array}{c}-y_i\cos\delta_{i0}\\ +x_i\sin\delta_{i0}\end{array} & \begin{array}{c}y_i\sin\delta_{i0}\\ +x_i\cos\delta_{i0}\end{array} & \cdots \end{array}\right]}_{A'(3\times 2n)} \underbrace{\begin{bmatrix} \vdots \\ T'_{ix} \\ T'_{iy} \\ \vdots \end{bmatrix}}_{T'(2n\times 1)} = \underbrace{\begin{bmatrix} X \\ Y \\ N \end{bmatrix}}_{\tau(3\times 1)}$$

最適化問題としての定式化

不等式制約付き２次計画問題

$$\min \ \|T'\|^2/2$$
such that
$$A'T' = \tau$$
$$T'_{ix} > 0, \ |T'_{iy}| \le T'_{ix} \tan \bar{\theta}_i$$

$$T'_{ix} = T_i \cos\theta_i, \ T'_{iy} = T_i \sin\theta_i$$
$$\theta_i = \delta_i - \delta_{i0} \ (|\theta_i| \le \bar{\theta}_i) \ (i = 1, \cdots, n)$$

$\Rightarrow$

$$\min \ \|c\|^2/2$$
such that
$$(T' = V'_1 \Sigma'^{-1}_1 U'^T \tau + V'_2 c)$$
$$B_{ex} c \le D_{ex} \tau$$

$$\begin{cases} (v^T_{2,2i} - v^T_{2,2i-1} \tan \bar{\delta}_i) c \le (-v^T_{1,2i} + v^T_{1,2i-1} \tan \bar{\delta}_i) \tau' \\ (-v^T_{2,2i} + v^T_{2,2i-1} \tan \underline{\delta}_i) c \le (v^T_{1,2i} - v^T_{1,2i-1} \tan \underline{\delta}_i) \tau' \\ -v^T_{2,2i-1} c \le v^T_{1,2i-1} \tau' \end{cases}$$
$$(i = 1, \cdots, n)$$

$$\underbrace{\begin{bmatrix} \sigma_1^{-1} u_1^T \\ \sigma_2^{-1} u_2^T \\ \sigma_3^{-1} u_3^T \end{bmatrix} \tau}_{\tau'}$$

近似解の検討

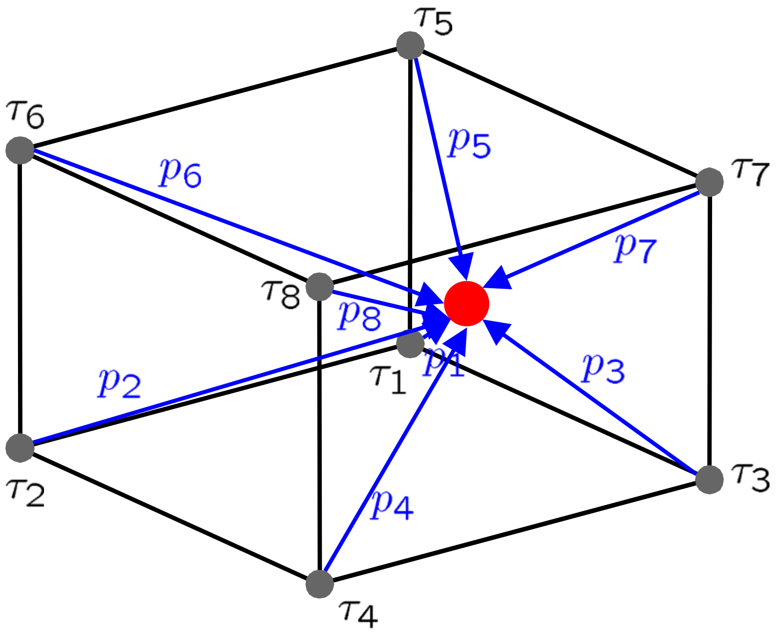
$$\boxed{\text{min } \|c\|^2/2 \text{ s.t. } B_{ex}c \leq D_{ex}\tau}$$

$$\Downarrow \tau(t) = p_1\tau_1 + \cdots + p_8\tau_8$$

$$\boxed{\text{min } \|c_i\|^2/2 \text{ s.t. } B_{ex}c_i \leq D_{ex}\tau_i \quad (i = 1, \cdots, 8)}$$

$$\Downarrow$$

$$c(t) = p_1c_1 + \cdots + p_8c_8$$



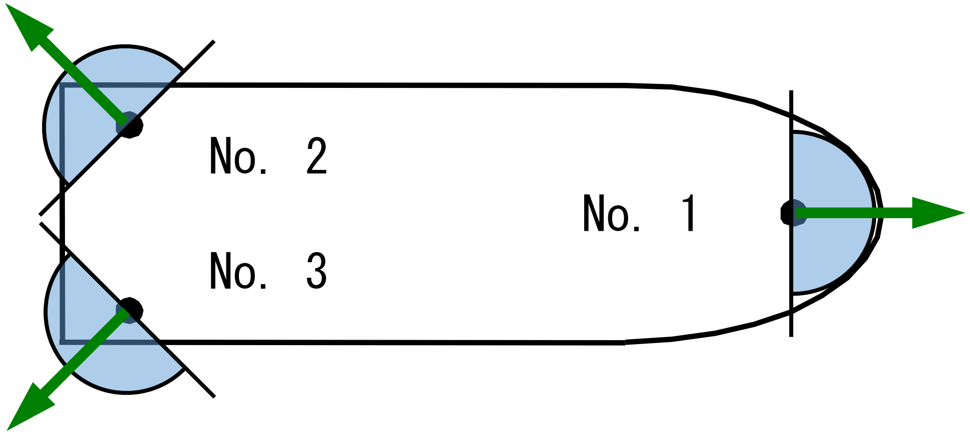
$$\underbrace{\frac{X_2-X}{X_2-X_1}\cdot\frac{Y_2-Y}{Y_2-Y_1}\cdot\frac{N_2-N}{N_2-N_1}}_{p_1(X,Y,N)}+\underbrace{\frac{X-X_1}{X_2-X_1}\cdot\frac{Y_2-Y}{Y_2-Y_1}\cdot\frac{N_2-N}{N_2-N_1}}_{p_2(X,Y,N)}+\underbrace{\frac{X_2-X}{X_2-X_1}\cdot\frac{Y-Y_1}{Y_2-Y_1}\cdot\frac{N_2-N}{N_2-N_1}}_{p_3(X,Y,N)}+\underbrace{\frac{X-X_1}{X_2-X_1}\cdot\frac{Y-Y_1}{Y_2-Y_1}\cdot\frac{N_2-N}{N_2-N_1}}_{p_4(X,Y,N)}+$$

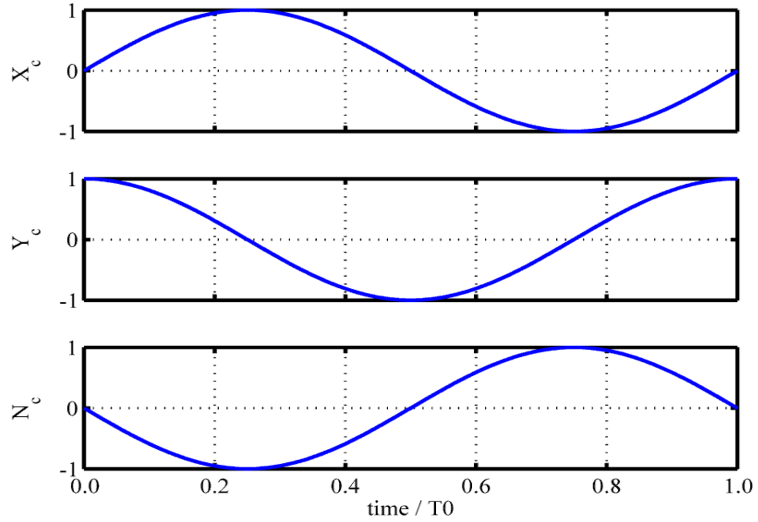
$$\underbrace{\frac{X_2-X}{X_2-X_1}\cdot\frac{Y_2-Y}{Y_2-Y_1}\cdot\frac{N-N_1}{N_2-N_1}}_{p_5(X,Y,N)}+\underbrace{\frac{X-X_1}{X_2-X_1}\cdot\frac{Y_2-Y}{Y_2-Y_1}\cdot\frac{N-N_1}{N_2-N_1}}_{p_6(X,Y,N)}+\underbrace{\frac{X_2-X}{X_2-X_1}\cdot\frac{Y-Y_1}{Y_2-Y_1}\cdot\frac{N-N_1}{N_2-N_1}}_{p_7(X,Y,N)}+\underbrace{\frac{X-X_1}{X_2-X_1}\cdot\frac{Y-Y_1}{Y_2-Y_1}\cdot\frac{N-N_1}{N_2-N_1}}_{p_8(X,Y,N)}=1$$

$$\tau_1=\begin{bmatrix}X_1\\Y_1\\N_1\end{bmatrix},\tau_2=\begin{bmatrix}X_2\\Y_1\\N_1\end{bmatrix},\tau_3=\begin{bmatrix}X_1\\Y_2\\N_1\end{bmatrix},\tau_4=\begin{bmatrix}X_2\\Y_2\\N_1\end{bmatrix}+,\tau_5=\begin{bmatrix}X_1\\Y_1\\N_2\end{bmatrix},\tau_6=\begin{bmatrix}X_2\\Y_1\\N_2\end{bmatrix},\tau_7=\begin{bmatrix}X_1\\Y_2\\N_2\end{bmatrix},\tau_8=\begin{bmatrix}X_2\\Y_2\\N_2\end{bmatrix}$$

$$\left(\begin{array}{lll} X_1=\begin{cases}X_{\text{min}} & (X<0)\\ 0 & (X\geq 0)\end{cases}, & Y_1=\begin{cases}Y_{\text{min}} & (Y<0)\\ 0 & (Y\geq 0)\end{cases}, & N_1=\begin{cases}N_{\text{min}} & (N<0)\\ 0 & (N\geq 0)\end{cases}\\ X_2=\begin{cases}0 & (X<0)\\ X_{\text{max}} & (X\geq 0)\end{cases}, & Y_2=\begin{cases}0 & (Y<0)\\ Y_{\text{max}} & (Y\geq 0)\end{cases}, & N_2=\begin{cases}0 & (N<0)\\ N_{\text{max}} & (N\geq 0)\end{cases}\end{array}\right)$$

推力配分シミュレーション

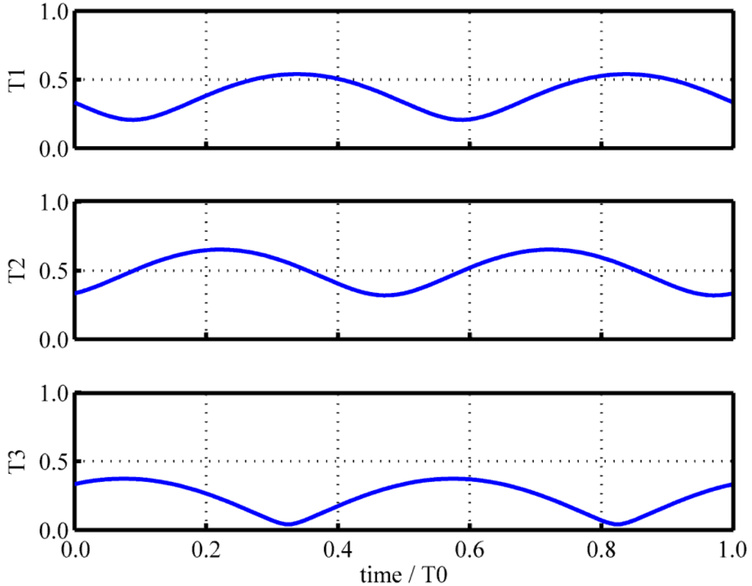


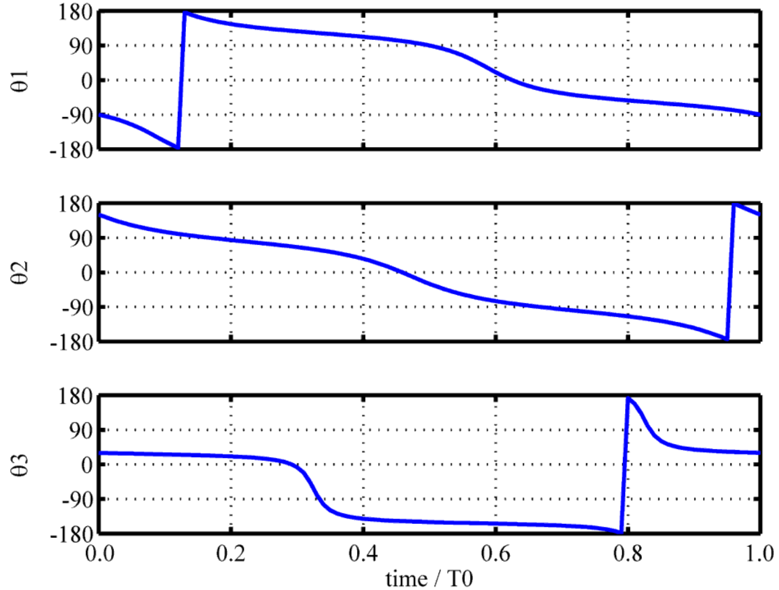


要求推力

従来法（首振角制限無しの推力最小化）

推力指令





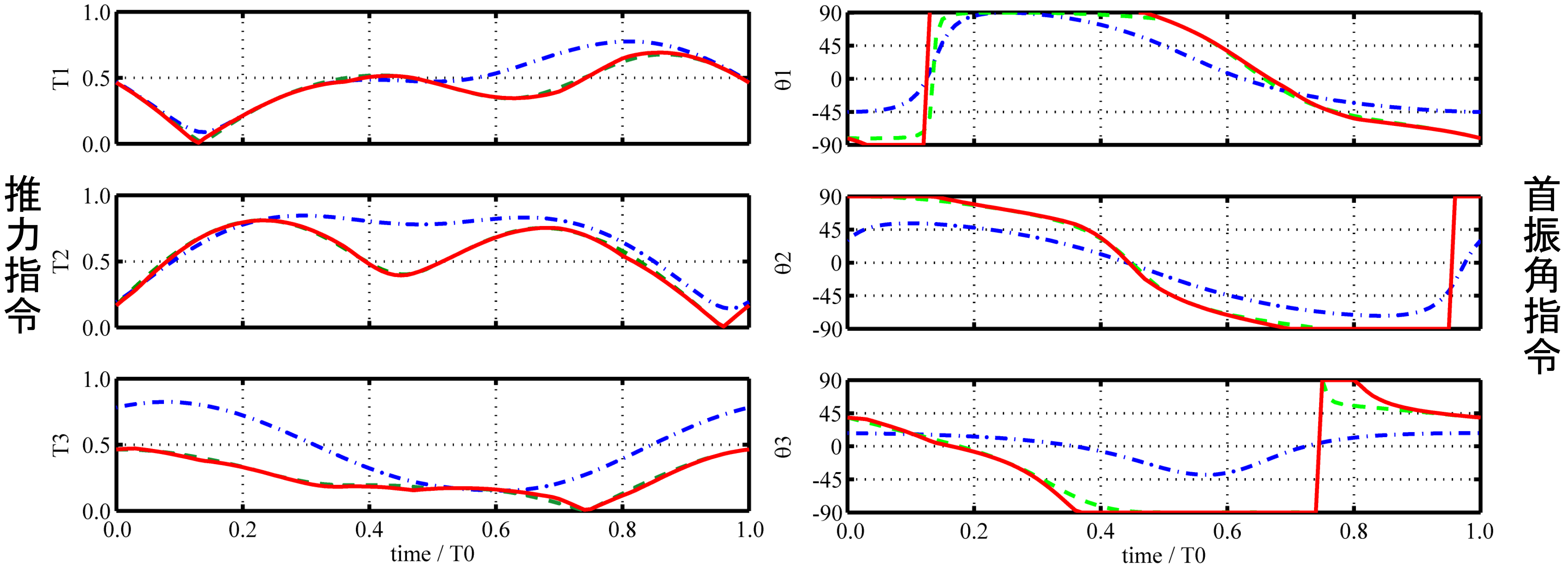
首振角指令

推力配分シミュレーション

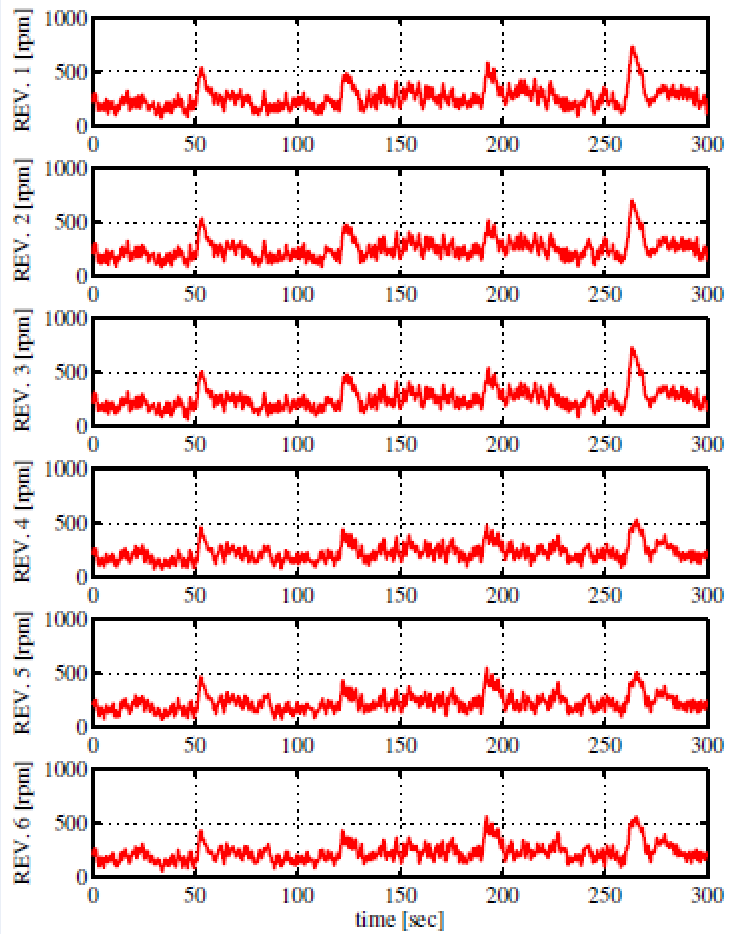
①近似解 $\min\ \|c_i\|^2/2 \text{ s.t. } B_{ex}c_i \leq D_{ex}\tau_i\ (i=1,\cdots,8)$

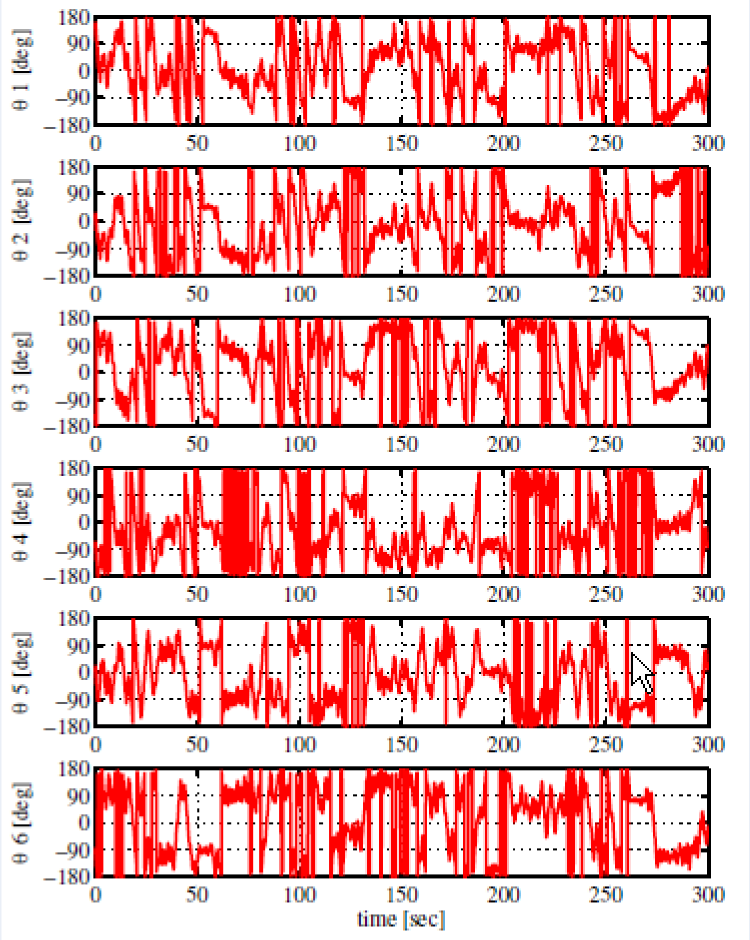
②暫定解 $\min\ \|c\|^2/2 \text{ s.t. } \begin{bmatrix} B_{ex} \\ \vdots \\ B_{ex} \end{bmatrix} c \leq \begin{bmatrix} D_{ex}\tau_1 \\ \vdots \\ D_{ex}\tau_8 \end{bmatrix}$

③最適解 $\min\ \|c\|^2/2 \text{ s.t. } B_{ex}c \leq D_{ex}\tau$



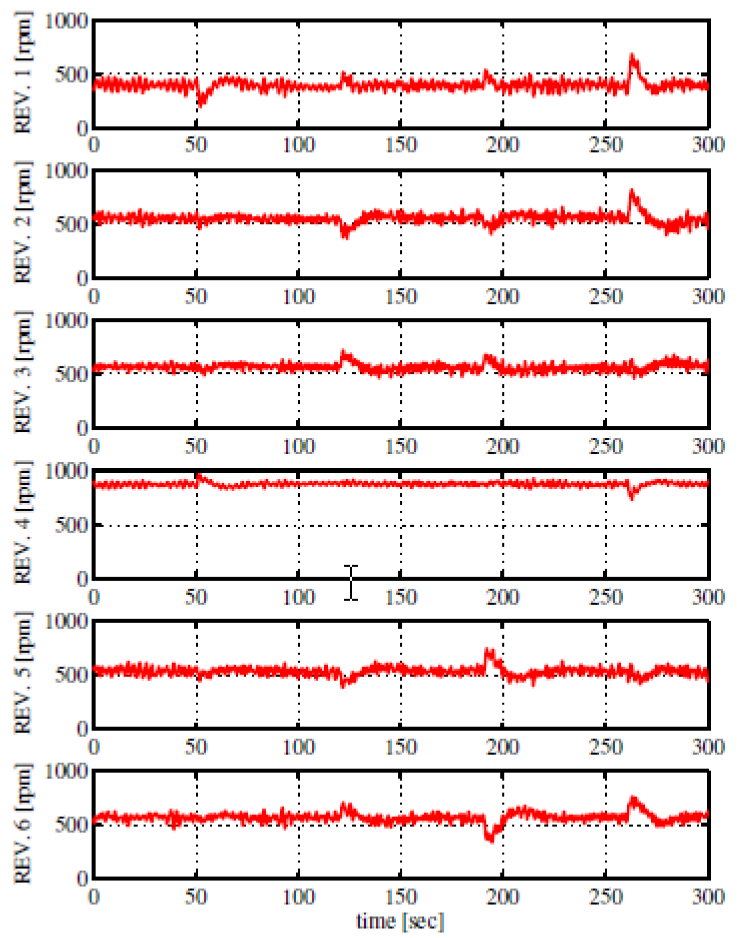
従来法(首振角の制約なし)

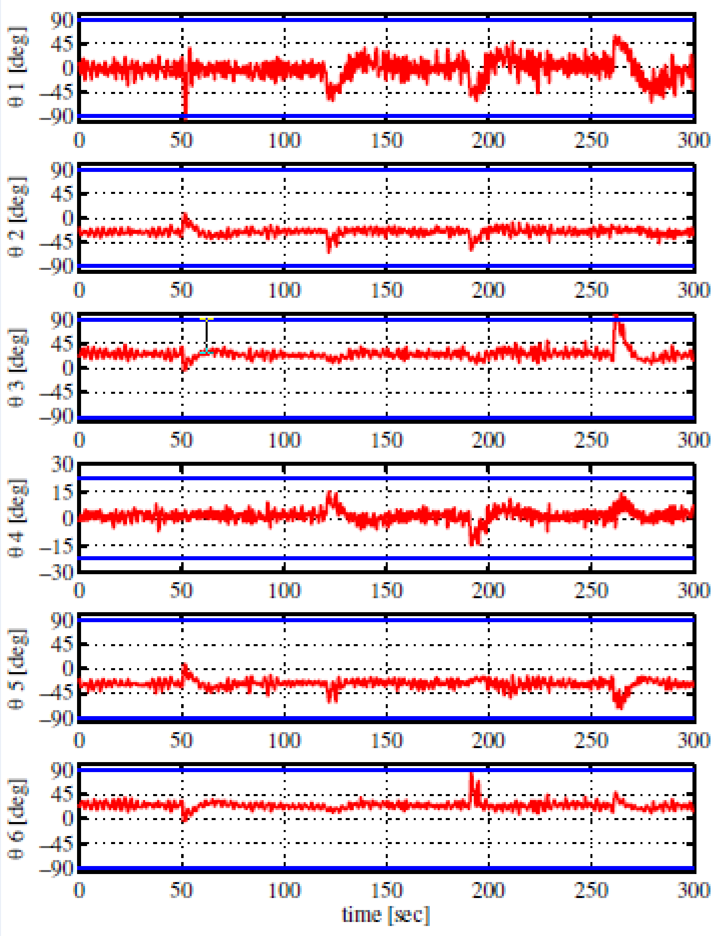




スラストカは適切	スラスタが回転する

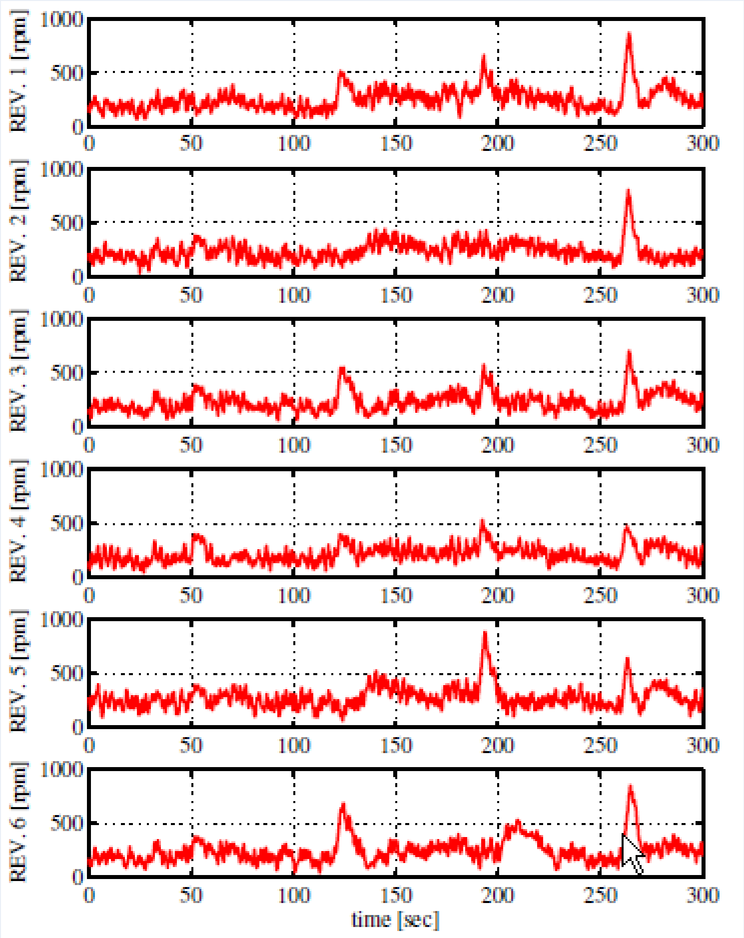
近似解（首振角の制約有り）

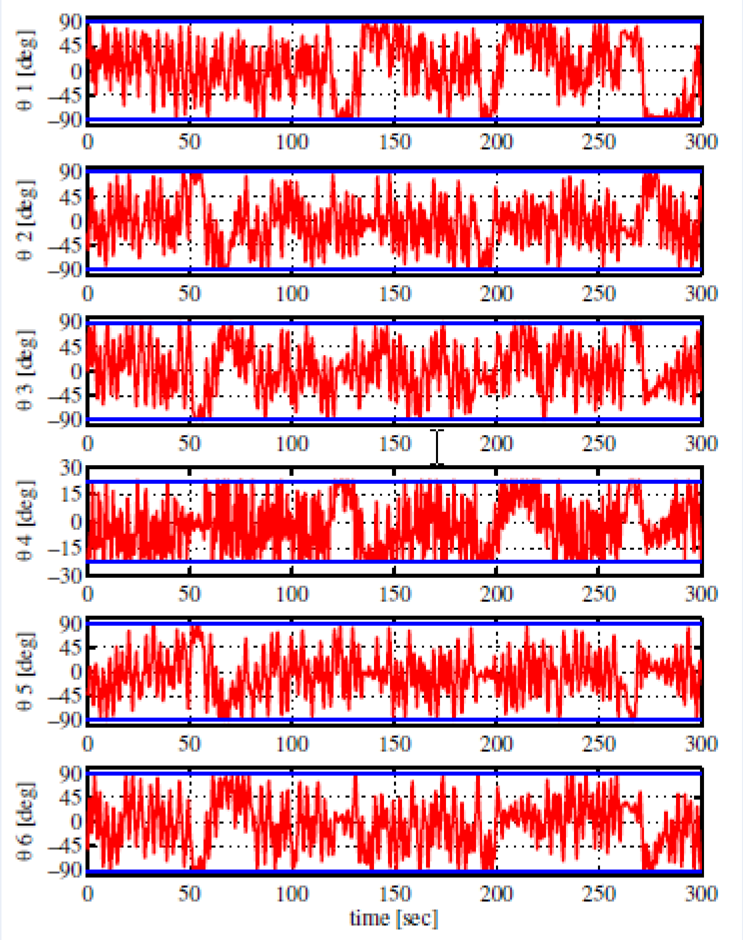




スラストカが過大	首振りは余り行わない

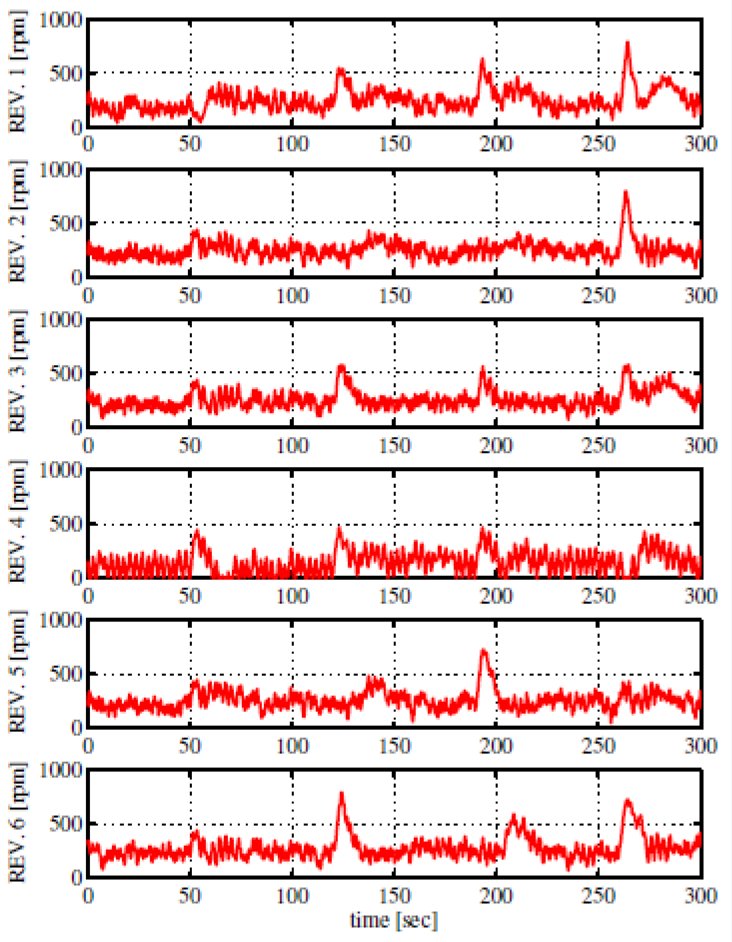
暫定解（首振角の制約有り）

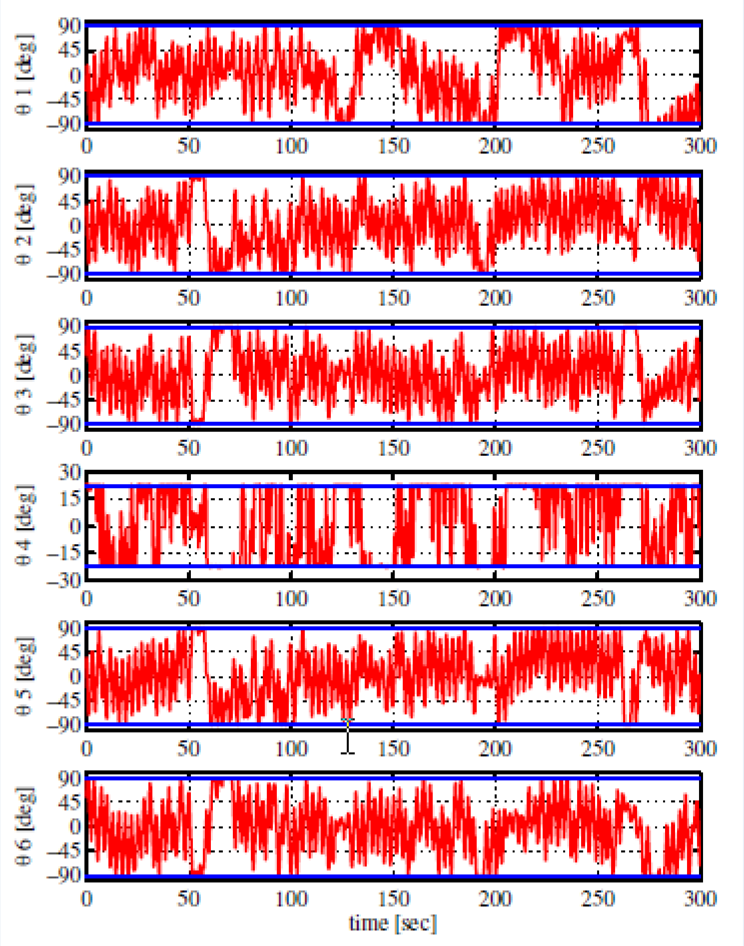




スラスト力は適切	首振角制約をほぼ満たす

最適解（首振角の制約有り）





スラスト力は暫定法と同様適切	首振角制約を厳守

模型実験

三角形軌道

楕円軌道

最適解の検討

等式制約付き２次計画問題

$$\min \ \|c\|^2/2$$
such that
$$Bc - D\tau + s = 0 \ (s \geq 0)$$

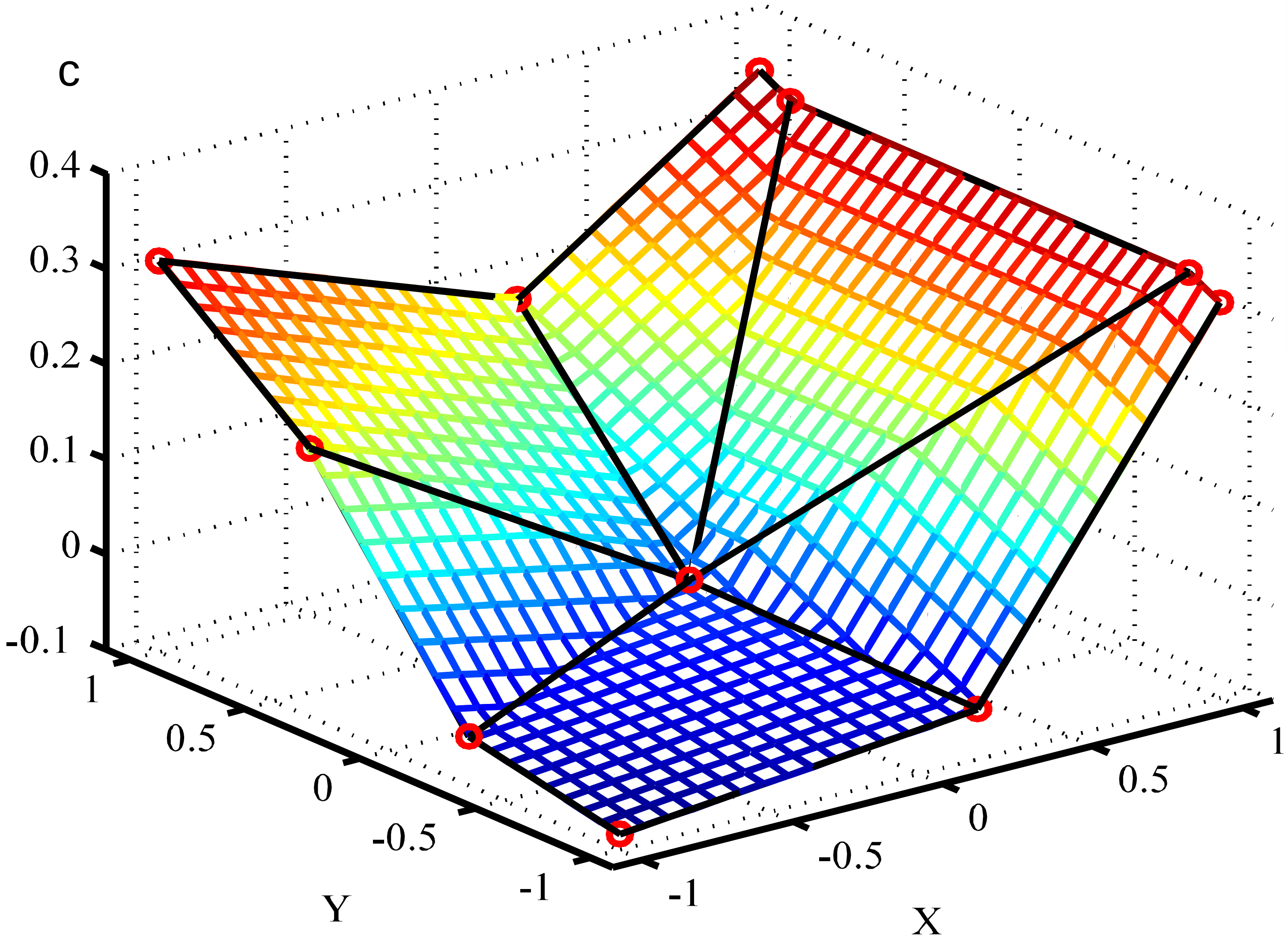
$$\min \ \|c\|^2/2$$
such that
$$Bc \leq D\tau$$

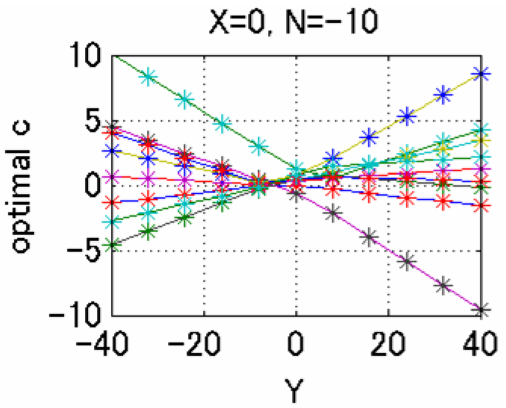
２次計画問題におけるKKT(Karush-Kuhn-Tucker)条件

$$L = \frac{1}{2}c^T c + y^T(Bc - d + s) - z^T s$$
$$\nabla_c L = c + B^T y = 0$$
$$\nabla_s L = y - z = 0$$
$$\nabla_y L = Bc - d(\tau) + s = 0$$
$$\nabla_z L = -s \leq 0$$
$$z \geq 0$$
$$z^T s = 0$$

$$c = -B^T z$$
$$\begin{cases} \begin{bmatrix} -BB^T & I \end{bmatrix} \begin{bmatrix} z \\ s \end{bmatrix} = D\tau \\ z^T s = 0, \ z \geq 0, \ s \geq 0 \end{cases}$$

最適解の区分的線形性





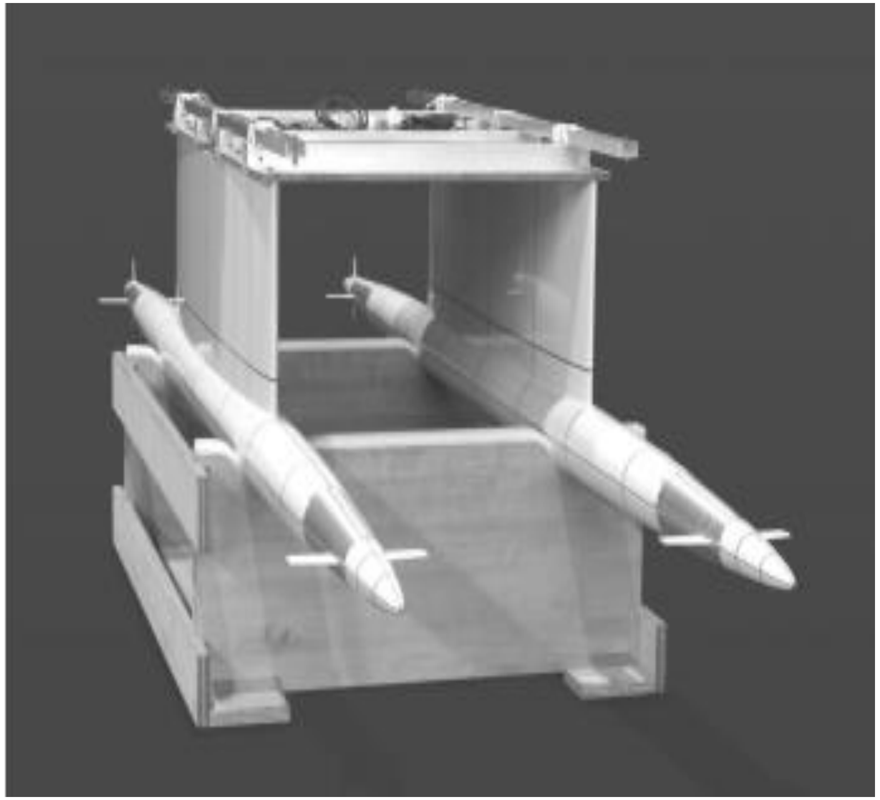
N=1としたときのX-Y平面上における最適解の区分的線形性

【5】ロバスト性への対応（SM制御）

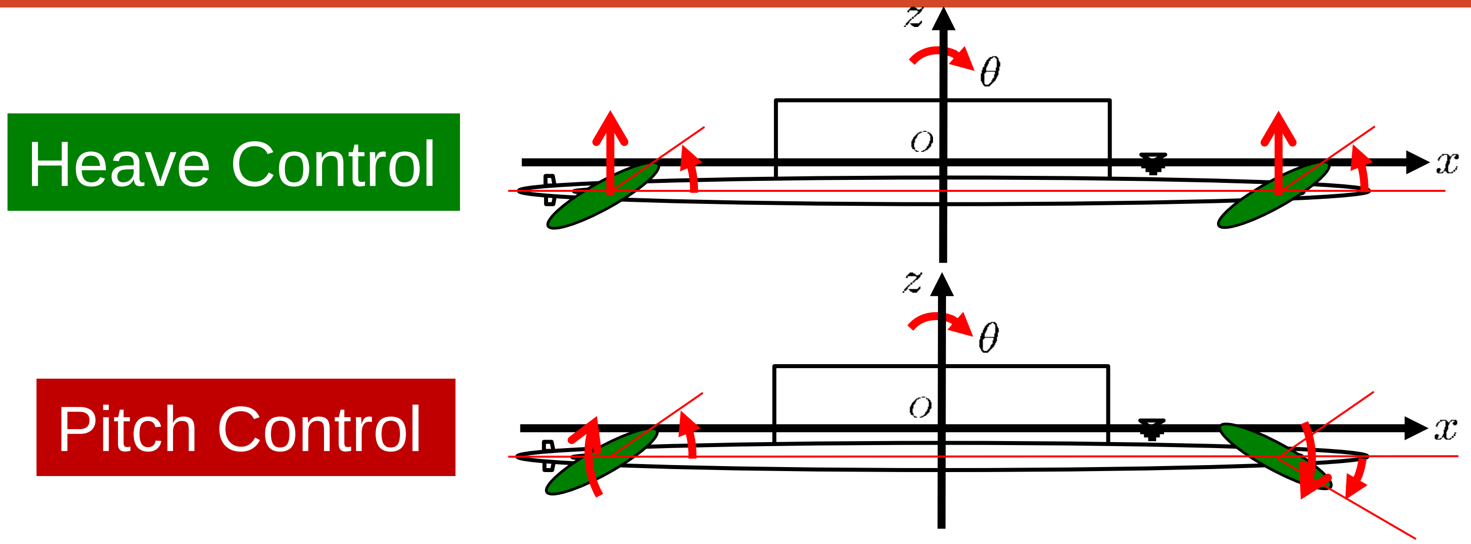
- 揺れが小さい船の実現のために
- スライディングモード制御とは？
- 内部モデル原理の適用？

SWATH(Small Waterplane-Area Twin Hull)

1) 吉田基樹, 岩下英嗣, 木原 一, 木下 健: Resonance-Free SWATHの概念設計と耐航性能, 日本船舶海洋工学会論文集, 第10号, pp. 73-81, 2009.
2) 木原 一, 吉田基樹, 岩下英嗣, 木下 健: Resonance-Free SWATHの運動応答解析, 日本船舶海洋工学会論文集, 第10号, pp. 83-96, 2009.
3) 吉田基樹, 梶 正和, 木原 一, 岩下英嗣, 木下 健: Resonance-Free SWATHの駆動finによる運動制御, 日本船舶海洋工学会論文集, 第12号, pp. 79-88, 2010.
4) 吉田基樹, 岩下英嗣, 木原 一, 木下 健: Resonance-Free SWATHの波浪中運動性能, 日本船舶海洋工学会論文集 第12号, pp. 89-99, 2010.
5)吉田基樹, 梶原宏之, 神田雅光：波浪中Resonance-Free SWATHの水中翼による非線形制御,日本船舶海洋工学会論文集, 第15号, pp. 119-128, 2015.



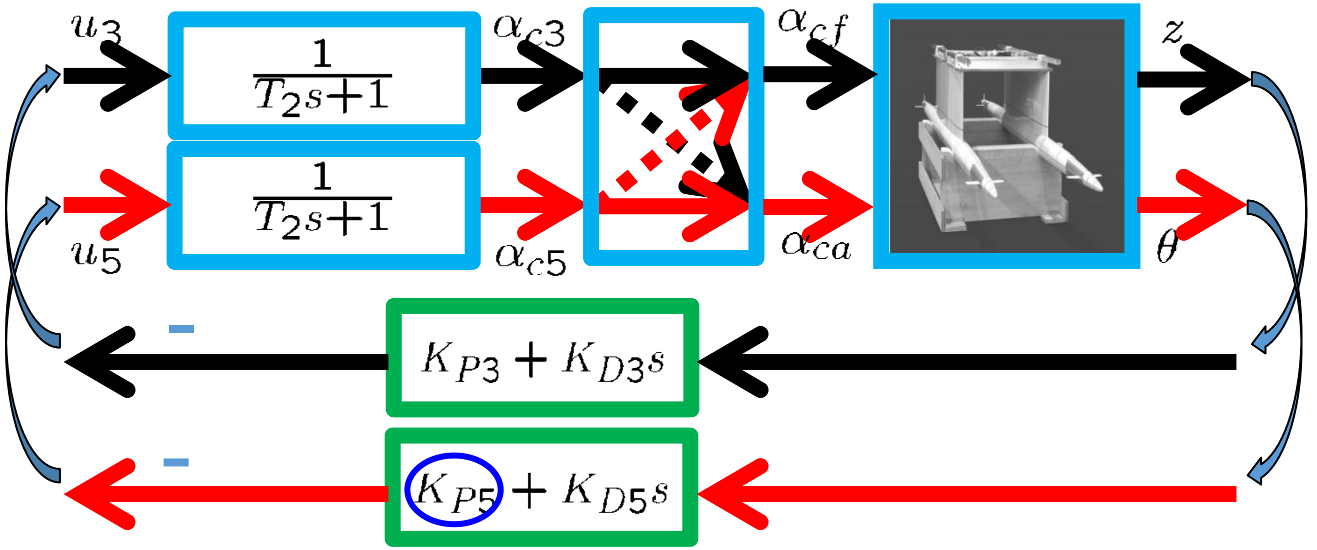
PD Contr. of RFS(Resonance Free SWATH)



Restoring Coefficients		
C33	N/m	464.01
C35	N/rad	0
C53	Nm/m	0
C55	Nm/rad	-4.25

How to control RFS by using the fins?

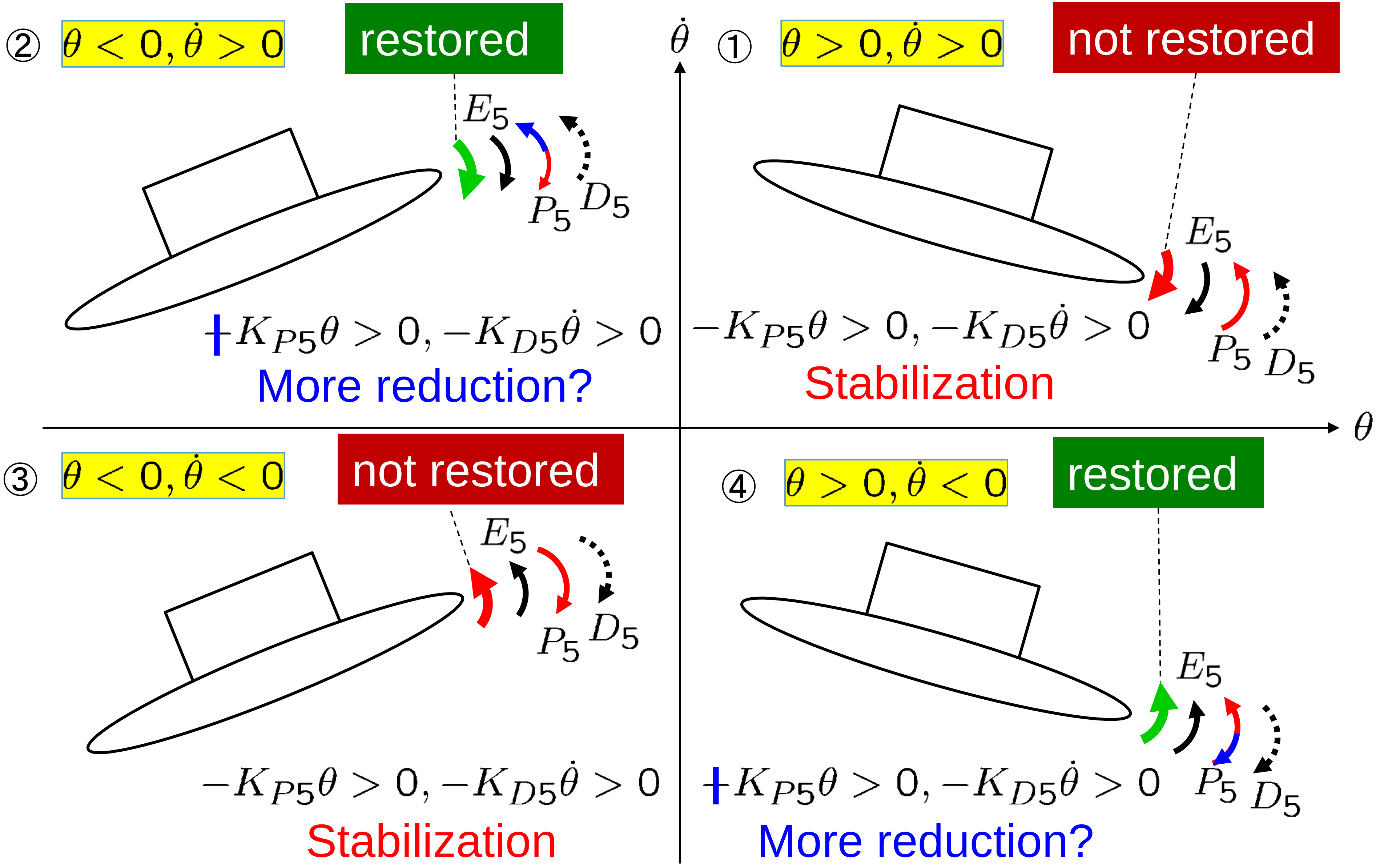
$$\begin{bmatrix} \alpha_{cf} \\ \alpha_{ca} \end{bmatrix} = -\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha_{c3} \\ \alpha_{c5} \end{bmatrix}$$



Control Gains		
KP3	kg/s^2	0
KD3	kg/s	214
KP5	kg/s^2	99
KD5	kg/s	112
T2	s	0.07

Pitch Reduction by Switching

	θ	$\dot{\theta}$	$\theta\dot{\theta}$	stability	E_5	$-K_P\theta$	$-K_D\dot{\theta}$
1	>0	>0	>0	diverge	>0	<0	<0
2	<0	>0	<0	restored	>0	$\underline{>0 \Rightarrow <0}$	<0
3	<0	<0	>0	diverge	<0	>0	>0
4	>0	<0	<0	restored	<0	$\underline{<0 \Rightarrow >0}$	>0



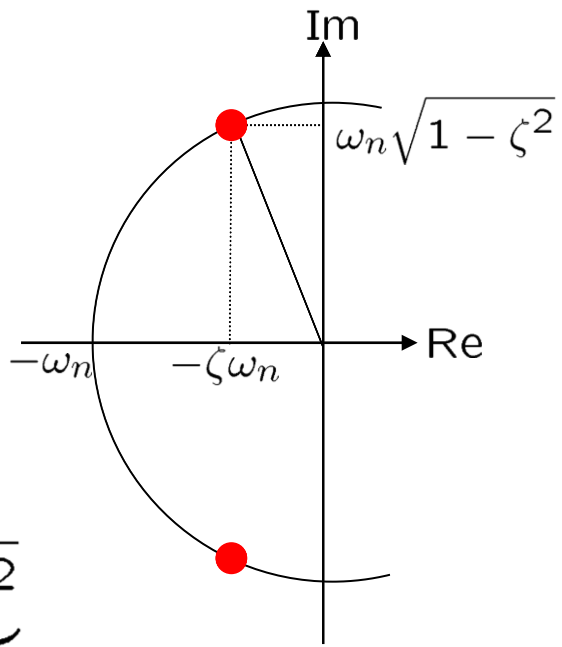
Oscillated 2nd–order system

- 2nd-order system

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}}_{B} u \quad (\zeta < 1)$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x$$

ζ:damping coef., ω_n:natural angular freq.



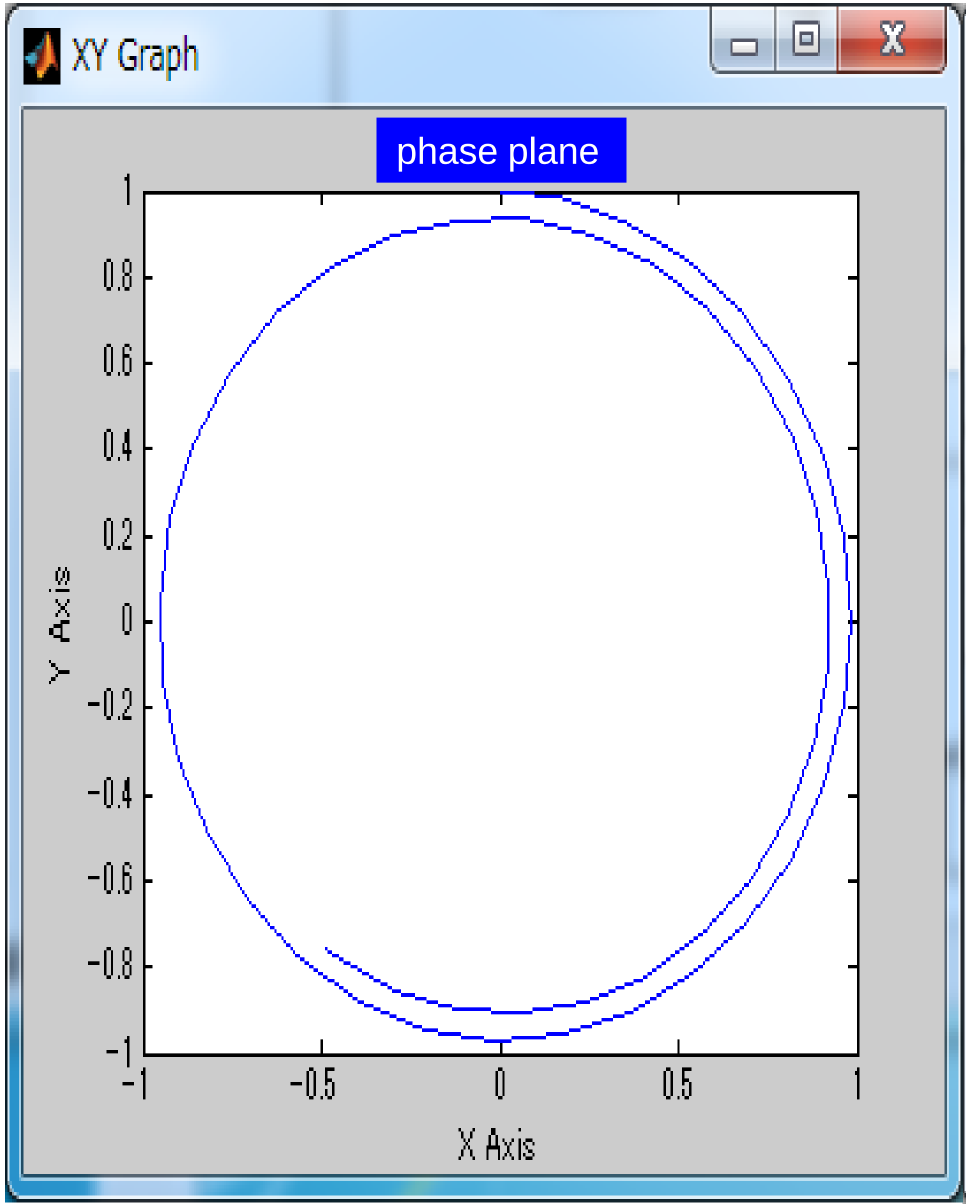
- eigenvalues of A: $\lambda = \underbrace{-\zeta\omega_n}_{\lambda_R} \pm j \underbrace{\omega_n\sqrt{1-\zeta^2}}_{\lambda_I}$

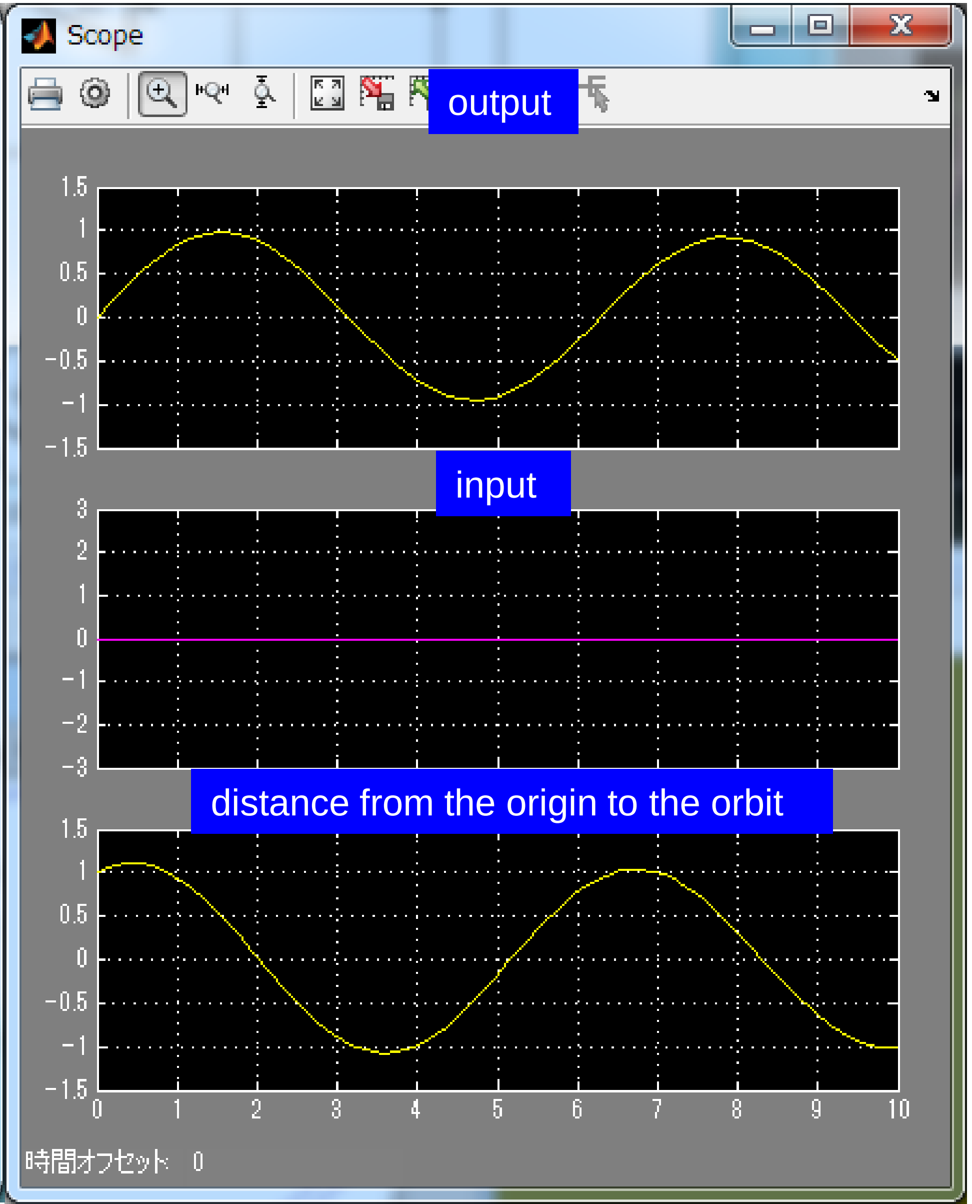
- impluse resp.: $G(t) = \frac{\omega_n^2}{\lambda_I} e^{\lambda_R t} \sin \lambda_I t$

- step resp.: $y(t) = 1 - \frac{\omega_n}{\lambda_I} e^{\lambda_R t} \sin(\lambda_I t - \tan^{-1} \frac{\lambda_I}{\lambda_R})$

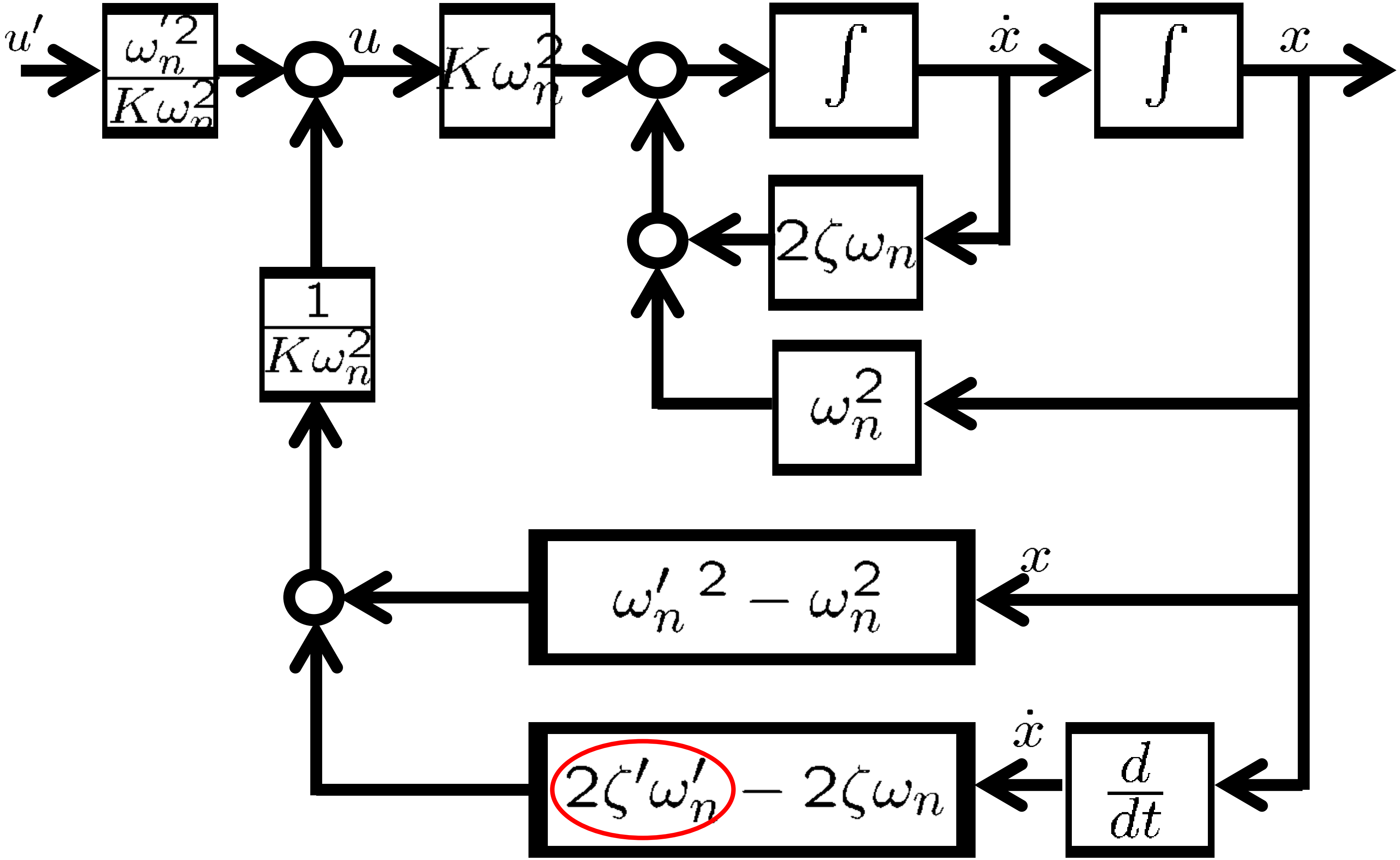
- $(T_p, 1 + p_0) = \left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, 1 + \exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) \right)$

Simulation Under No Control



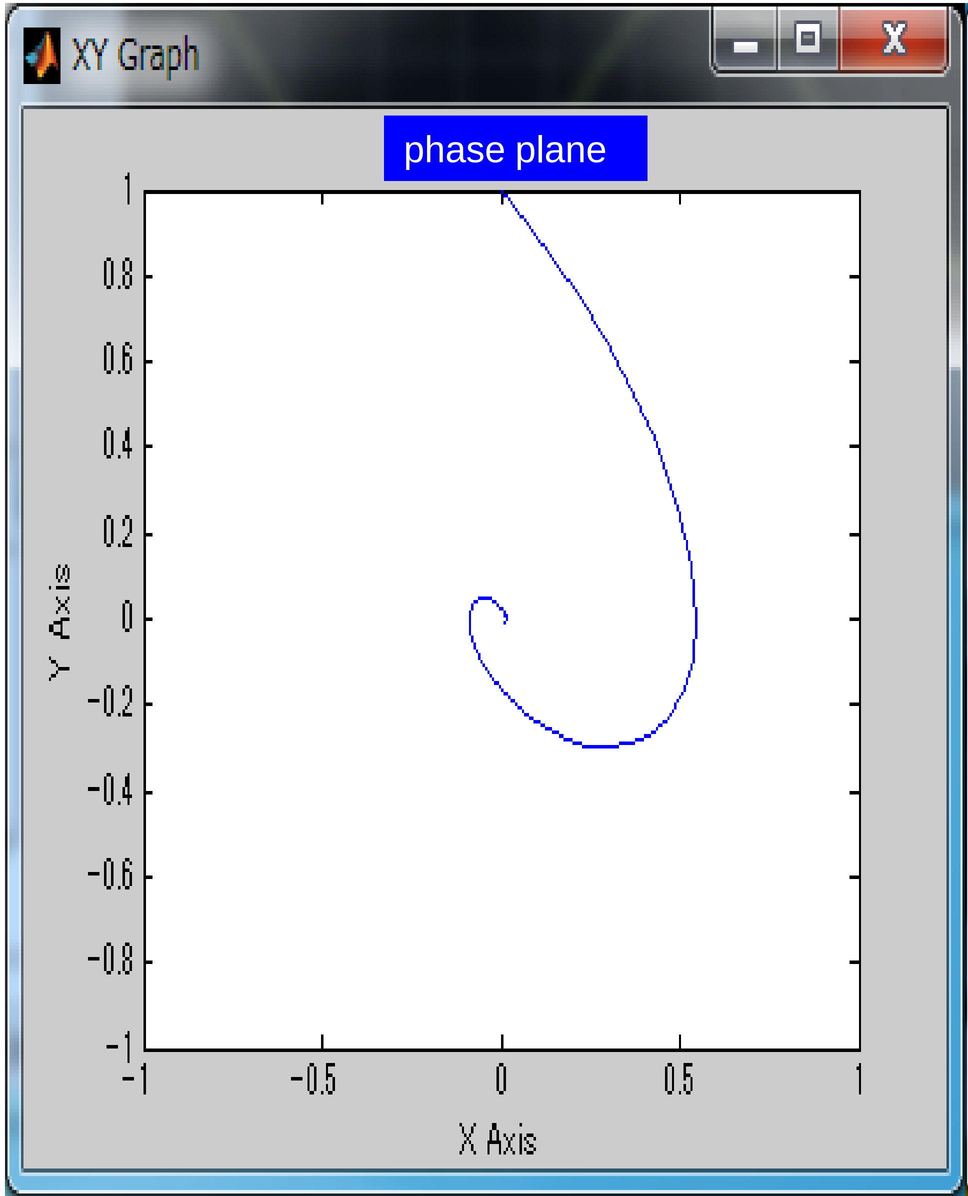


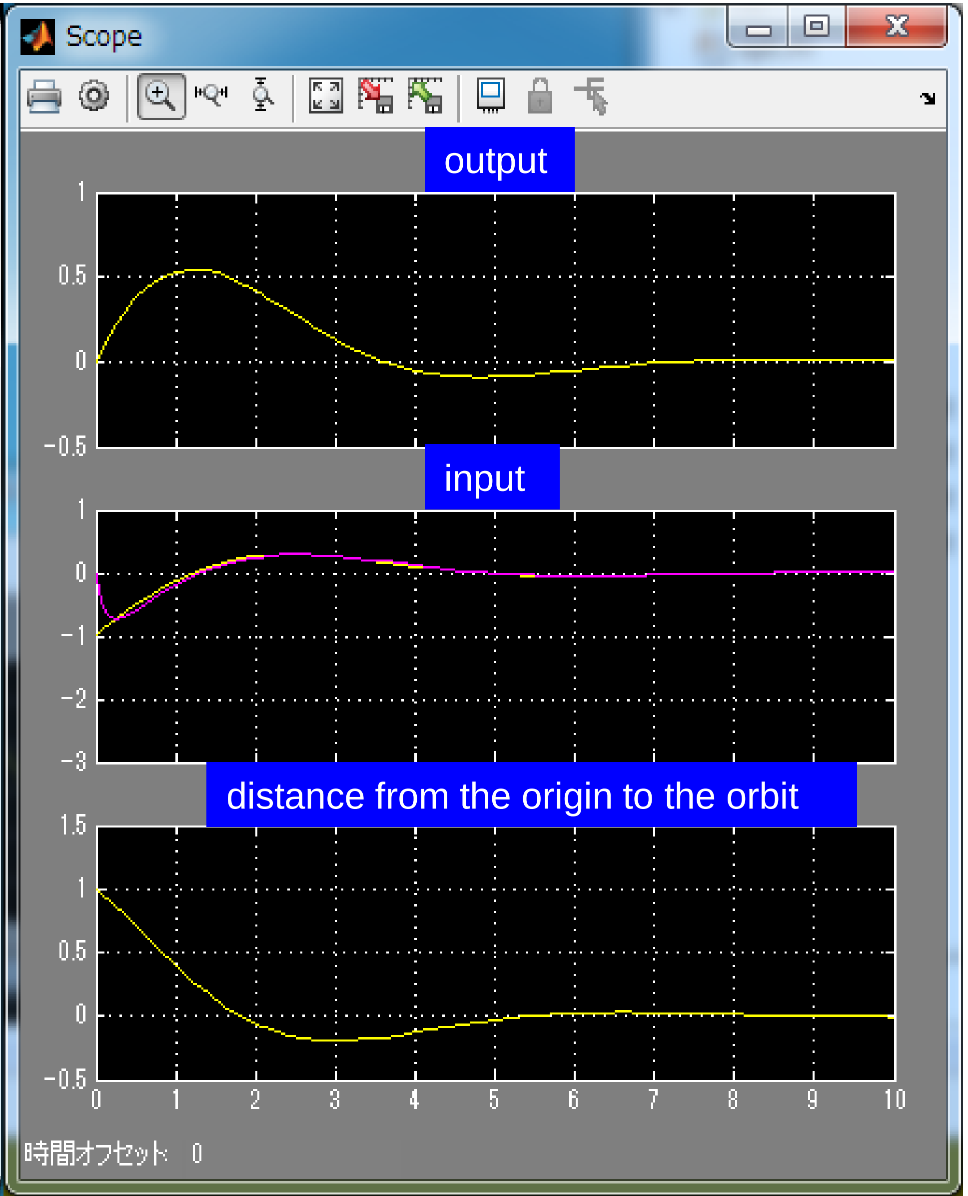
PD Control



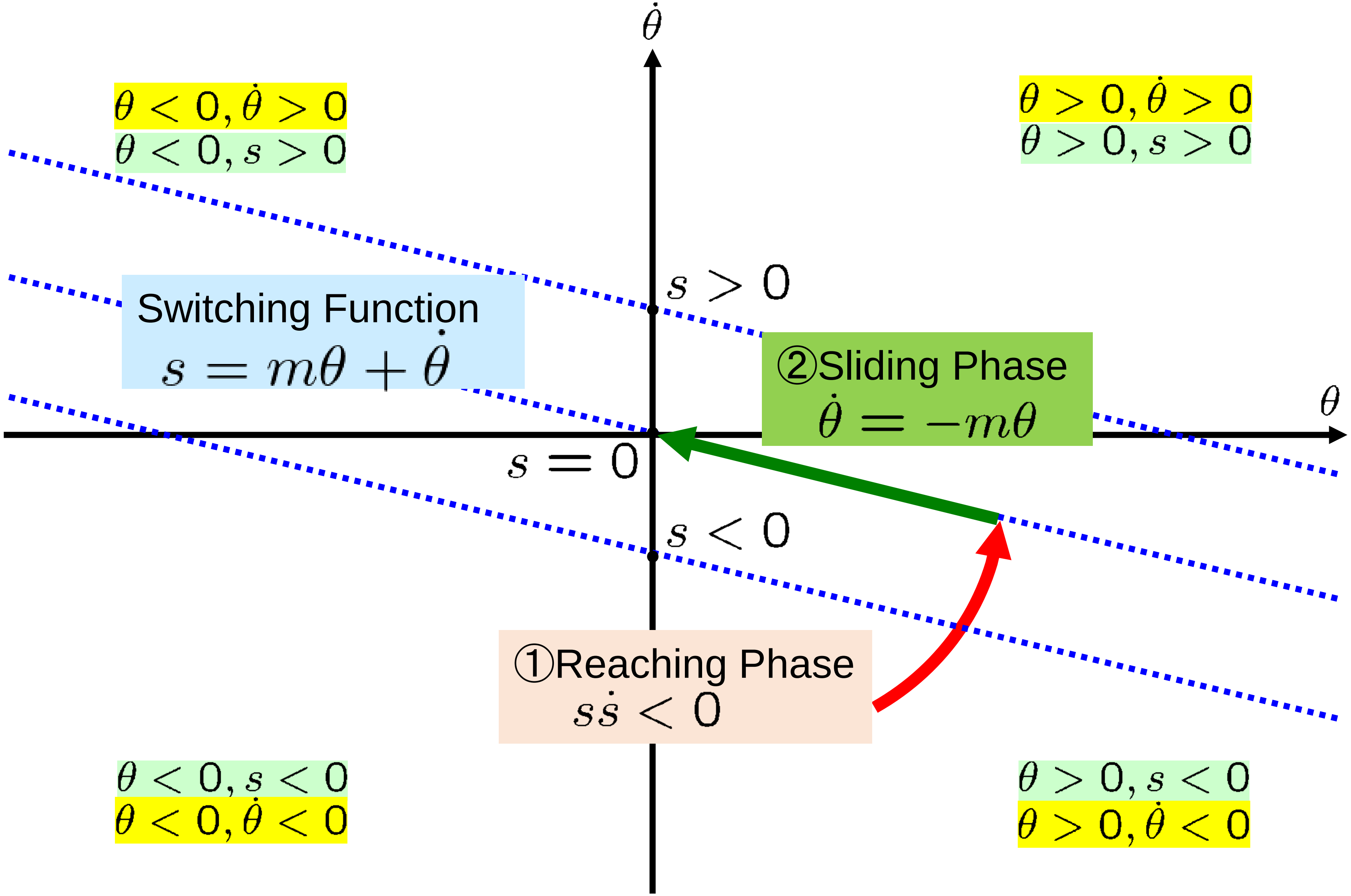
u'
$\frac{\omega_n'^2}{K\omega_n^2}$
u
$K\omega_n^2$
$\int$
$\dot{x}$
$\int$
x
$2\zeta\omega_n$
$\frac{1}{K\omega_n^2}$
ω_n^2
$\omega_n'^2 - \omega_n^2$
x
$\dot{x}$
$2\zeta'\omega_n' - 2\zeta\omega_n$
$\frac{d}{dt}$

Simulation Under PD Control





SMC (Sliding Mode Control)



SMC by PD Gain Switching

1° State Equation:

$$\underbrace{\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ w_2 \end{bmatrix}}_{w}$$

2° Swiching Function: $s = \underbrace{\begin{bmatrix} m & 1 \end{bmatrix}}_{S}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$

3° Sliding-Mode Control:

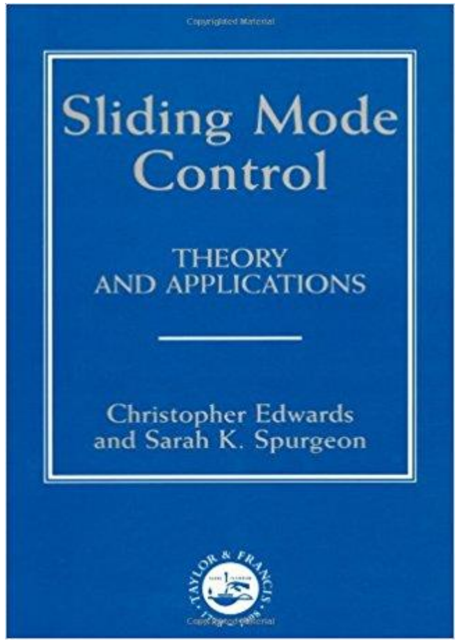
$$u = -\underbrace{1}_{(SB)^{-1}}\left(\underbrace{\begin{bmatrix} a_1 & m + a_2 \end{bmatrix}}_{SA} x - \underbrace{\begin{bmatrix} k_1 & k_2 \end{bmatrix}}_{K} x + \eta \mathrm{sgn}(s)\right)$$

where $k_i = \begin{cases} k_i^- < 0 & (sx_i > 0) \\ k_i^+ > 0 & (sx_i < 0) \end{cases} \quad (i = 1, 2), \;\; \eta > |w_2|$

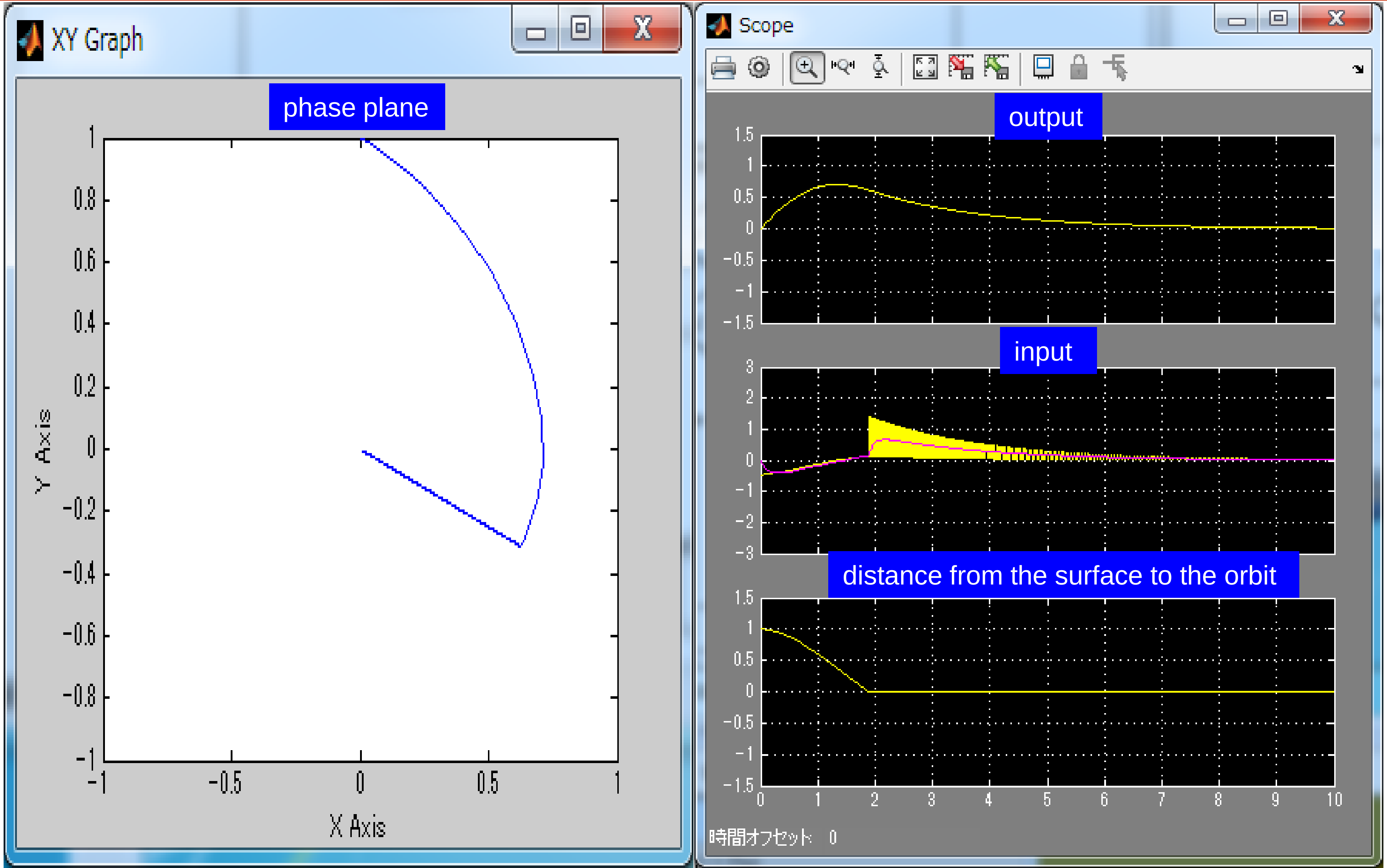
PD gain switching

4° Lyapnov Function: $V = \frac{1}{2}s^2$

$$\begin{aligned} \dot{V} &= s\dot{s} = sS(Ax + Bu + w) \\ &= s\{SAx - (SB)(SB)^{-1}(SAx - Kx + \eta \mathrm{sgn}(s))\} + Sw \\ &= s(k_1 x_1 + k_2 x_2 - \eta \mathrm{sgn}(s) + w_2) \\ &\leq \underbrace{k_1 \cdot s x_1}_{<0} + \underbrace{k_2 \cdot s x_2}_{<0} - \underbrace{(\eta - |w_2|)}_{<0} |s| < 0 \end{aligned}$$



SMC by Proportional Gain Switching



SMC by Switching Action

1° State Equation:

$$\underbrace{\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ w_2 \end{bmatrix}}_{w}$$

2° Switching Function: $s = \underbrace{\begin{bmatrix} m & 1 \end{bmatrix}}_{S}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$

3° Sliding-Mode Control:

$$u = -(SB)^{-1}\left((SA - \phi S)x + \eta \underbrace{\frac{p_2 s}{|p_2 s| + \epsilon}}_{\simeq \mathrm{sgn}(s)}\right)$$

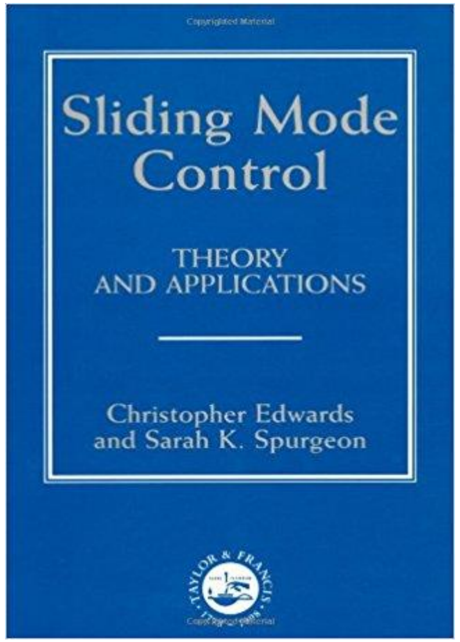
where $p_2 = -\dfrac{1}{2\phi} > 0,\ \ \eta > |w_2|$

Switching action

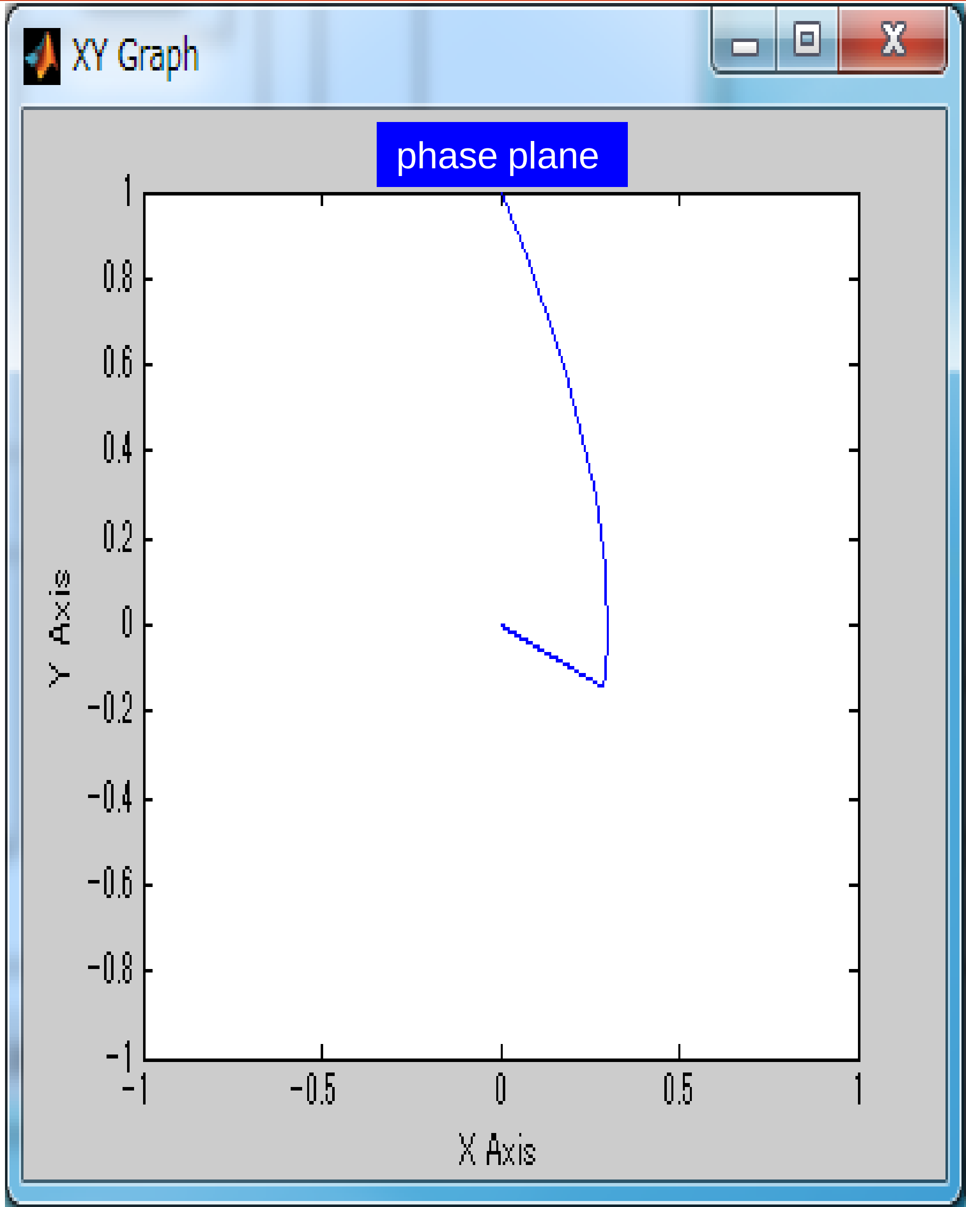
4° Lyapnov Function: $V = p_2 s^2$

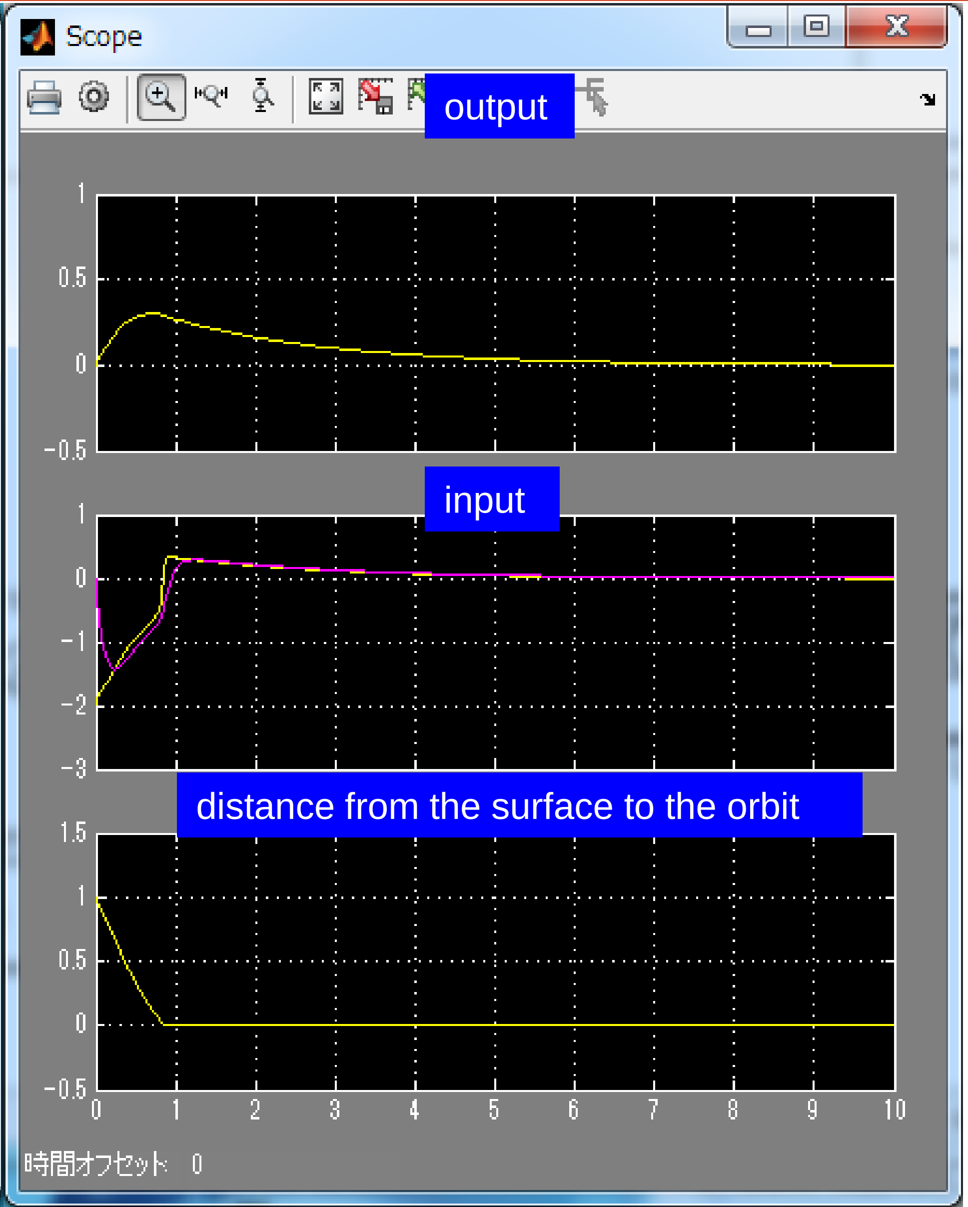
$$\dot{V} = 2p_2 s\dot{s} = 2p_2 s\left(\phi s - \eta\frac{p_2 s}{|p_2 s|} + w_2\right) = (2p_2\phi)s^2 - 2\eta\frac{p_2^2 s^2}{|p_2 s|} + 2p_2 s w_2$$

$$= -s^2 - 2\eta|p_2 s| + 2p_2 s w_2 \leq -s^2 - 2|p_2 s|\underbrace{(\eta - |w_2|)}_{<0} < 0$$



SMC by Switching Action





PDC/SMC1,2

PDC for the heave control loop :

$$T_2\dot{\alpha}_{c3} + \alpha_{c3} = -\frac{K_{P3}}{F_{fin}C_{La}}z - \frac{\rho_D K_{D3}}{F_{fin}C_{La}}\dot{z}$$

PDC for the pitch control loop :

$$T_2\dot{\alpha}_{c5} + \alpha_{c5} = -\frac{K_{P5}}{F_{fin}C_{La}\ell_0}\theta - \frac{\rho_D K_{D5}}{F_{fin}C_{La}\ell_0}\dot{\theta}$$

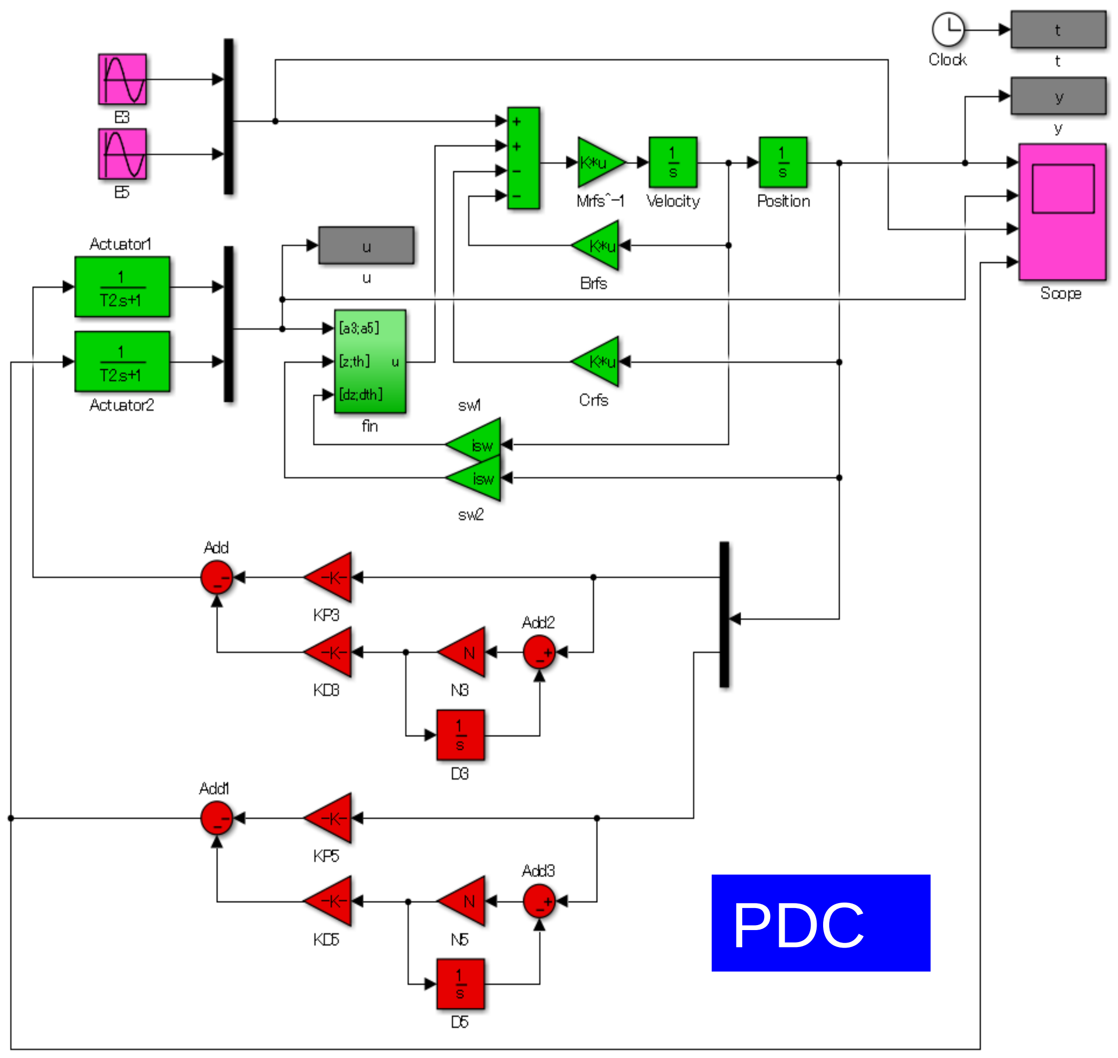
SMC1 for the pitch control loop :

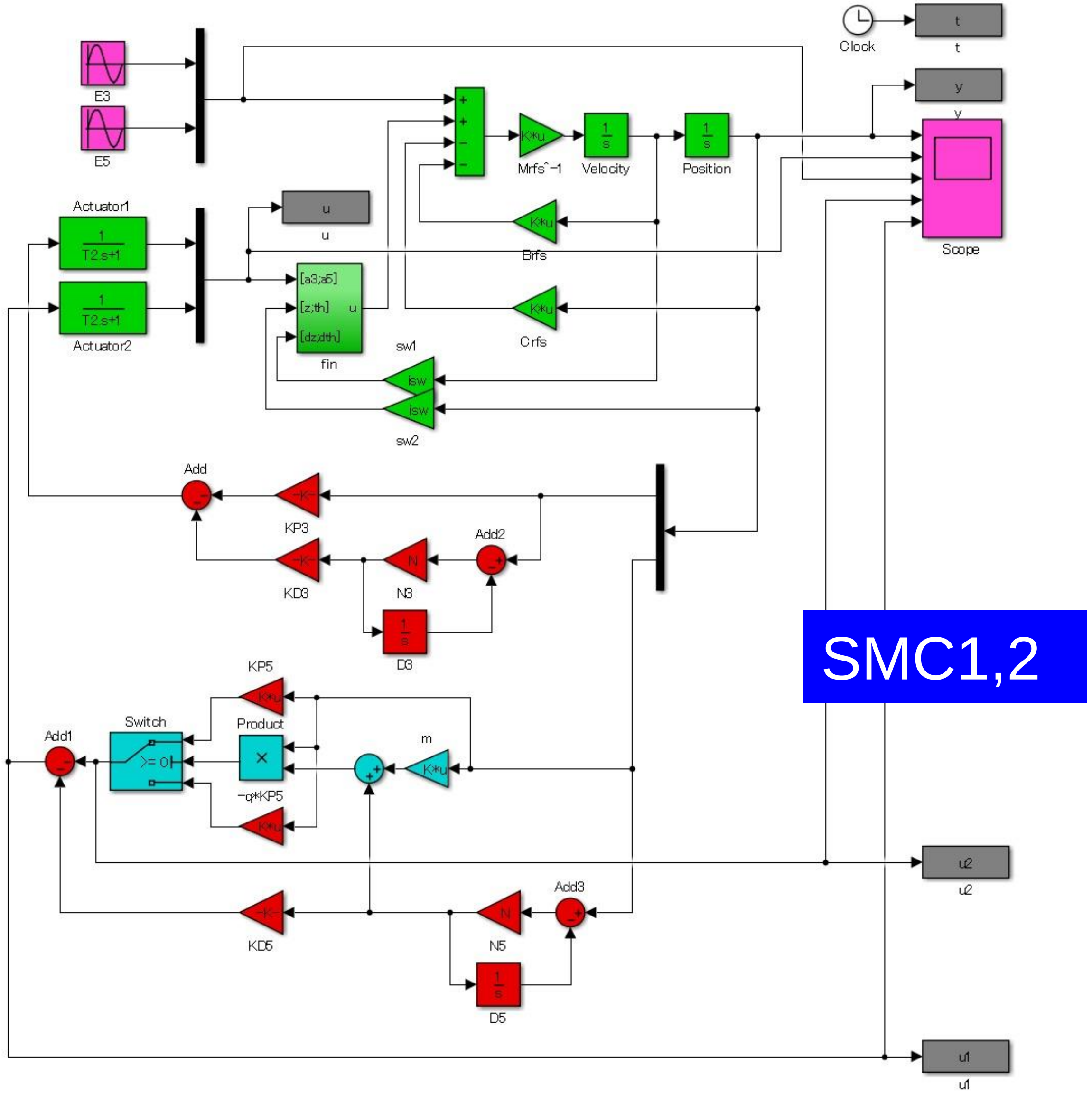
$$T_2\dot{\alpha}_{c5} + \alpha_{c5} = \begin{cases} -\frac{K_{P5}}{F_{fin}C_{La}\ell_0}\theta - \frac{K_{D5}}{F_{fin}C_{La}\ell_0}\dot{\theta} & (\theta s > 0) \\ +\frac{K_{P5}}{F_{fin}C_{La}\ell_0}\theta - \frac{K_{D5}}{F_{fin}C_{La}\ell_0}\dot{\theta} & (\theta s < 0) \end{cases}$$

SMC2 for the pitch control loop :

$$T_2\dot{\alpha}_{c5} + \alpha_{c5} = \begin{cases} -\frac{K_{P5}}{F_{fin}C_{La}\ell_0}\theta - \frac{\rho_D K_{D5}}{F_{fin}C_{La}\ell_0}\dot{\theta} & (\theta s > 0) \\ +\frac{K_{P5}}{F_{fin}C_{La}\ell_0}\theta - \frac{\rho_D K_{D5}}{F_{fin}C_{La}\ell_0}\dot{\theta} & (\theta s < 0) \end{cases}$$

RFS-PDC/RFS-SMC1,2 Simulator





Switching Function

1° Switching Function: $s = \underbrace{\begin{bmatrix} M & I \end{bmatrix}}_{S} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$

2° Cost Function:

$$J = \int_0^\infty \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T}_{x^T} \underbrace{\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}}_{T_r Q T_r^T} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} dt$$

$$= \int_0^\infty \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} I & Q_{12}Q_{22}^{-1} \\ & I \end{bmatrix} \begin{bmatrix} Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} & 0 \\ 0 & Q_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ Q_{22}^{-1}Q_{21} & I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} dt$$

$$= \int_0^\infty \begin{bmatrix} x_1 \\ Q_{12}Q_{22}^{-1}x_1 + x_2 \end{bmatrix}^T \begin{bmatrix} Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} & 0 \\ 0 & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ Q_{12}Q_{22}^{-1}x_1 + x_2 \end{bmatrix} dt$$

3° Sliding Motion:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 = \underbrace{(A_{11} - A_{12}Q_{22}^{-1}Q_{21})}_{\hat{A}} x_1 + \underbrace{A_{12}}_{\hat{B}} \underbrace{(Q_{22}^{-1}Q_{21}x_1 + x_2)}_{\hat{u}},$$

$$x_2 = -Mx_1 \;\Rightarrow\; \underbrace{Q_{22}^{-1}Q_{21}x_1 + x_2}_{\hat{u}} = -\underbrace{(M - Q_{22}^{-1}Q_{21})}_{\hat{F}} x_1$$

4° Determination of Switching Function:

$$\boxed{\begin{aligned} &\Pi\hat{A} + \hat{A}^T\Pi - \Pi\hat{B}Q_{22}^{-1}\hat{B}^T\Pi + (Q_{11} - Q_{12}Q_{22}^{-1}Q_{21}) = 0 \\ &\hat{F} = Q_{22}^{-1}\hat{B}^T\Pi \;\Rightarrow\; M = Q_{22}^{-1}(Q_{21} + A_{12}^T\Pi) \end{aligned}}$$

SMC Design

1° Sliding-Mode Control:

$$u = -\underbrace{S_2B_2}_{(SB)^{-1}}\Big(\underbrace{\begin{bmatrix} S_2\bar{A}_{21} & S_2\bar{A}_{22}S_2^{-1} \end{bmatrix} x' - \Phi s}_{(SA-\Phi S)x} + \eta\frac{P_2 s}{||P_2 s||}\Big)$$

where $P_2 > 0 : P_2\Phi + \Phi^T P_2 = -I, \ \eta > ||S_2 w_2||$

2° Closed-loop System:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & A_{12}S_2^{-1} \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} x_1 \\ s \end{bmatrix} + \begin{bmatrix} 0 \\ -\eta\frac{P_2 s}{||P_2 s||} + S_2 w_2 \end{bmatrix}$$

3° Lyapnov Function: $V = s^T P_2 s$

$$\dot{V} = 2s^T P_2 \dot{s} = 2s^T P_2(\Phi s - \eta\frac{P_2 s}{||P_2 s||} + S_2 w_2)$$

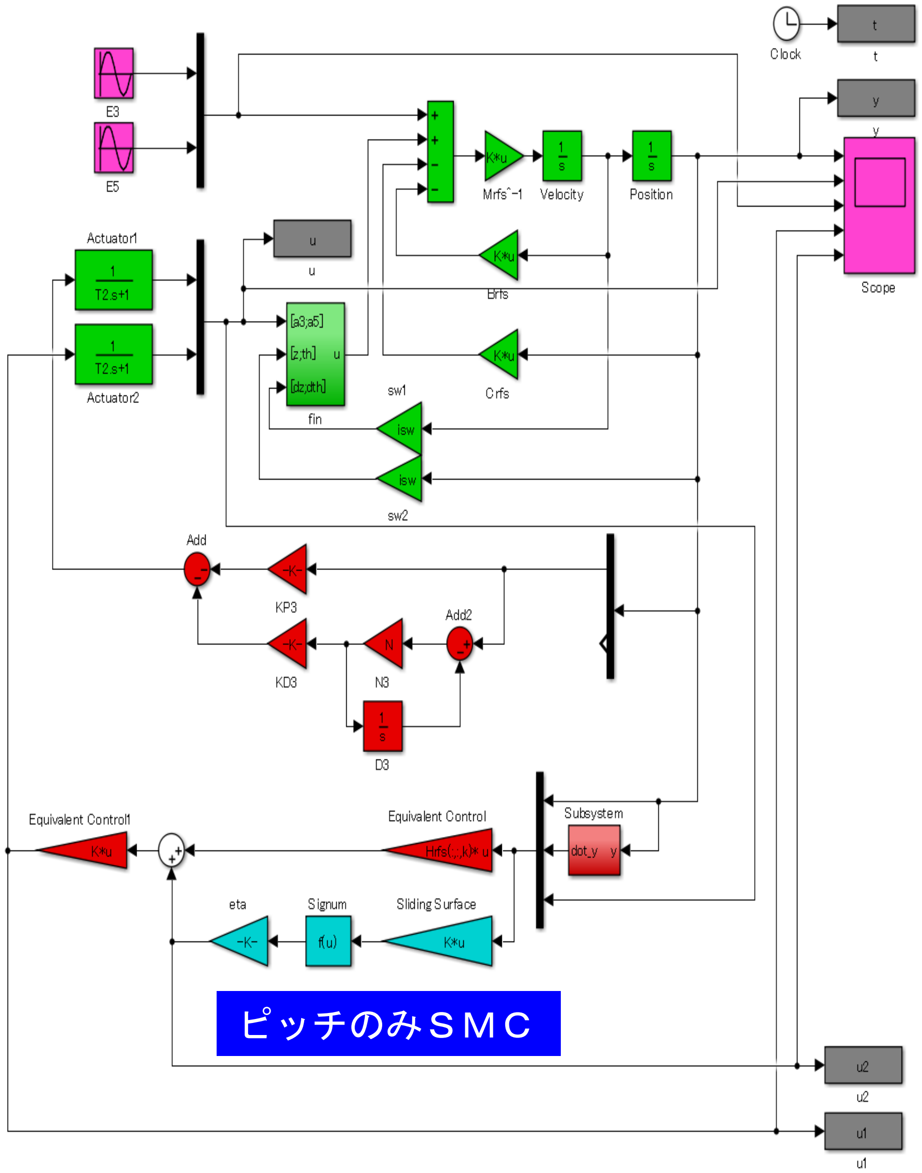
$$= s^T(P_2\Phi + \Phi^T P_2)s - 2\eta\frac{s^T P_2 P_2 s}{||P_2 s||} + 2s^T P_2 S_2 w_2$$

$$= -||s||^2 - 2\eta||P_2 s|| + 2s^T P_2 S_2 w_2$$

$$\leq -||s||^2 - 2||P_2 s||\underbrace{(\eta - ||S_2 w_2||)}_{<0} < 0$$

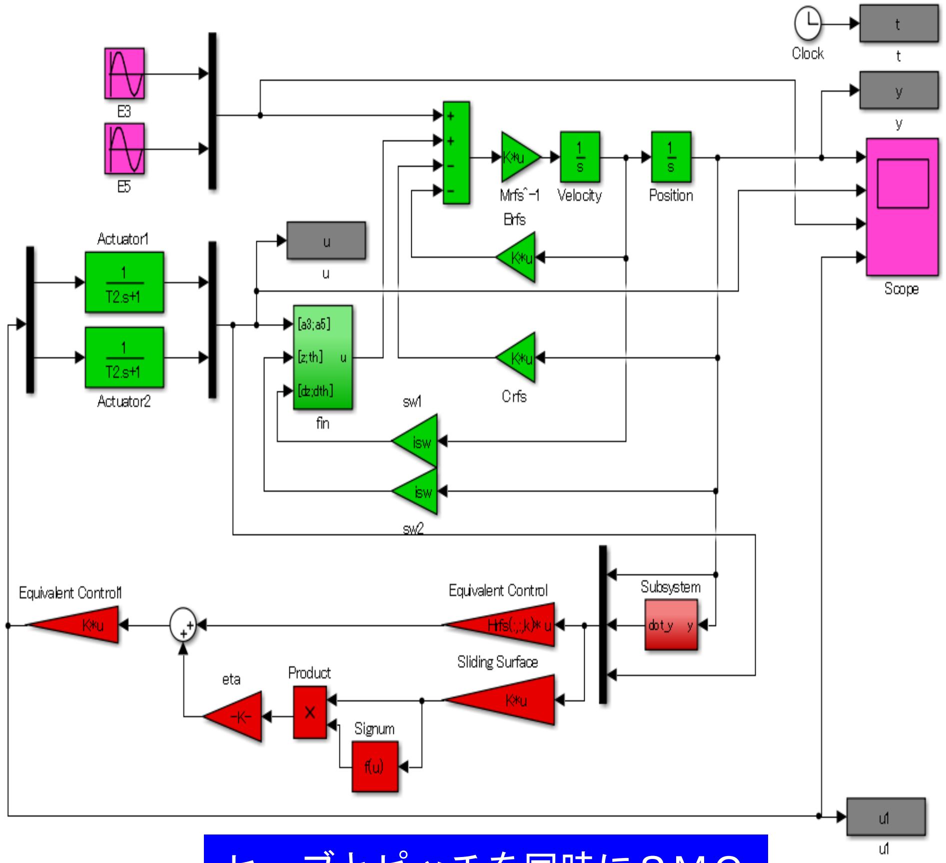
RFS-SMC3/4 Simulators

SMC3



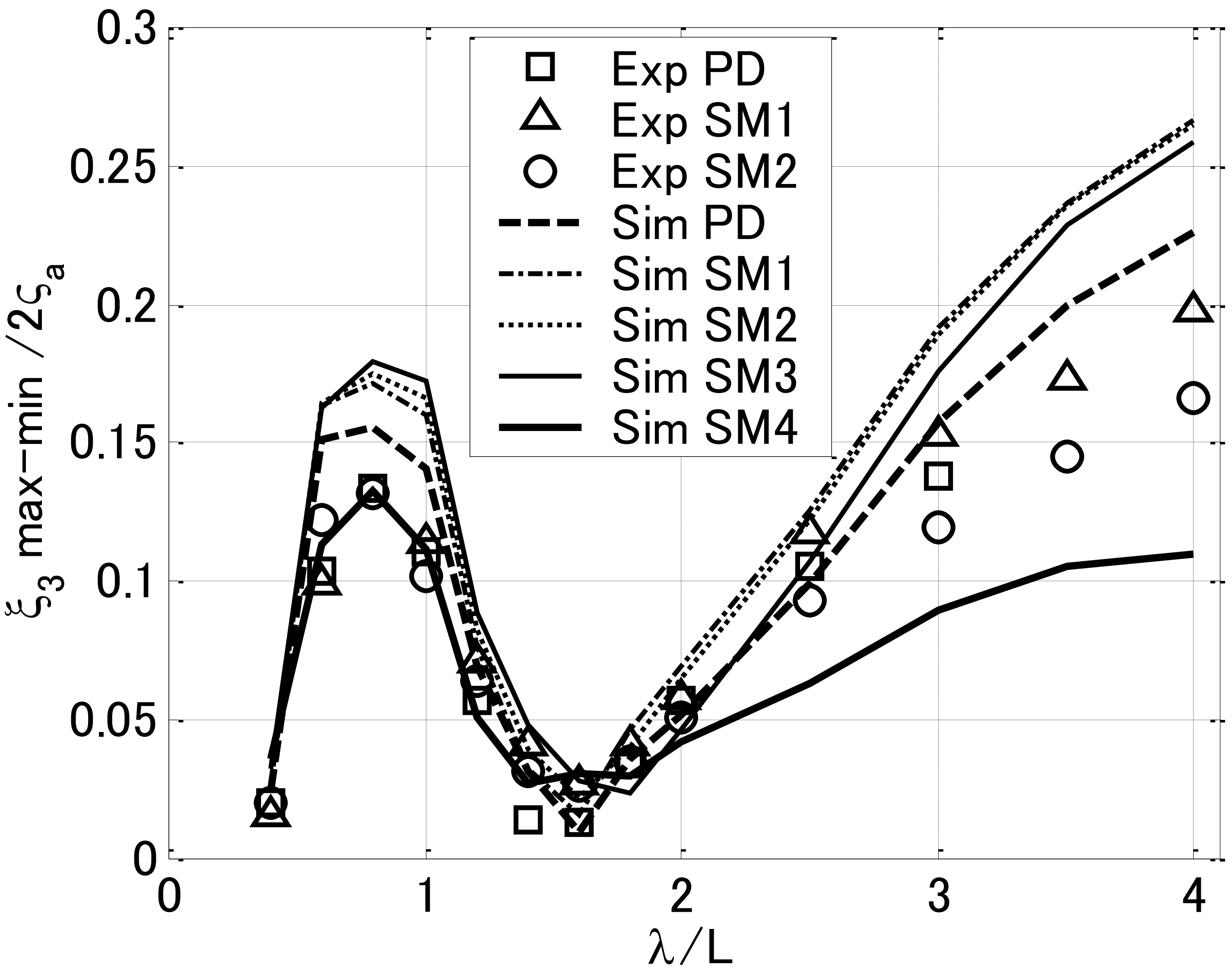
ピッチのみＳＭＣ

SMC4

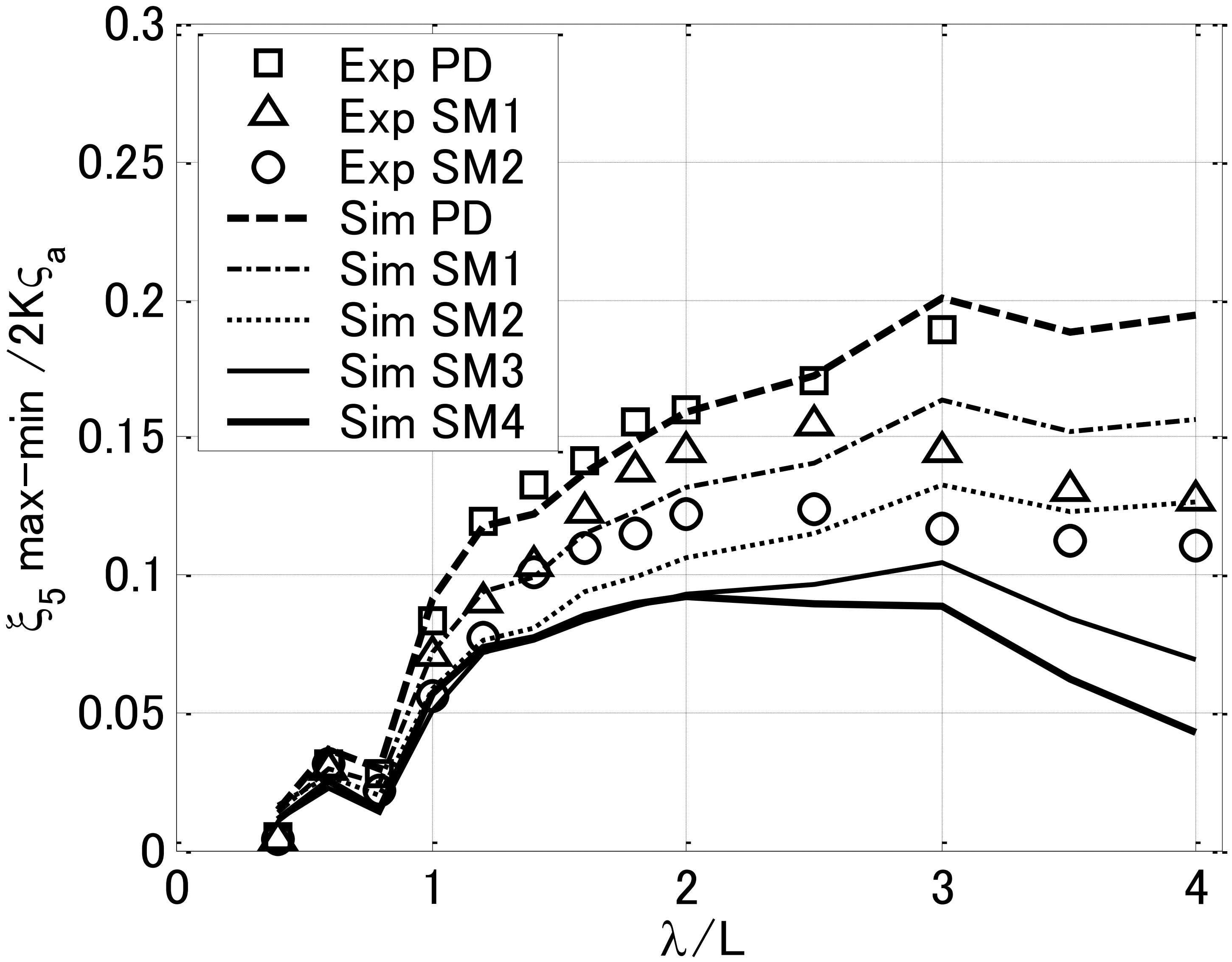


ヒーブとピッチを同時にＳＭＣ

Heave Motion



Pitch Motion



Internal Model Principle

2.4 INCORPORATION OF INTERNAL MODEL PRINCIPLE FOR ASYMPTOTIC DISTURBANCE REJECTION

Dynamical systems are sometimes subject to persistent external disturbances. A state–space approach of the internal model principal [Chen, 1984; Wie and Liu, 1992 and 1993] is presented here for asymptotic disturbance rejection.

Consider a persistent disturbance with one or more frequency components represented as

$$d(t) = \sum_i A_i \sin(p_i t + \phi_i) \tag{2.53}$$

with unknown magnitudes A_i and phases ϕ_i but known frequencies p_i. Active disturbance rejection for the measured output is achieved by introducing a model

of the disturbance inside the control loop. A disturbance rejection filter for $d_i(t)$ at a particular frequency p_i is modeled as

$$\dot{x}_{d_i} = A_{d_i} x_{d_i} + B_{d_i} y \tag{2.54}$$

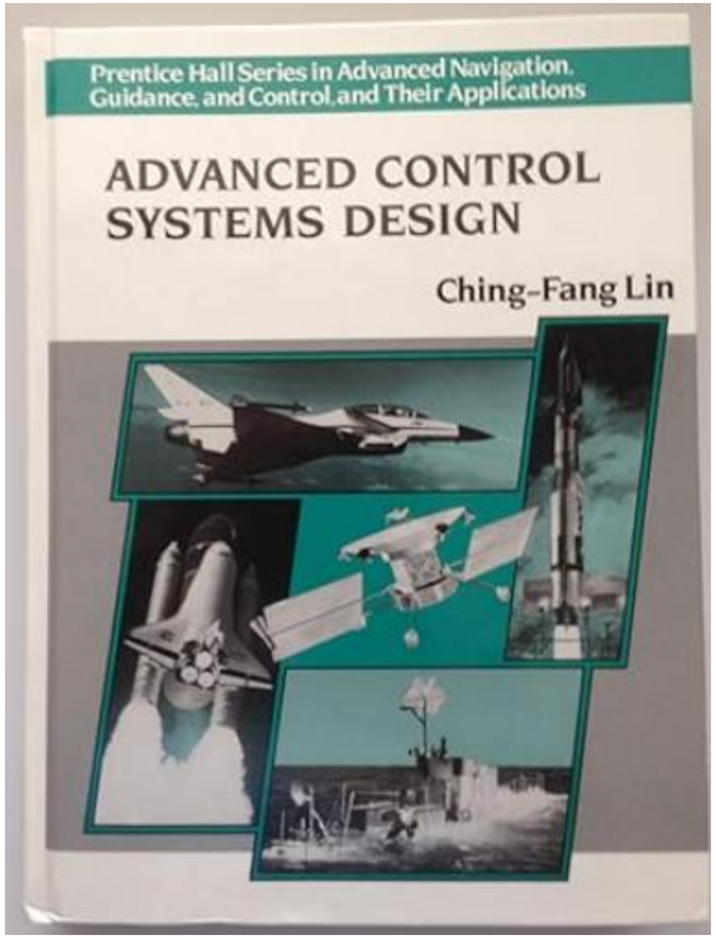
where

$$A_{d_i} = \begin{bmatrix} 0 & 1 \\ -p_i^2 & 0 \end{bmatrix}; \qquad B_{d_i} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

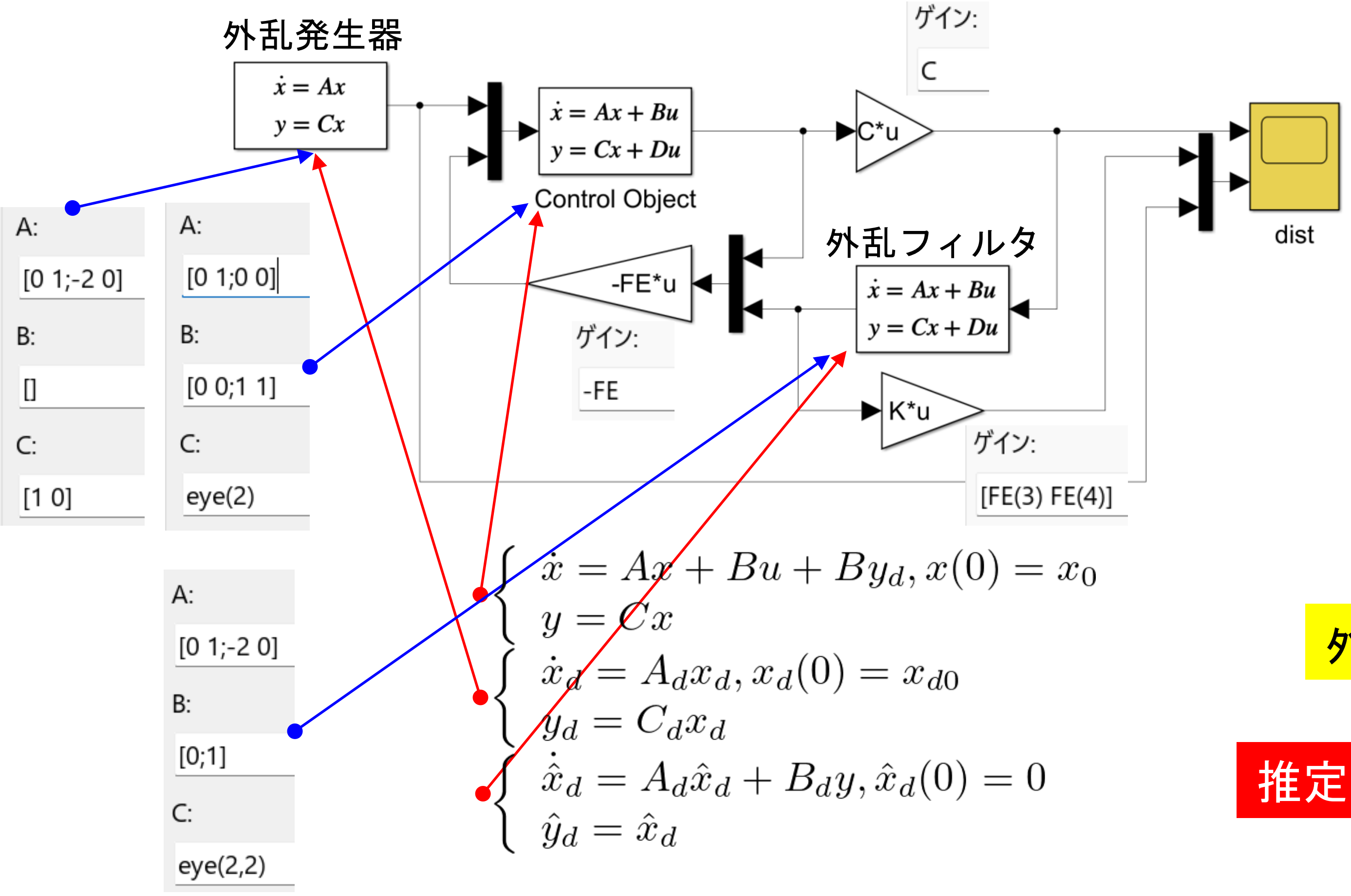
The internal model then includes as many frequencies as the given disturbance, and is driven by the measured output y of the plant. This procedure is equivalent to the one used in the classical approach with the disturbance model now consisting of a state–space model. The disturbance rejection filter is then described by

$$\dot{x}_d = A_d x_d + B_d y \tag{2.55}$$

where x_d is the state vector introduced by the internal disturbance model, A_d is block–diagonal and contains A_{d_i} for each disturbance $d_i(t)$, and B_d also contains B_{d_i}'s for each disturbance.



外乱推定・抑制の例



$$\begin{cases}\dot{x}=Ax+Bu+By_d,\ x(0)=x_0\\ y=Cx\end{cases}$$

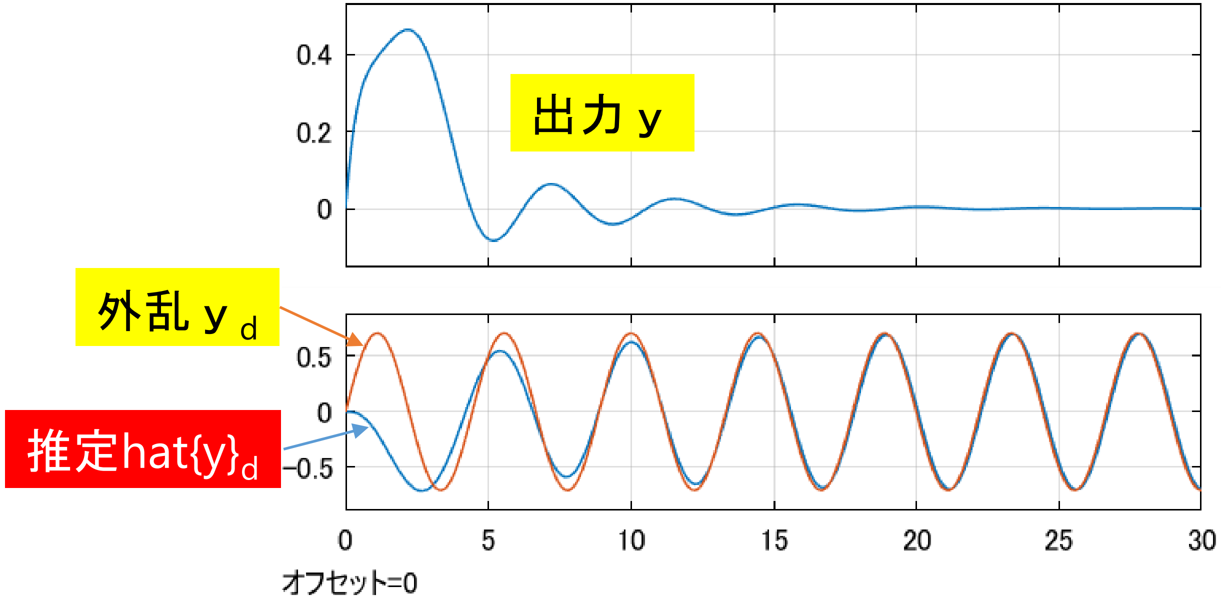
$$\begin{cases}\dot{x}_d=A_dx_d,\ x_d(0)=x_{d0}\\ y_d=C_dx_d\end{cases}$$

$$\begin{cases}\dot{\hat{x}}_d=A_d\hat{x}_d+B_dy,\ \hat{x}_d(0)=0\\ \hat{y}_d=\hat{x}_d\end{cases}$$

$$\begin{bmatrix}\dot{x}\\ \dot{\hat{x}}_d\end{bmatrix}=\begin{bmatrix}A & 0\\ B_dC & A_d\end{bmatrix}\begin{bmatrix}x\\ \hat{x}_d\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}(u+y_d)$$

$$u=-\begin{bmatrix}F & F_d\end{bmatrix}\begin{bmatrix}x\\ \hat{x}_d\end{bmatrix}$$

A=[0 1;0 0]; B=[0;1]; C=[1 0];
Ad=[0 1;-2 0]; Bd=[0;1];
AE=[A zeros(2,2);Bd*C Ad];
BE=[0;1;0;0];
FE=opt(AE,BE,eye(4,4),eye(4,4),1)



$$u=-Fx-F_d\hat{x}_d$$

第２項で外乱を打ち消す

まとめ

- 無定位系は単位フィードバックによって2次振動系になり、これはステップ応答の第1頂点の座標から同定可能
- LPVモデルにおける端点モデルを同時安定化することにより、スケジューリング制御が可能
- 特異値分解を利用した連立1次方程式の解表現を利用して、回転角制約付きの推力配分問題を解ける
- ２次振動系のPD制御則にスイッチング制御則を加えてスライディングモードを達成し、安定化問題低次元化（非振動化）やロバスト制御が可能
- 規則波外乱下では、外乱の内部モデルを導入することにより、外乱を推定・抑制するのに役立つはずであるが、前例がない模様

謝辞

- 古田勝久先生
- 藤井隆雄先生
- 貴島勝郎先生
- 小寺山亘先生、中村昌彦先生
- 木下 健 先生、吉田基樹先生

ご清聴ありがとうございました

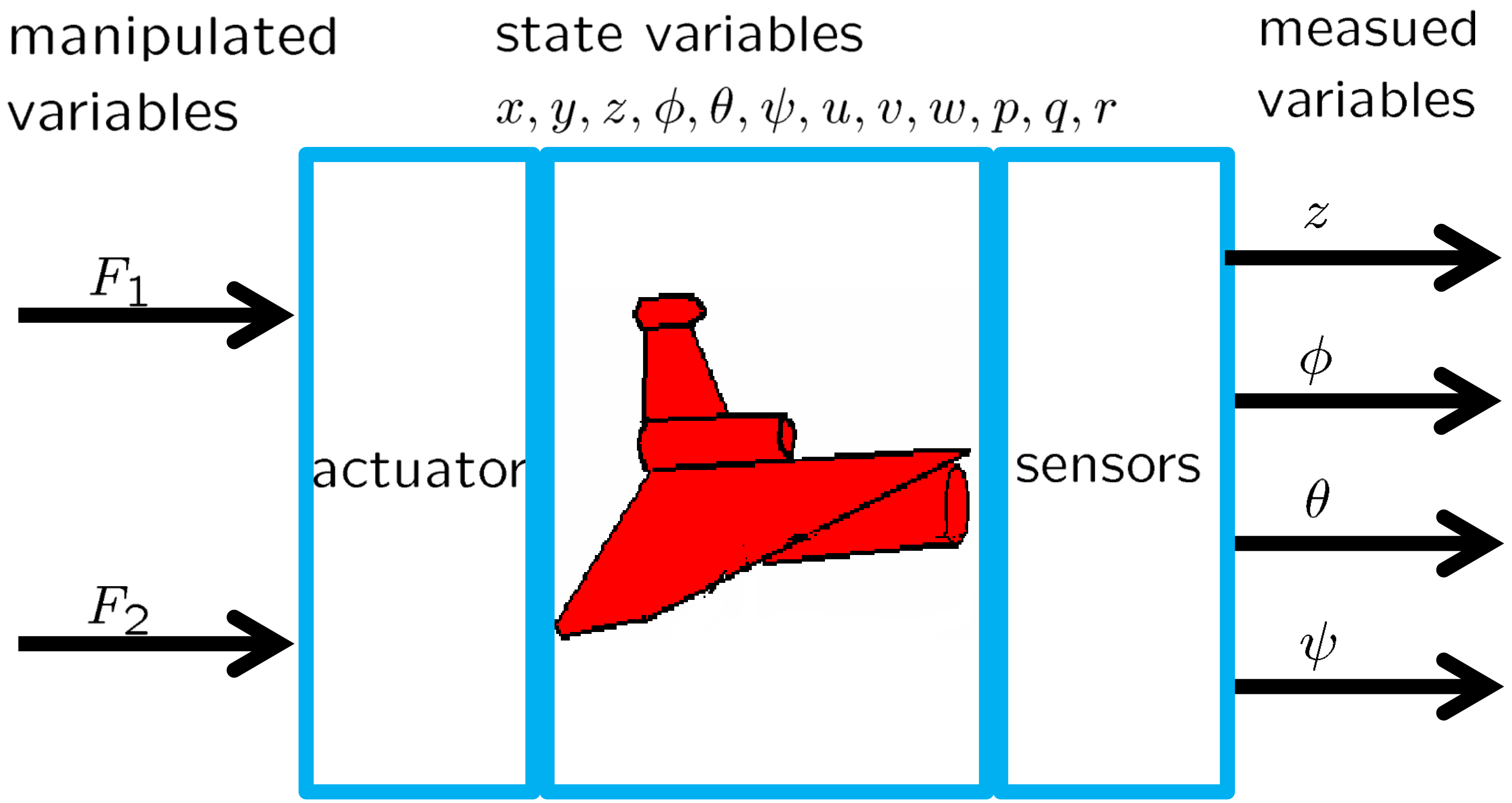
【2】多変数制御への対応（LQI制御）

- 水中ビークルDELTAを手動で操作してみる
- 運動に対応した操作変数の導入し、多変数を同時に操作
- 行列の2次方程式を解いて、ゲインを求める！

16. Hiroyuki KAJIWARA Wataru KOTERAYAMA, Masahiko NAKAMURA, H.TERADA, T.MORITA: "Control System Design of a ROV Operated Both as Towed and Self-Propulsive Vehicle", Proc. of 3rd International Offshore and Polar Engineering Conference, paper no. ISOPE-I-93-163, pp.451-454, 1993

水中ビークルDELTA

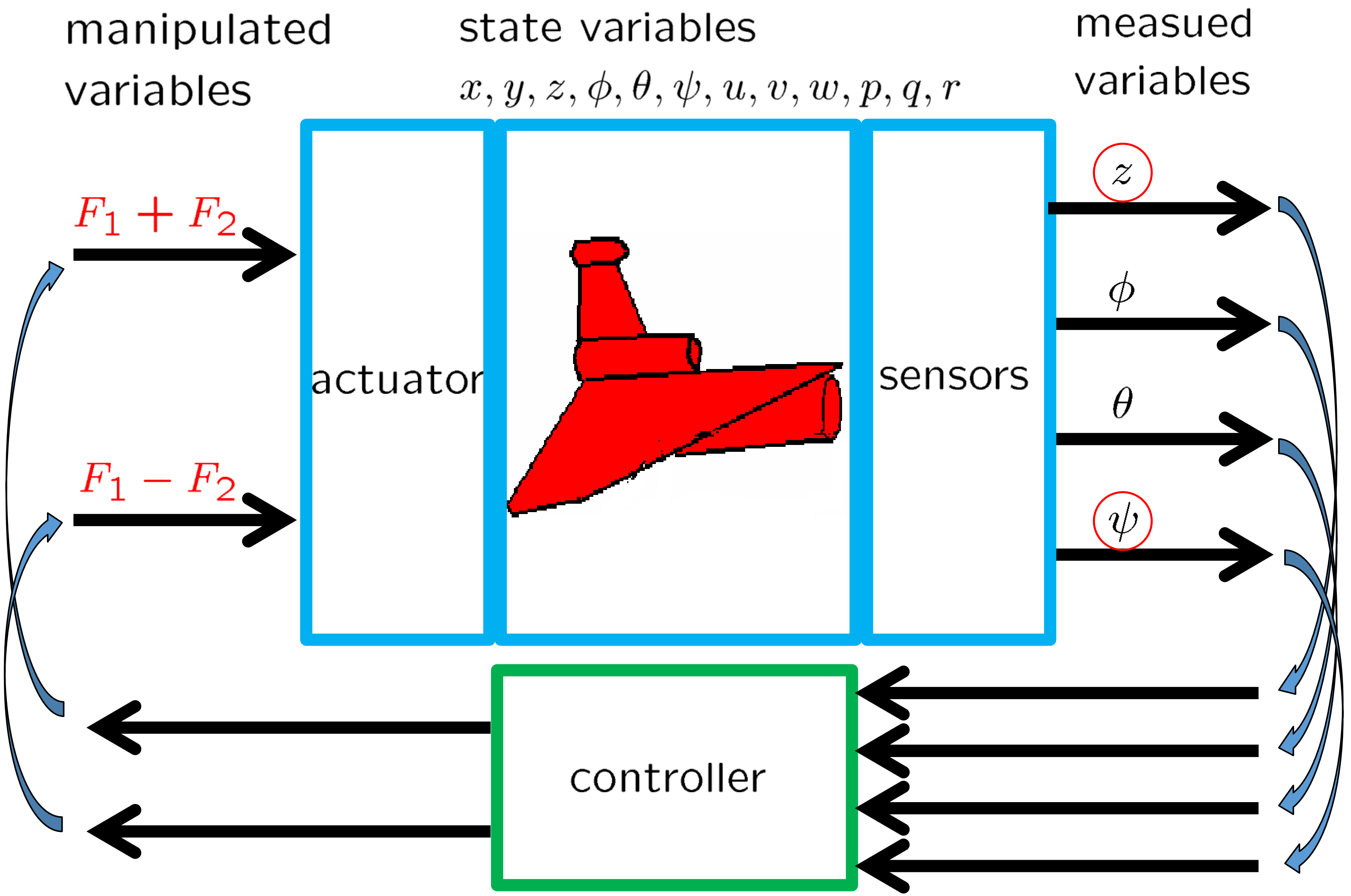
DELTAの運動方程式



$$\dot{\xi}_E = J(\xi_E)\xi_B \quad (\xi_E = \left[\ x, y, z, \phi, \theta, \psi\ \right]^T,\ \xi_B = \left[\ u, v, w, p, q, r\ \right]^T)$$

$$M\dot{\xi}_B + (C(\xi_B) + D(\xi_B))\xi_B + G(\xi_E) = F$$

ヒーブ駆動 とヨー駆動


manipulated variables
state variables
$x, y, z, \phi, \theta, \psi, u, v, w, p, q, r$
measued variables
$F_1 + F_2$
$F_1 - F_2$
actuator
sensors
z
ϕ
θ
ψ
controller

LQ（Linear Quadratic）制御

- **可制御かつ可観測なn次系**

  $\dot{x} = Ax + Bu,\ \ y = Cx$

- **安定化状態フィードバック**　$u = -Fx$

- **閉ループ系**

  $\dot{x} = A_F x, \qquad A_F = A - BF$ は安定行列

- **２次形式評価関数**

  $$J = \int_0^\infty (y^T Q y + u^T R u)\, dt \quad (Q > 0, R > 0)$$

- **最適制御問題**　Jを最小化するFを求めよ。

- **LQ制御（最適制御問題の解）**

  リッカチ方程式

  $\Pi A + A^T \Pi - \Pi B R^{-1} B^T \Pi + C^T Q C = 0$

  の解$\Pi > 0$を用いて，次式で与えられる。

  $F = R^{-1} B^T \Pi$

手動制御と多変数制御

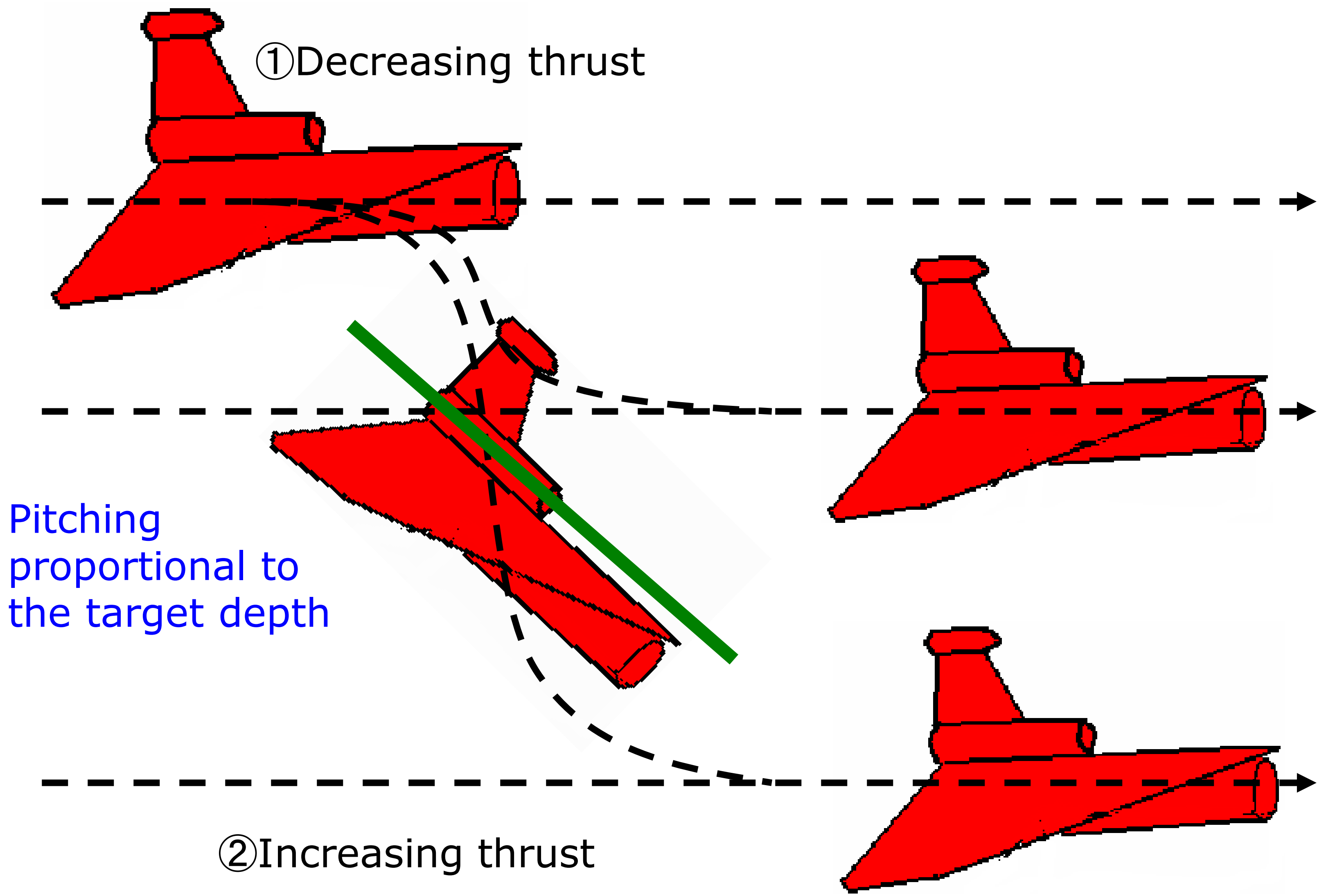
【3】速度変動への対応（LPV制御）

- 目の前に障害物が現れたらどうする
- 速度が変わると流体力微係数が大きく変わる
- 速度を変動パラメータとしてスケジューリングを行う！

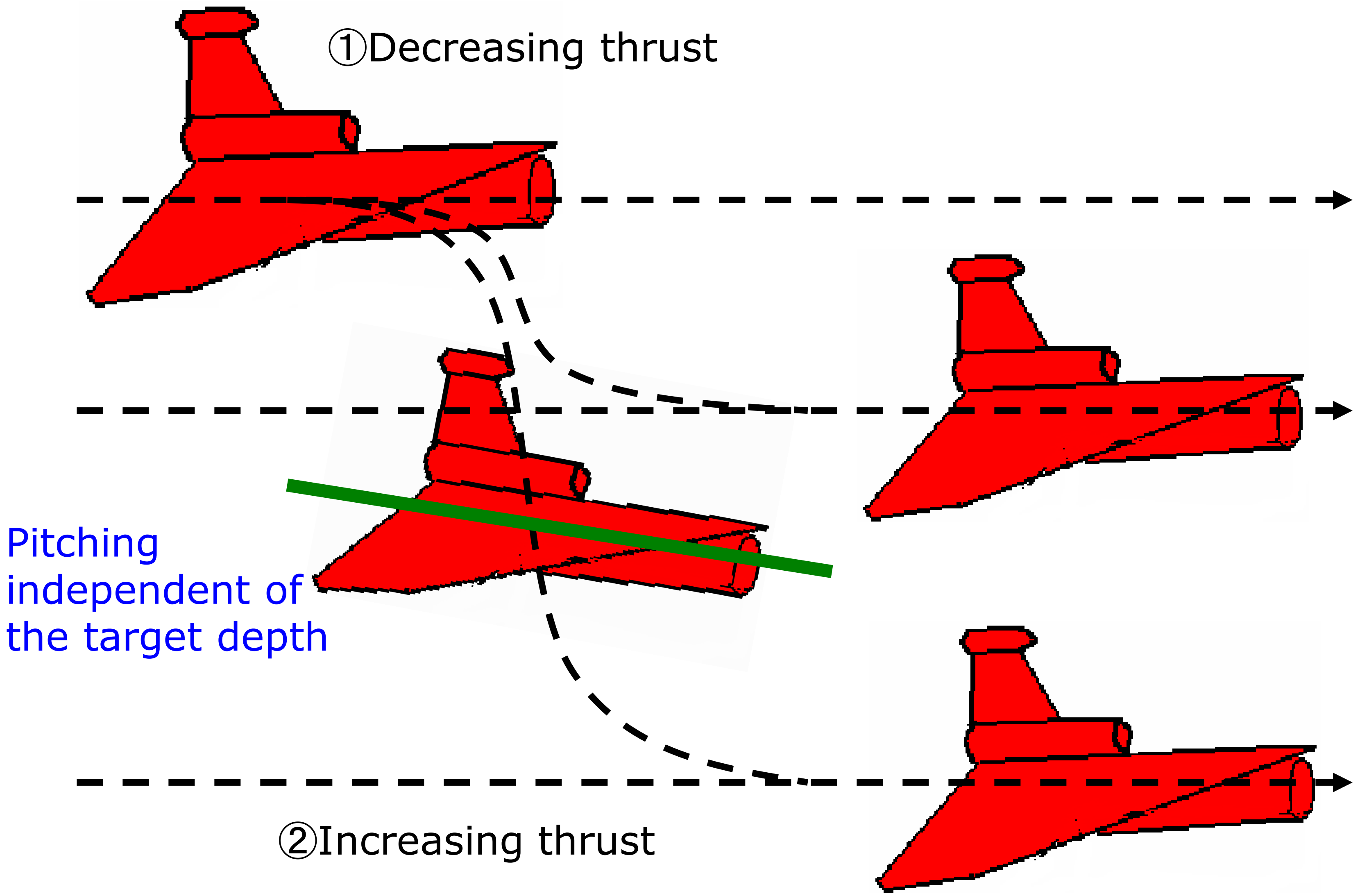
28. Hiroyuki KAJIWARA, Wataru KOTERAYAMA, Masahiko NAKAMURA, S.YUGAWA: "LMI-Based Design of Robust Controllers for an Underwater Vehicle ", Proc. of 7th International Offshore and Polar Engineering Conference, paper no. ISOPE-I-97-161, pp.51-56, 1997

LTI Control, 1m to 3m/4m

Diving by **Linear** Control



Diving by Scheduling Control



Physical Parameters of DELTA

$L=1.13,\ d=0.185,\ \rho=102,\ g=9.8$

$\nabla=53.24/1000,\ x_T=L\times(-0.088495),\ z_T=L\times(-0.04956)$

$x_B=-0.10555+0.0015,\ z_B=-0.04254$

$m=(51.82+w_1+w_2)/9.8,\ dm=0.5\rho L^2 d\times 0.02434$

$x_{G**}=(-5.1359+0.4787w_1-0.4368w_2)/(48.9469+w_1+w_2)$

$x_G=(m-dm)/m\,x_{G**}+dm/m\,x_{dm},\ \ z_G=(-1.5536+0.075w_1+0.075w_2)/(51.82+w_1+w_2)$

$I_{xx}=0.32323+0.075^2(w_1+w_2)/9.8$

$I_{yy*}=0.54778+((0.4787^2+0.075^2)w_1+(0.4368^2+0.075^2)w_2)/9.8,\ I_{yy}=I_{yy*}+dm\,x_{dm}{}^2$

$I_{zz*}=0.86779+(0.4787^2 w_1+0.4368^2 w_2)/9.8,\ I_{zz}=I_{zz*}+dm\,x_{dm}{}^2,\ I_{xz}=0$

Hydrodynamic Coefficients

$A_{11}=0.5\rho L^2 d\times 0.1278,\ \ A_{22}=0.5\rho L^2 d\times 0.0,\ \ A_{33}=0.5\rho L^2 d\times 0.5981$

$A_{44}=0.5\rho L^4 d\times 0.0843,\ \ A_{55}=0.5\rho L^4 d\times 0.4499,\ \ A_{66}=0.5\rho L^4 d\times 0.0$

$X_{uu}=0.5\rho Ld\times(-0.4062),\ \ X_{uuk}=0.5\rho Ld\times(-0.173\times 1.0)),\ \ X_v=0.5\rho ULd\times 0.4944,\ \ X_{ww}=0.5\rho Ld\times 0.2017$

$Y_v=0.5\rho ULd\times(-9.901),\ \ Y_{vv}=0.5\rho Ld\times 7.88,\ \ Y_p=0.5\rho UL^2 d\times 0.24618,\ \ Y_r=0.5\rho UL^2 d\times 4.7369,\ \ Y_{rr}=0.5\rho L^3 d\times 17.695$

$Z_w=0.5\rho ULd\times(-7.726)\times 0.9,\ \ Z_q=0.5\rho UL^2 d\times Z_{q*}$

$K_v=0.5\rho UL^2 d\times(-0.3254),\ \ K_p=0.5\rho UL^3 d\times(-0.3336),\ \ K_r=0.5\rho UL^3 d\times 0.0029953$

$M_w=0.5\rho UL^2 d\times(-0.8822),\ \ M_q=0.5\rho UL^3 d\times M_{q*}$

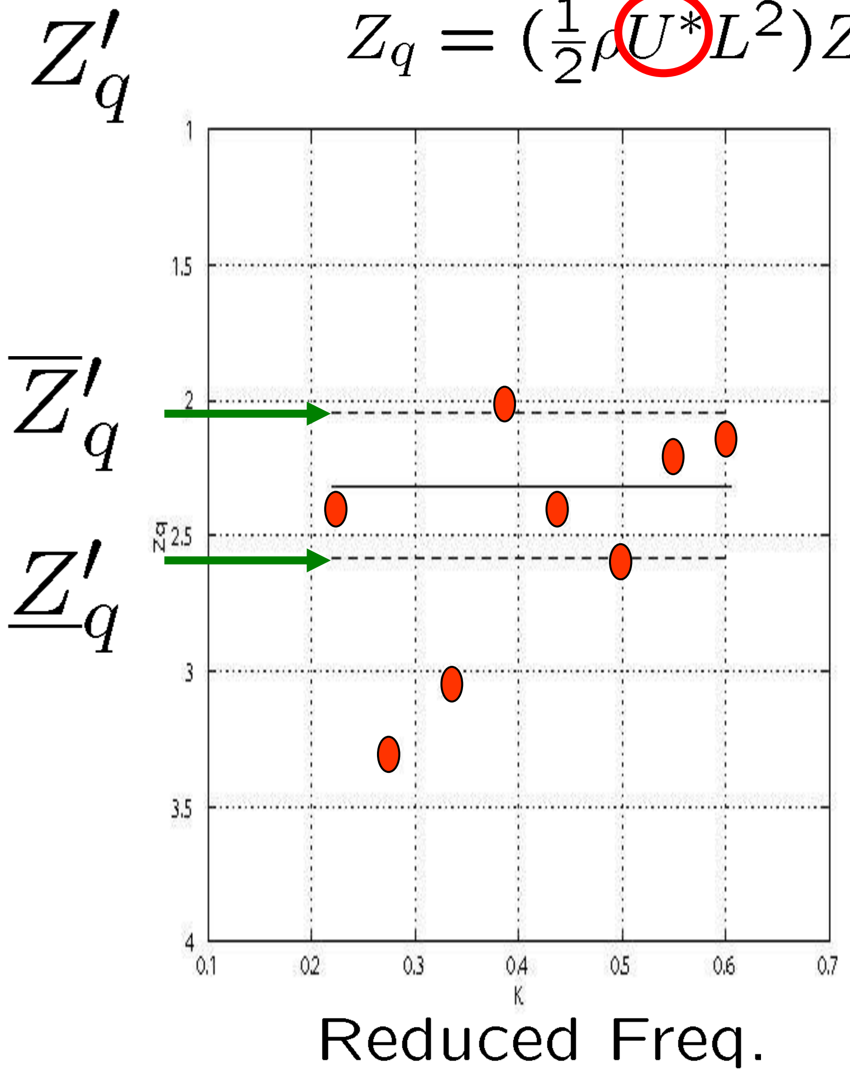
$N_v=0.5\rho UL^2 d\times 0.9939,\ \ N_{vv}=0.5\rho L^2 d\times(-6.9564),\ \ N_r=0.5\rho UL^3 d\times(-0.7028),\ \ N_{rww}=0$

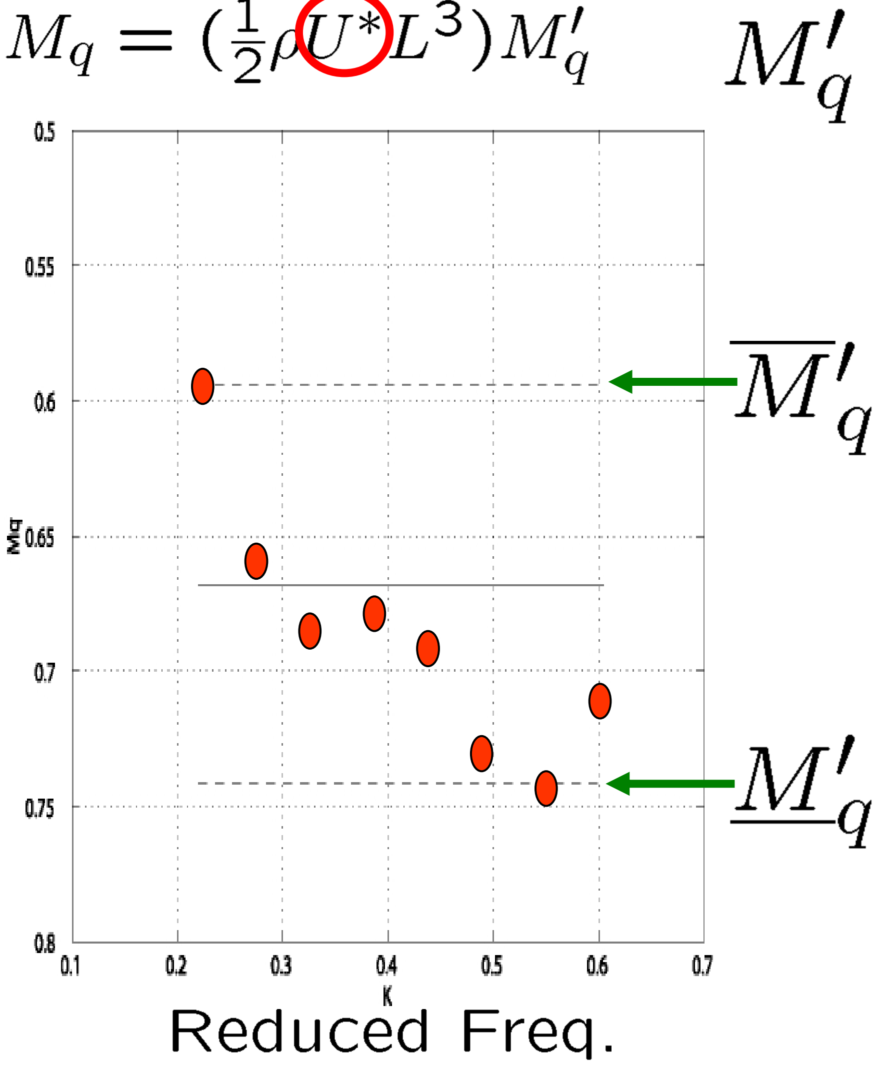
$\ell_{pTHy}=0.306,\ \ \ell_z=L\times(-0.1211),\ \ \ell_k=L\times(-0.1593),\ \ z_{TH}=L\times 0.08496,\ \ \ell_{pTHx}=L\times(-0.1209),\ \ y_{TH}=0.306$

$b_t=-0.4352,\ \ c_t=0.9383,\ \ e_t=0.8662,\ \ f_t=-0.1054;$

Parameter Uncertainties

$$Z_q = (\tfrac{1}{2}\rho U^* L^2) Z'_q, \quad M_q = (\tfrac{1}{2}\rho U^* L^3) M'_q$$





LPV Model of DELTA

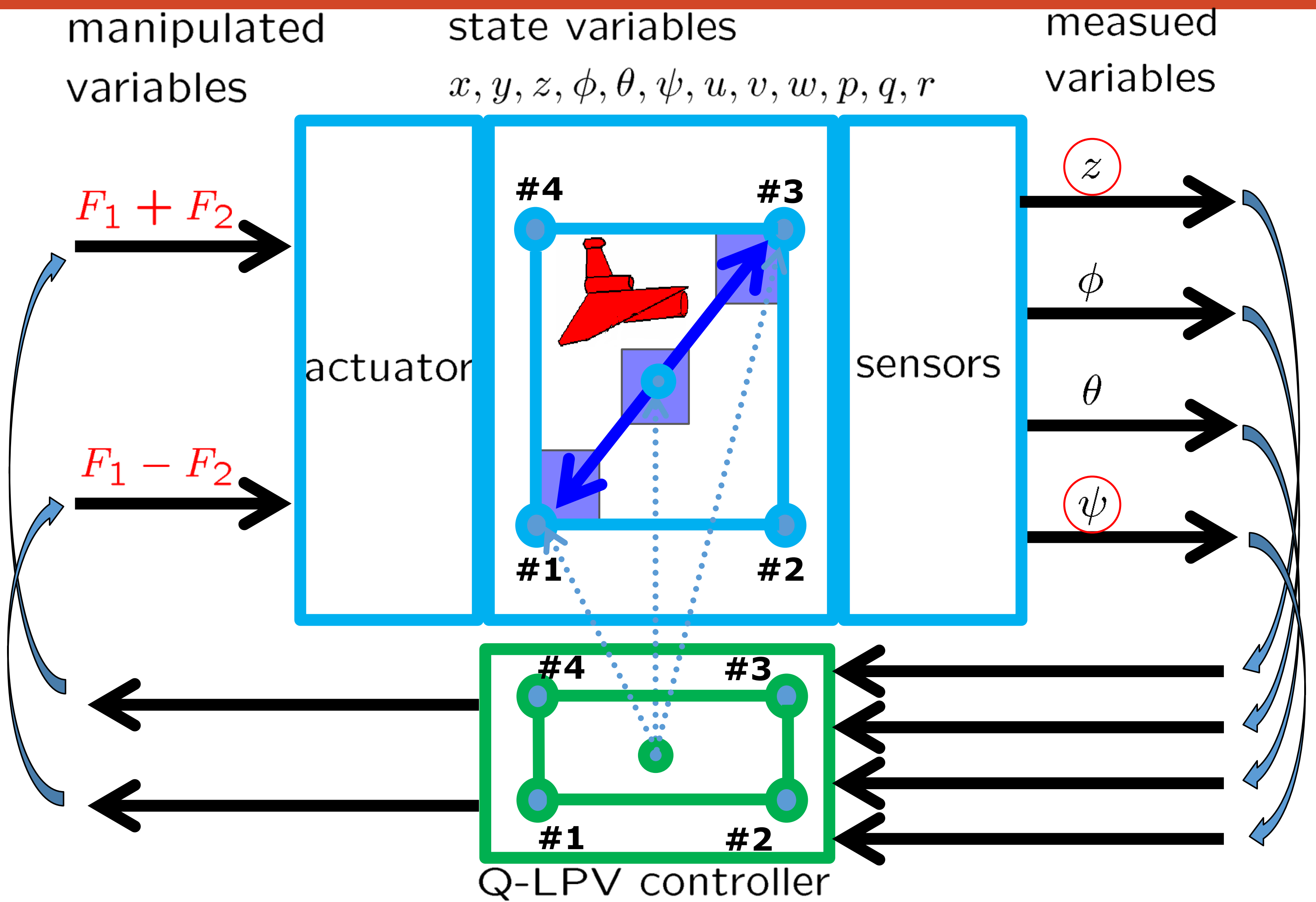
$$\frac{d}{dt}\begin{bmatrix}\xi_E-\xi_E^*\\ \xi_B-\xi_B^*\end{bmatrix} = \begin{bmatrix} E & J(\xi_E)\\ M^{-1}\bar{G} & M^{-1}(\bar{C}+\boxed{\bar{D}})\end{bmatrix}\begin{bmatrix}\xi_E-\xi_E^*\\ \xi_B-\xi_B^*\end{bmatrix} + \begin{bmatrix}0\\ M^{-1}\bar{F}\end{bmatrix}(\zeta-\zeta^*)$$

where

$$A(Z_q, M_q) = \begin{bmatrix} E & J(\xi_E)\\ M^{-1}\bar{G} & M^{-1}(\bar{C}+\boxed{\bar{D}_0})\end{bmatrix} + \boxed{U^*}\begin{bmatrix}0 & 0\\ 0 & M^{-1}\boxed{\bar{D}_1}\end{bmatrix}$$

$$(\underline{Z}_q \le Z_q \le \overline{Z}_q,\ \ \underline{M}_q \le M_q \le \overline{M}_q)$$

LPV Model of DELTA



準LPV Control, 1m to 3m/4m

LTI/準LPV Control, 5m to 1m

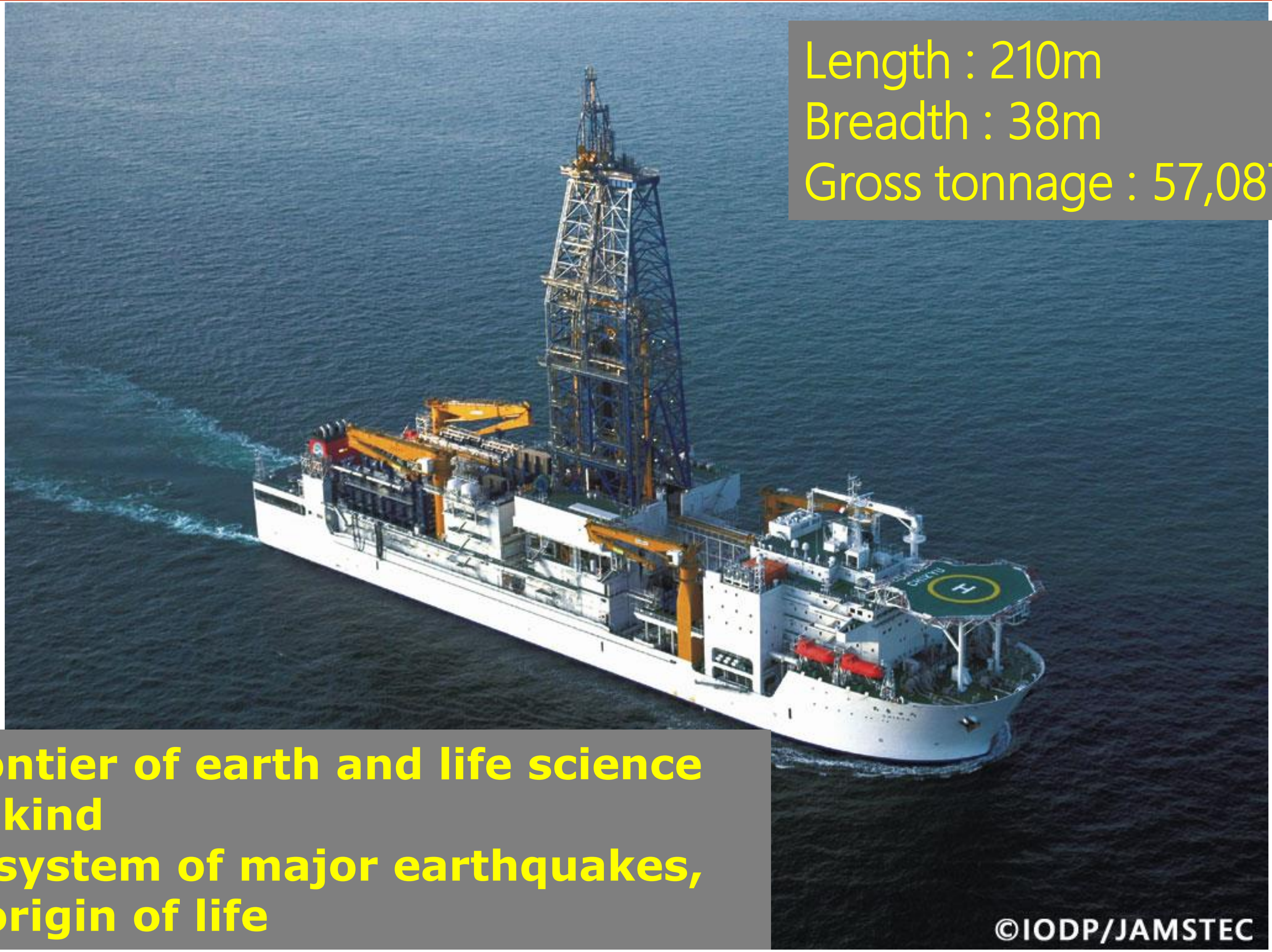
準LPV Control, 1m to 5m

【4】DPS関連の手法（HILS手法）

- CA(Control Allocation)問題とは
- 連立方程式求解問題（方程式の数より未知変数の数が多い）
- 特異値分解の利用！

81. Wataru KOTERAYAMA, Masahiko NAKAMURA, Hiroyuki KAJIWARA, Hitoshi MATSUI, Kenzo NORIDOMI: "Development of a Training Simulator for Dynamic Reentry Operations of a Riser Pipe Hanged off", Proc. of 18th International Offshore and Polar Engineering Conference, paper no. ISOPE-I-08-410, pp.67-70, 2008

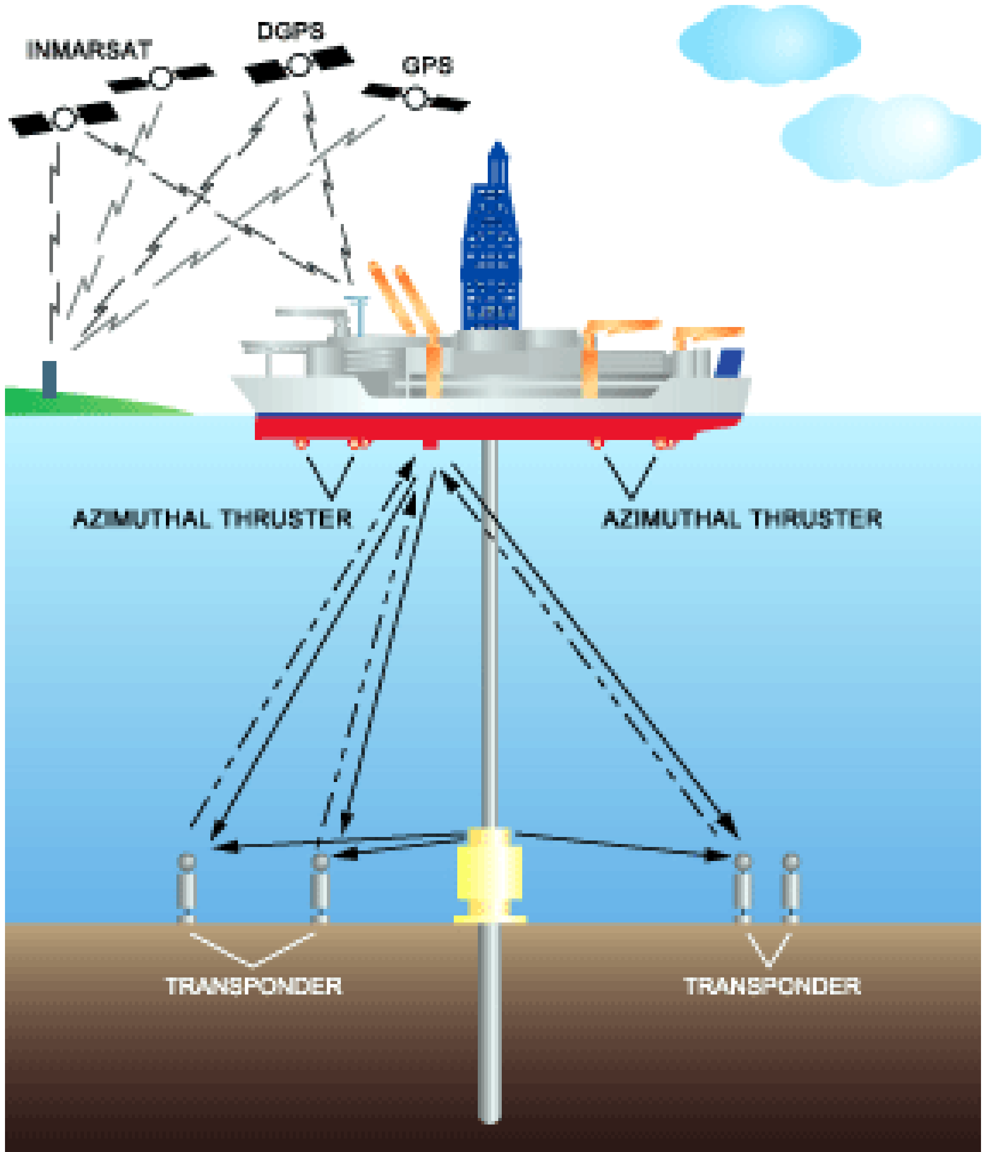
CHIKYU: Deep-sea Drilling Vessel



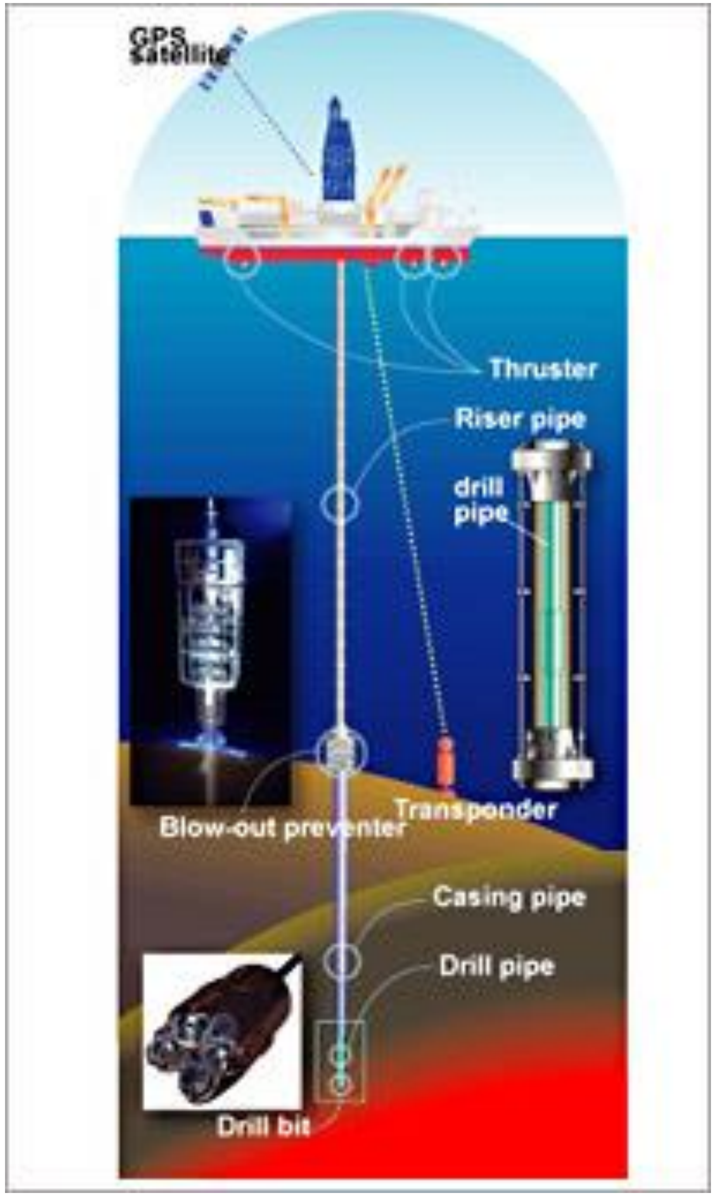
Length : 210m
Breadth : 38m
Gross tonnage : 57,087tons

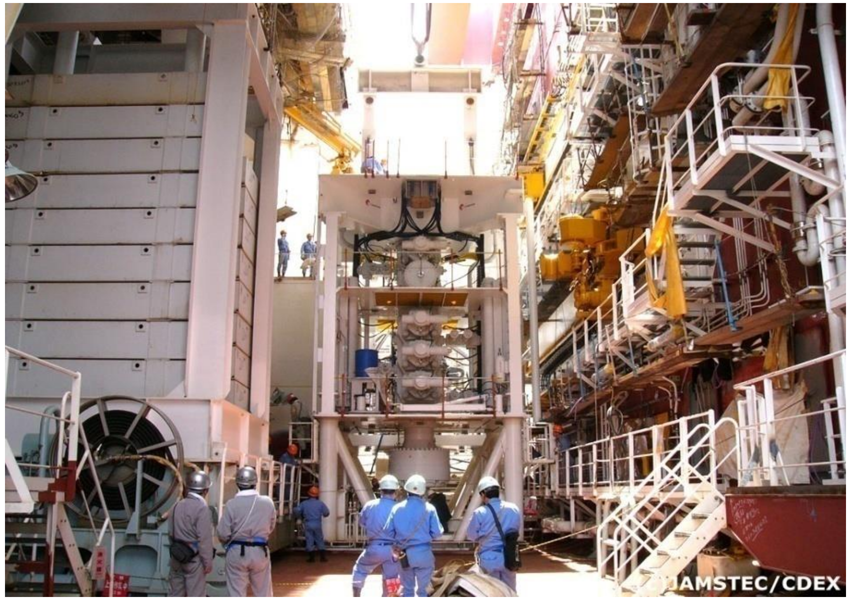
Open the new frontier of earth and life science for future of mankind
by revealing the system of major earthquakes, global changes, origin of life

CHIKYU: DPS & Azimuth Thrusters



CHIKYU: Riser Pipe





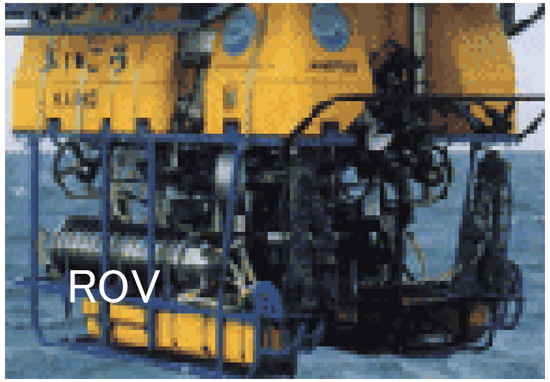
Length :27m
Diameter :1.2m

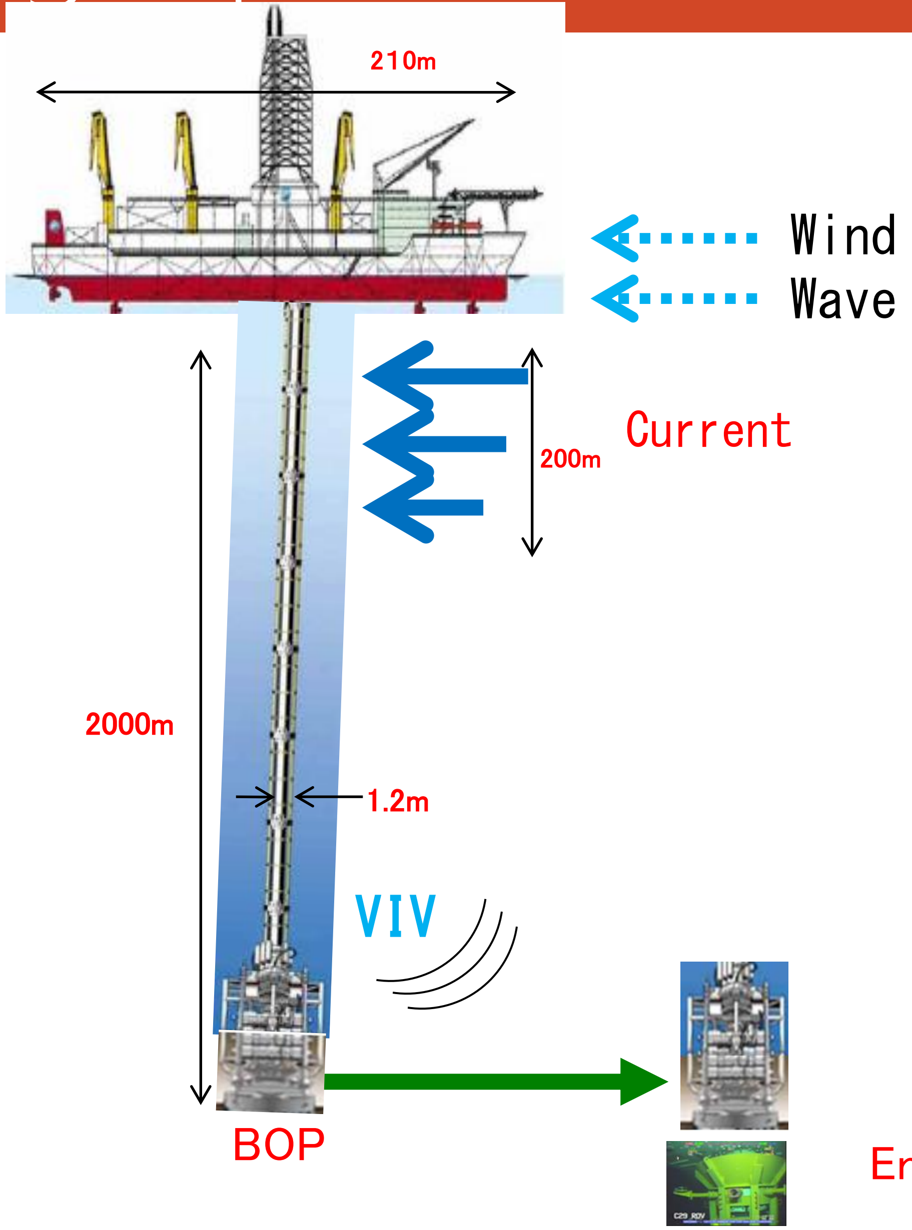
CHIKYU: Drill House

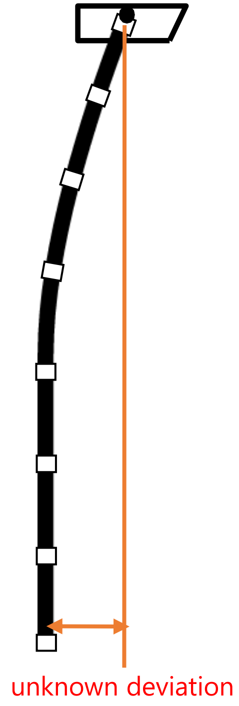
ドリルフロアにある金網で保護されたドリラーズハウス。いくつもの掘削装置を動かすコントロールルームです。
降下中のドリルビットを海中の無人探査機(ROV)でモニタリングしています。

The drill house, a room within a protective steel cage on the drill floor, is where many drilling operations are controlled. Here the controllers are monitoring the drill bit during its descent to the sea floor, using the Remotely Operated Vehicle (ROV).

CHIKYU: Landing Operation

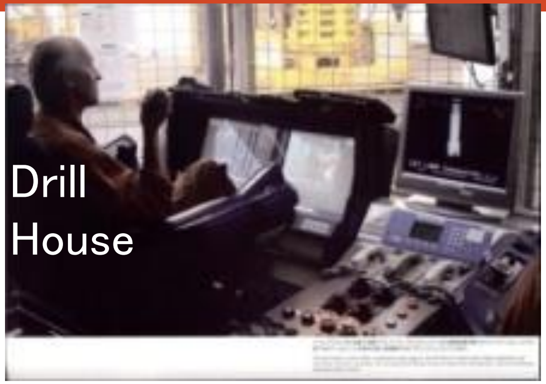


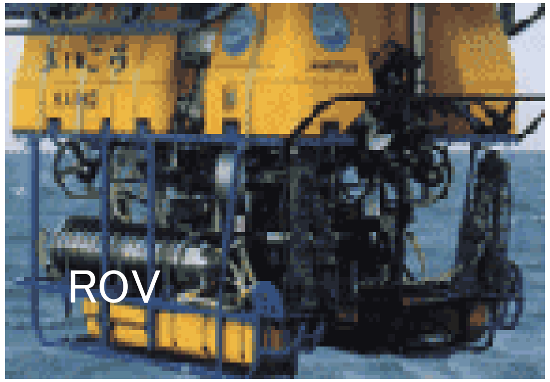


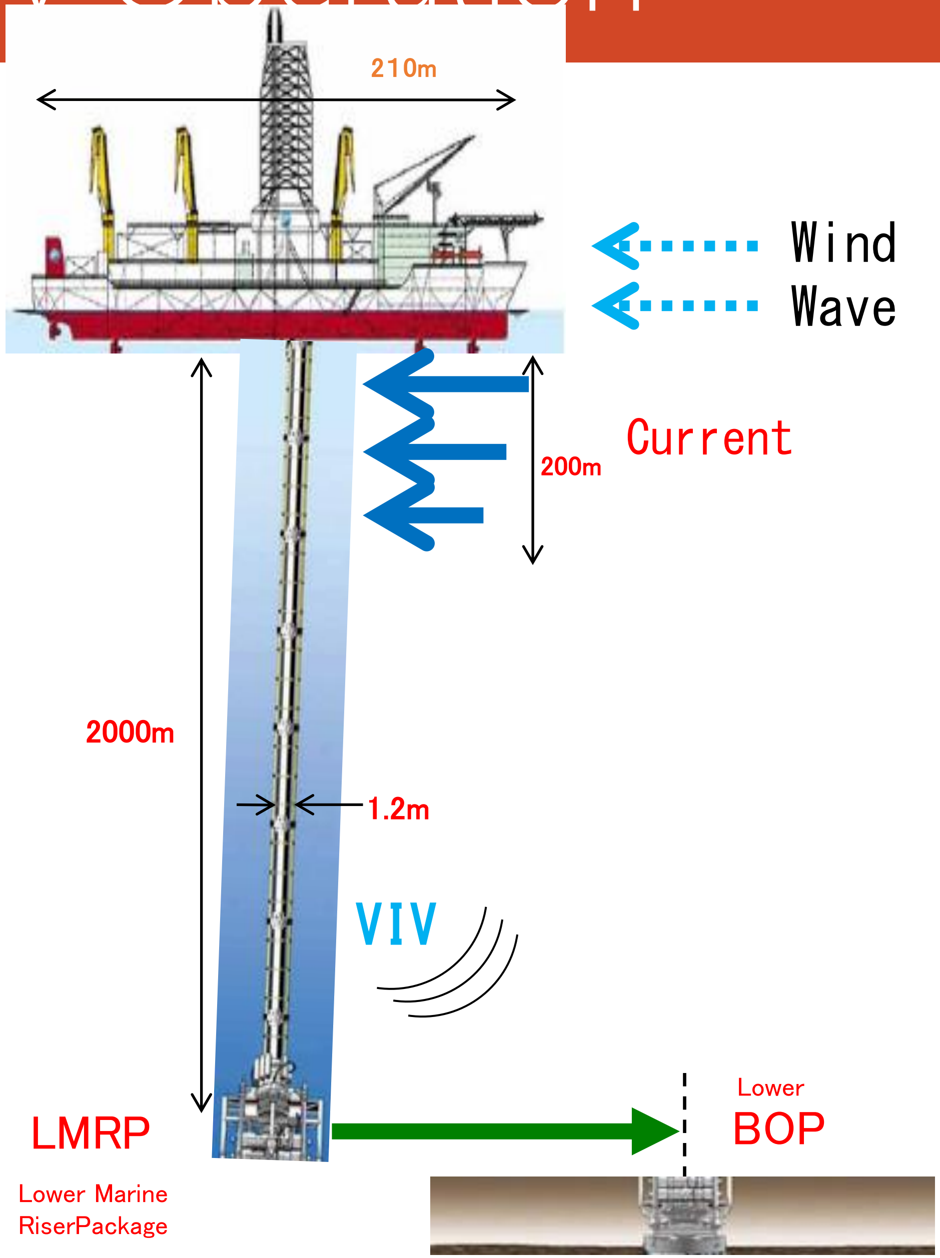


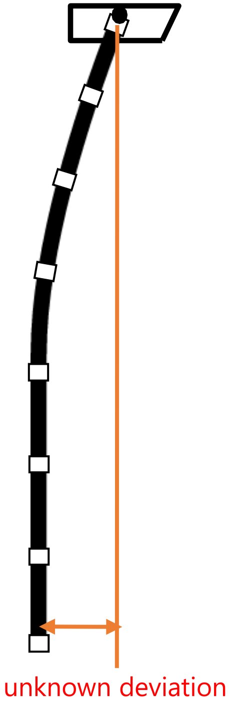
Entry Corn

CHIKYU: Reentry Operation

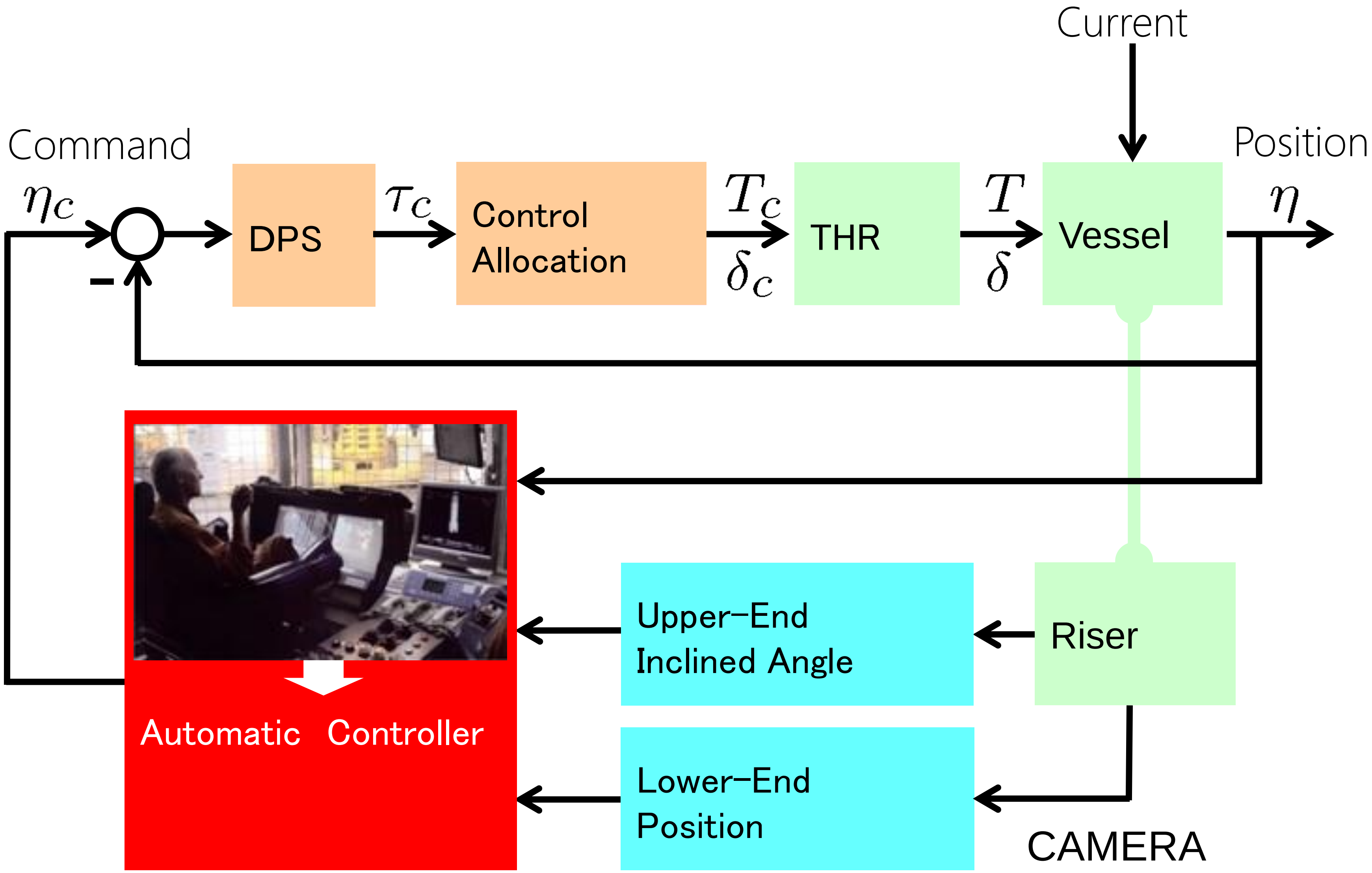




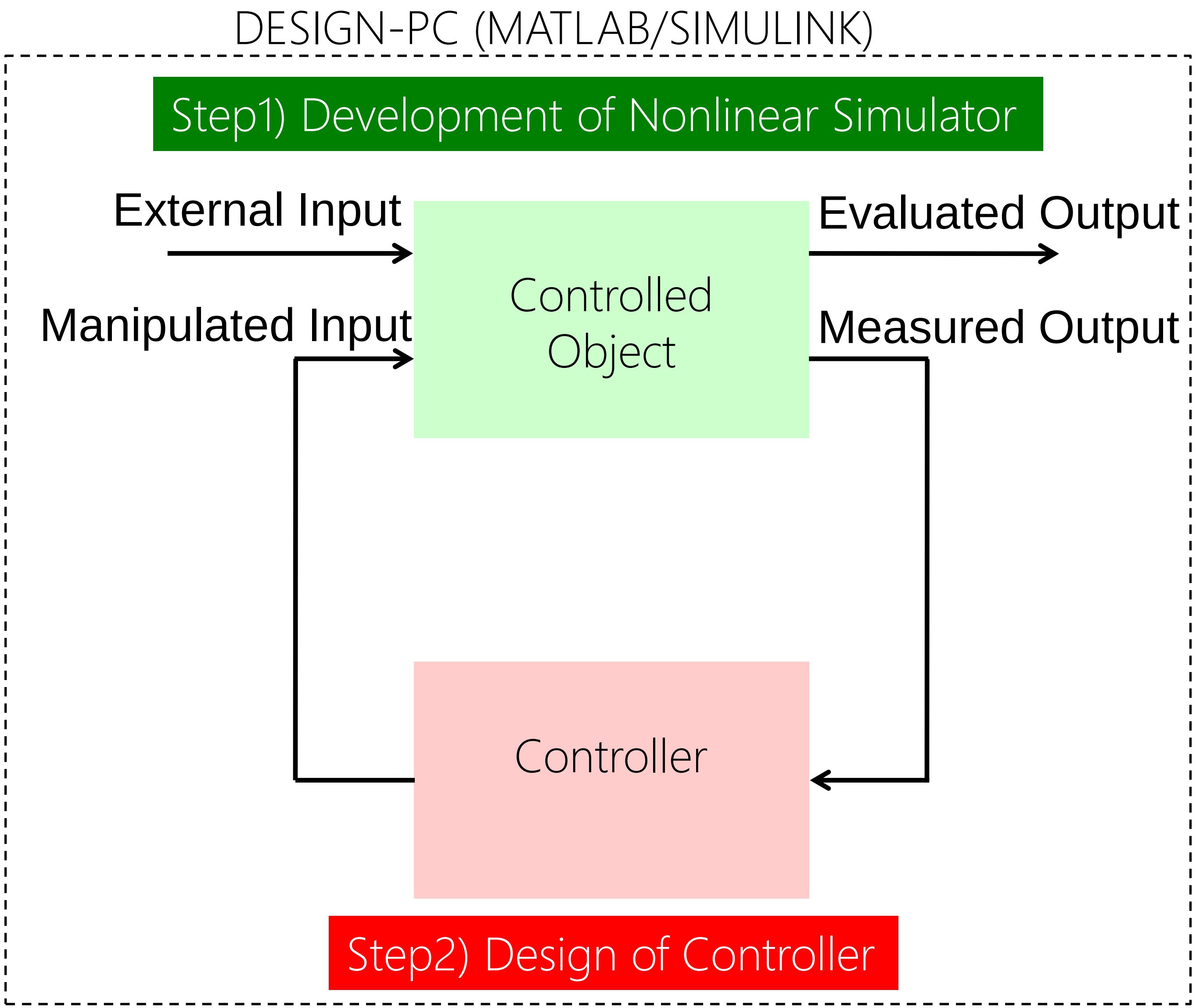




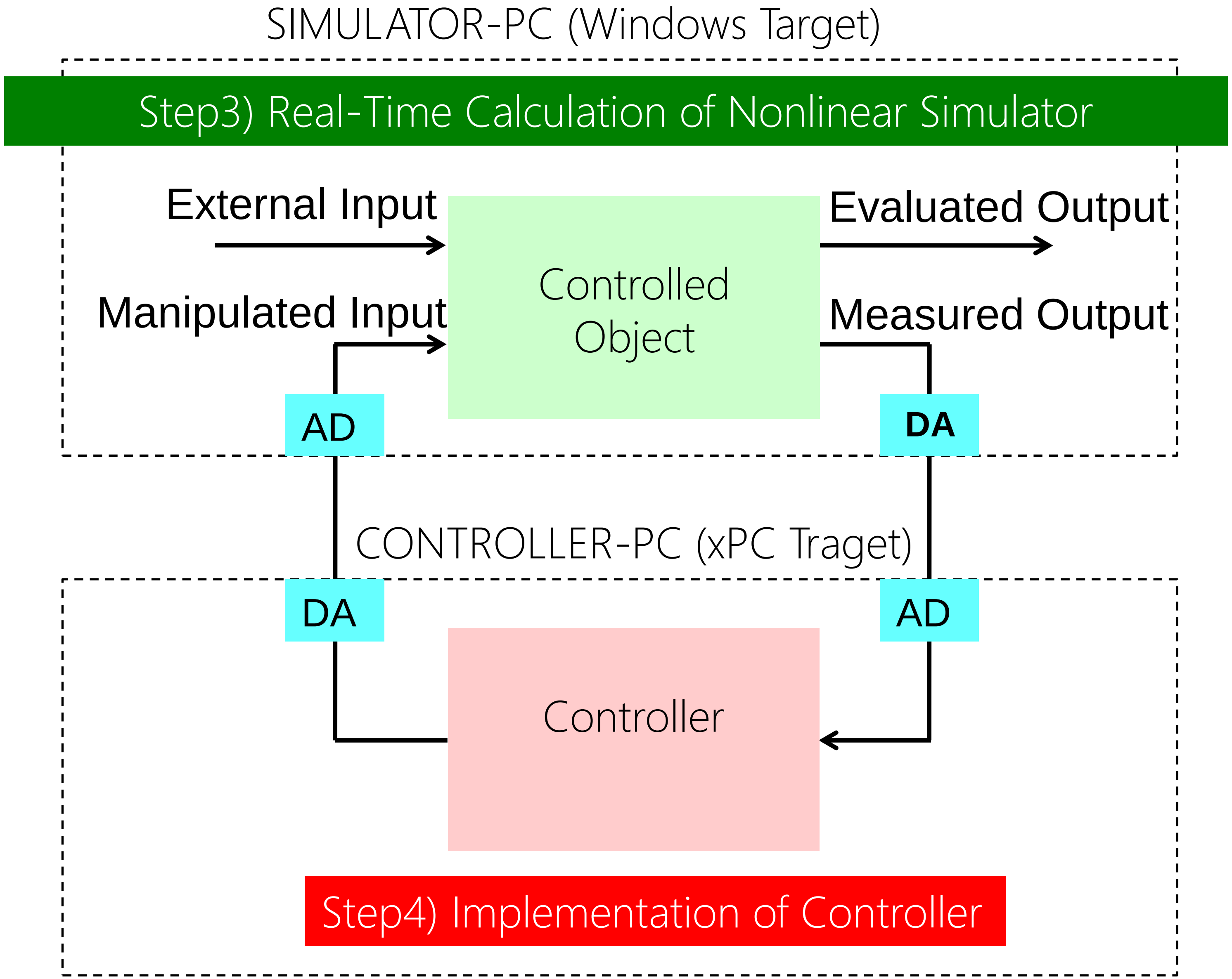
CHIKYU: Reentry Control System



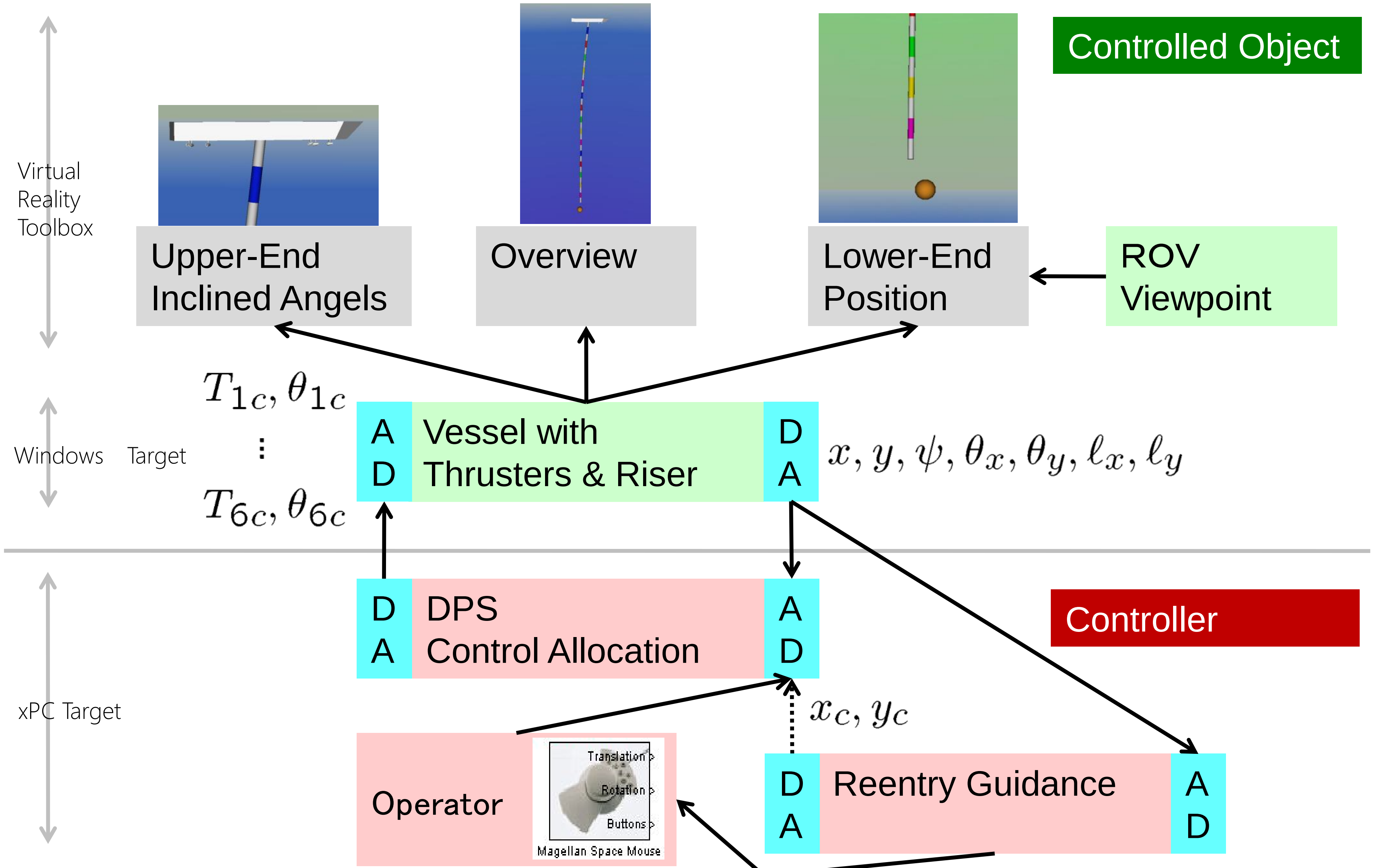
HILS for Control System Design



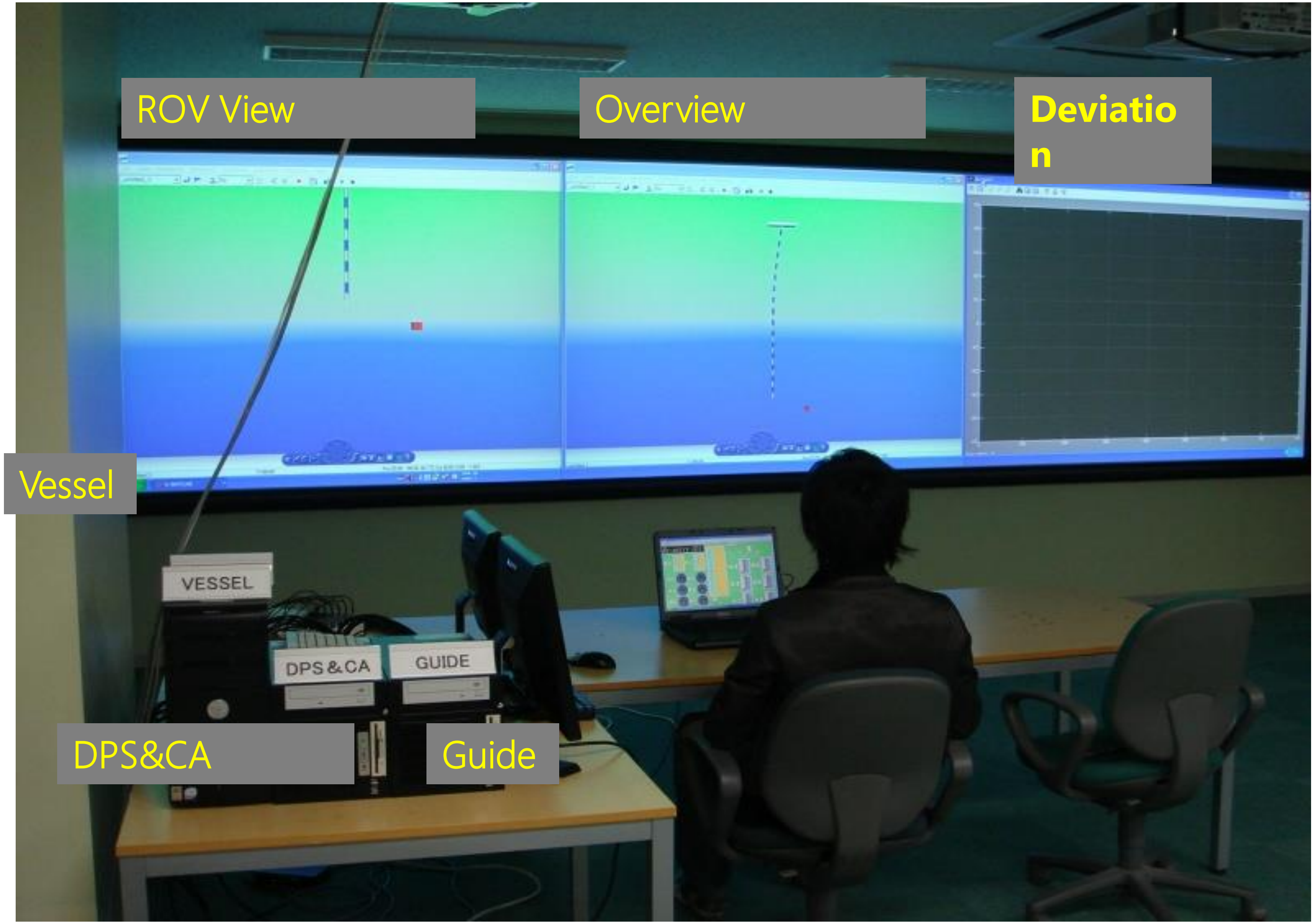
Virtual Control Experiment



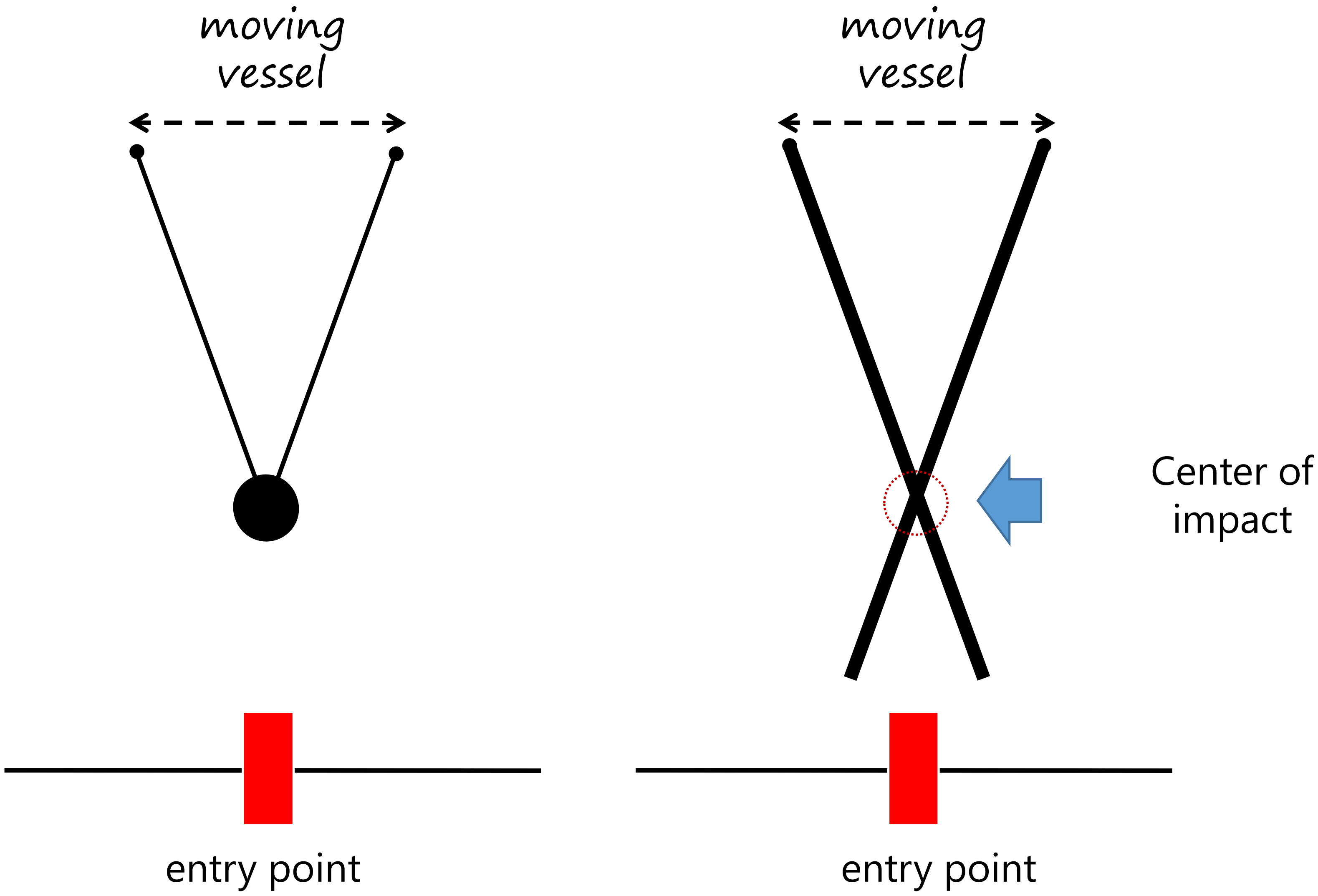
Developing Training Simulator



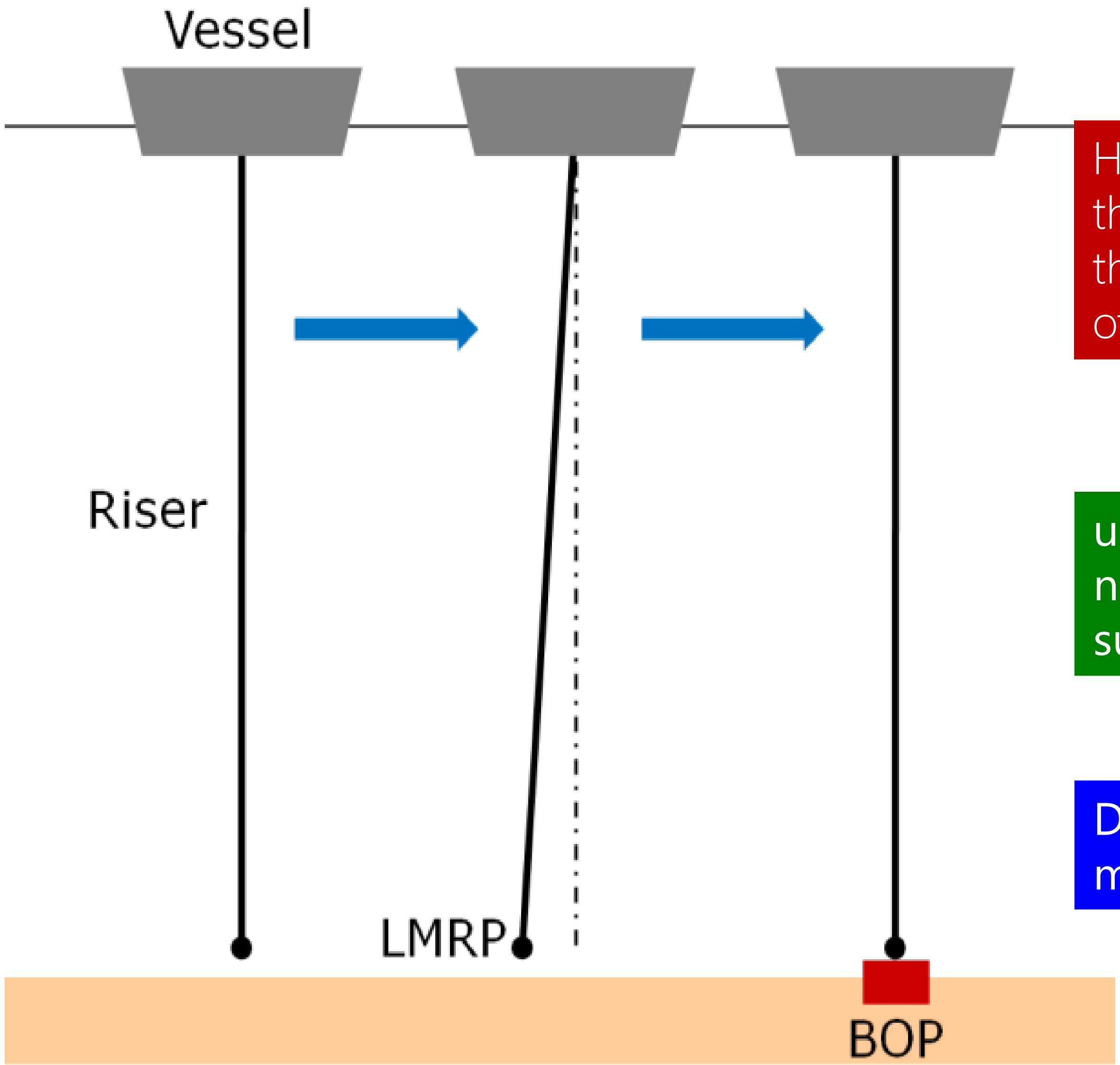
Training Somulator



Why the reentry op is difficult?



Reentry Control Problem

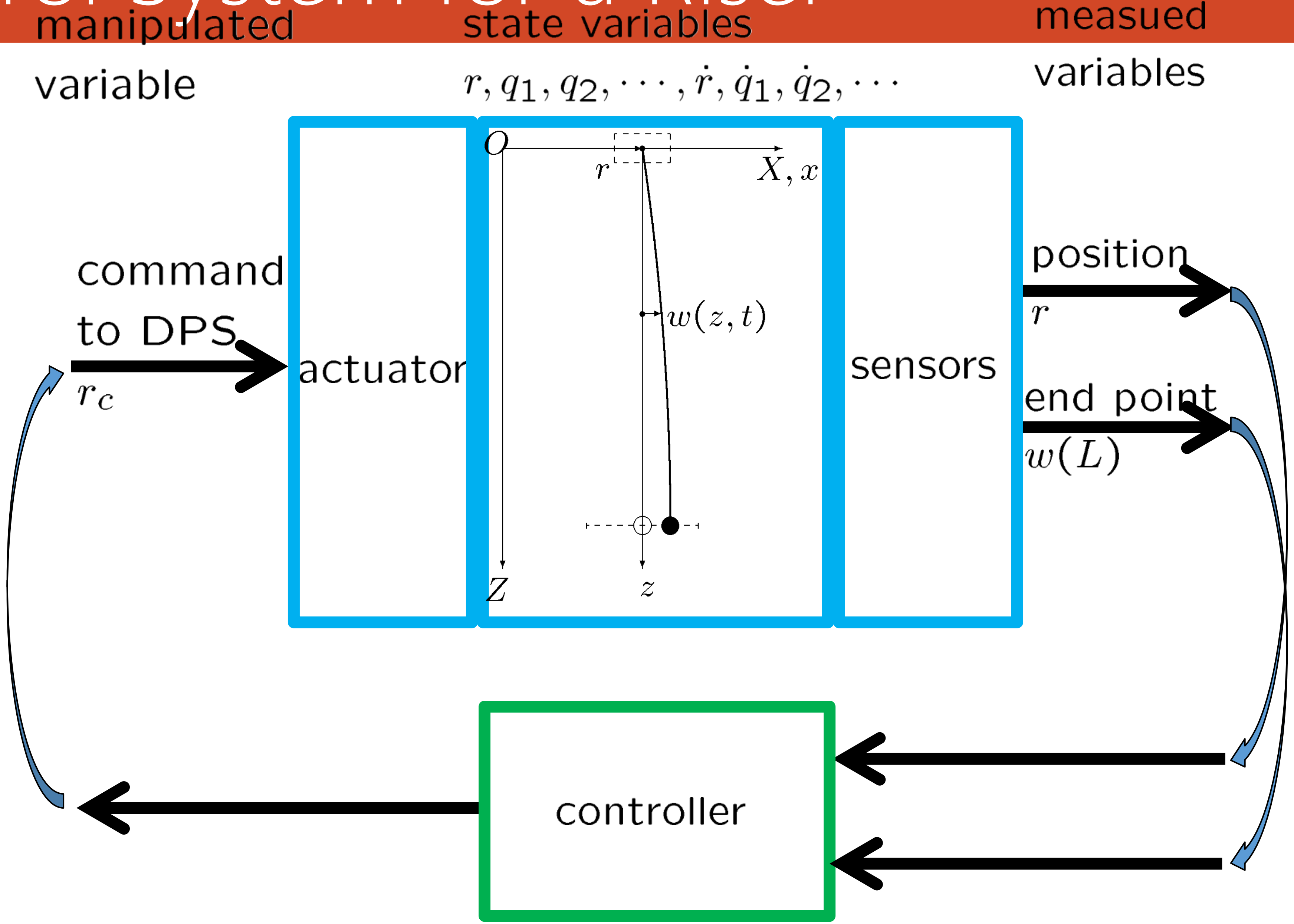


How to realize
the stability and
the performance
of reentry control

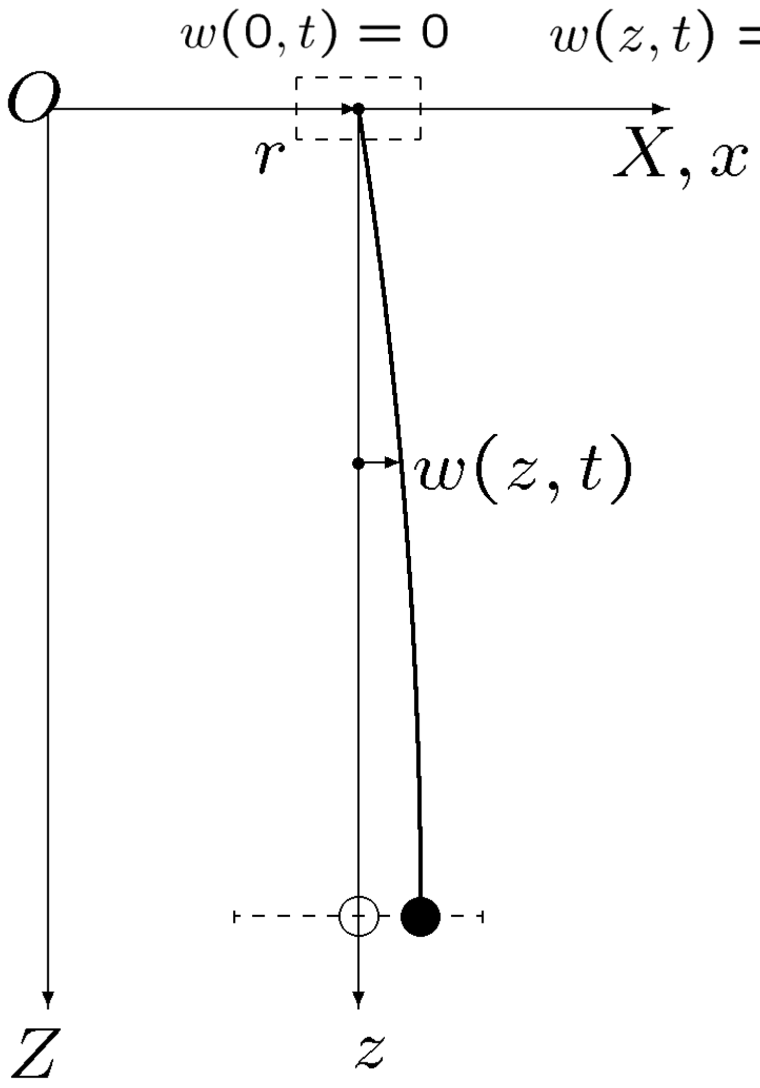
under no wind,
no current and
surface current

DPS not to be
modified

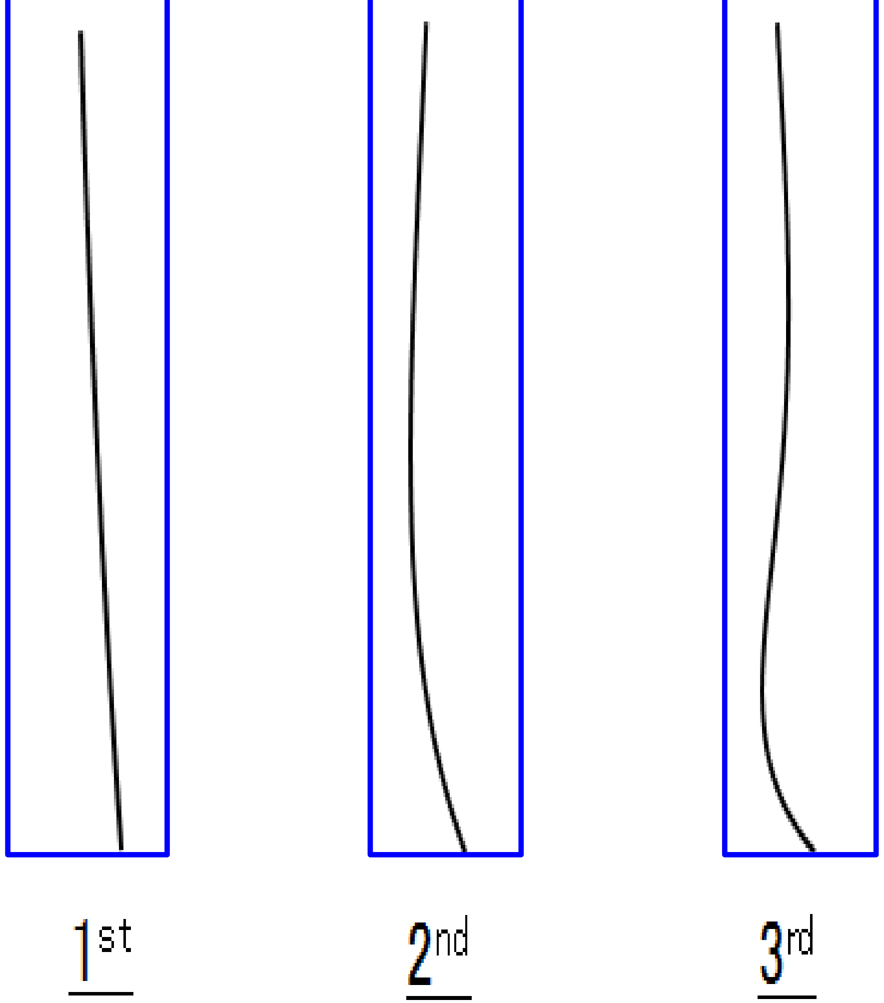
Control System for a Riser



State Variables q1,q2,...



$$w(z,t) = \boxed{\phi_1(z)} q_1(t) + \boxed{\phi_2(z)} q_2(t) + \boxed{\phi_3(z)} q_3(t) + \cdots$$



Motion Equation

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{r} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}\begin{bmatrix} \dot{r} \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{22} \end{bmatrix}\begin{bmatrix} r \\ q \end{bmatrix} = \begin{bmatrix} F_r \\ F_q \end{bmatrix}$$

$$M_{11} = M_s + M_b + (\tilde{m} + m_a)L$$

$$M_{12} = \int_0^L (\tilde{m} + m_a)\phi^T(z)dz + M_b\phi^T(L)$$

$$M_{21} = \int_0^L (\tilde{m} + m_a)\phi(z)dz + M_b\phi(L)$$

$$M_{22} = \int_0^L (\tilde{m} + m_a)\phi(z)\phi^T(z)dz + M_b\phi(L)\phi^T(L)$$

$$D_{11} = \int_0^L \zeta_d|V_{rel}|dz$$

$$D_{12} = \int_0^L \zeta_d|V_{rel}|\phi^T(z)dz \qquad (V_{rel}(z) = \dot{r} + \dot{w}(z) - V_c(z))$$

$$D_{21} = \int_0^L \zeta_d|V_{rel}|\phi(z)dz$$

$$D_{22} = \int_0^L \zeta_d|V_{rel}|\phi(z)\phi^T(z)dz$$

$$K_{22} = M_b g\phi(L)\phi'^{\,T}(L) + \int_0^L \phi(z)(\mu(z - \tilde{L})\phi'^{\,T}(z))'dz$$

Velocity Input Model

$$M_{21}\ddot{r} + M_{22}\ddot{q} + D_{21}\dot{r} + D_{22}\dot{q} + K_{22}q = F_q$$

$$\Downarrow \qquad \mathcal{M}_{21} = \text{diag}\{M_{21}(1), \cdots, M_{21}(N)\}, \mathbf{1}_N = \left[\, 1, \cdots, 1 \,\right]^T$$

$$\underbrace{\mathbf{1}_N\ddot{r} + \mathcal{M}_{21}^{-1}M_{22}\ddot{q}}_{\ddot{\xi}} + \underbrace{\mathcal{M}_{21}^{-1}D_{22}M_{22}^{-1}\mathcal{M}_{21}}_{-A_{22}}\underbrace{(\mathbf{1}_N\dot{r} + \mathcal{M}_{21}^{-1}M_{22}\dot{q})}_{\dot{\xi}}$$

$$+ \underbrace{\mathcal{M}_{21}^{-1}K_{22}M_{22}^{-1}\mathcal{M}_{21}}_{-A_{21}}\boxed{\underbrace{(\mathbf{1}_N r + \mathcal{M}_{21}^{-1}M_{22}q)}_{\xi}} - \underbrace{\mathcal{M}_{21}^{-1}K_{22}M_{22}^{-1}\mathcal{M}_{21}\mathbf{1}_N}_{A_{23}} r$$

$$= \underbrace{(\mathcal{M}_{21}^{-1}D_{22}M_{22}^{-1}\mathcal{M}_{21}\mathbf{1}_N - \mathcal{M}_{21}^{-1}D_{21})}_{B_2}\dot{r} + \underbrace{\mathcal{M}_{21}^{-1}F_q}_{w_2}$$

velocity input

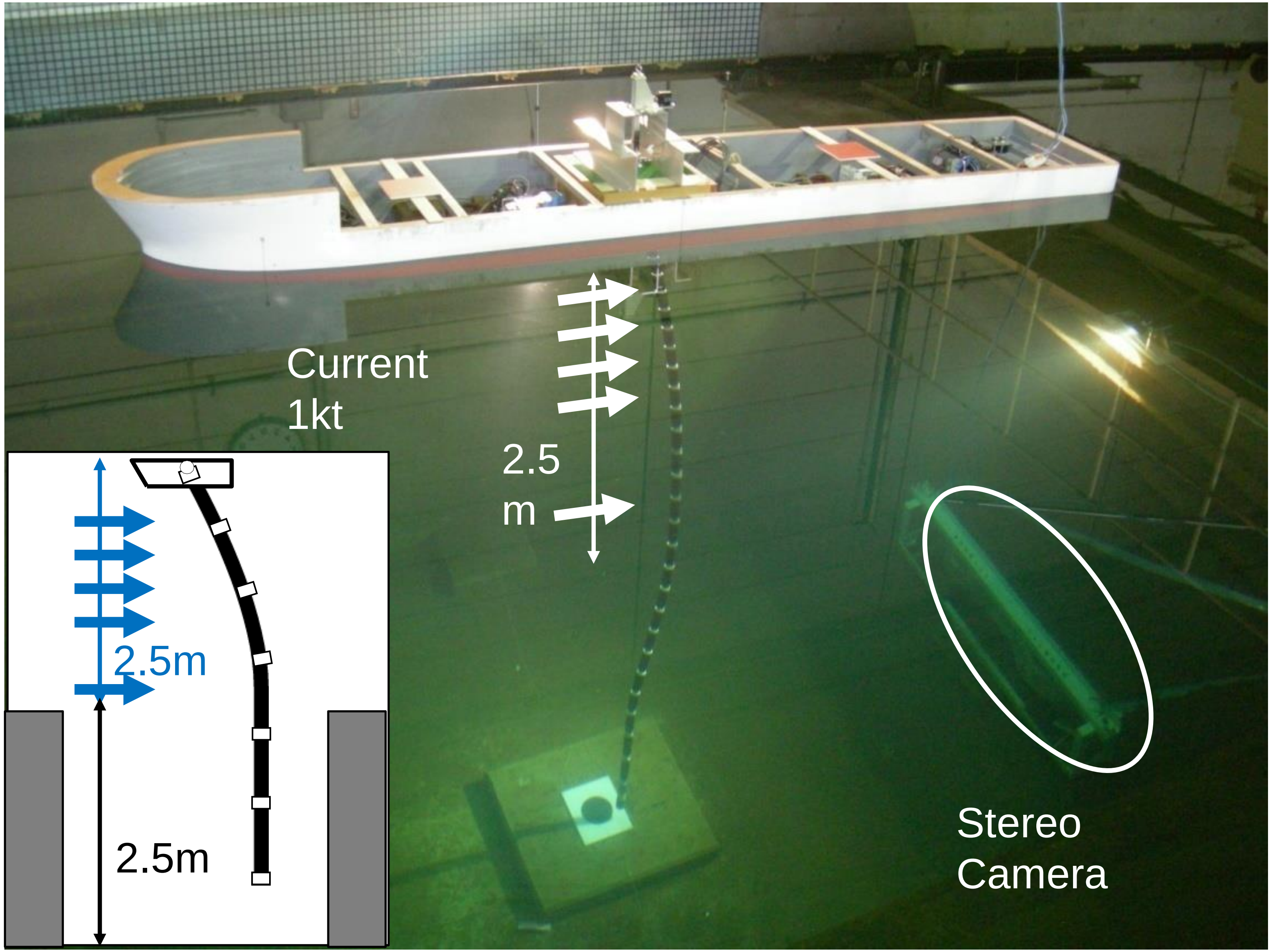
$$\ddot{\xi} = A_{21}\xi + A_{22}\dot{\xi} + A_{23}r + B_2\dot{r} + w_2$$

varying parameter

$$\Downarrow \qquad q = M_{22}^{-1}\mathcal{M}_{21}(\xi - \mathbf{1}_N r)$$

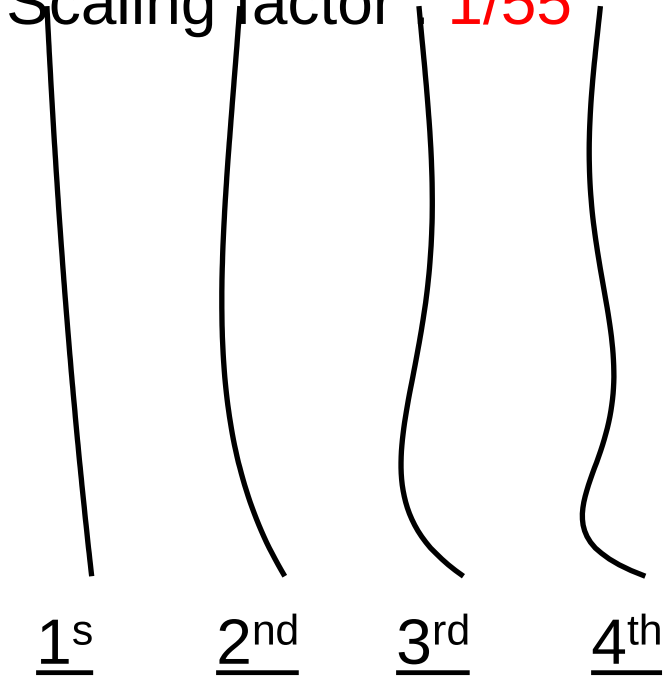
$$\boxed{\underbrace{\frac{d}{dt}\begin{bmatrix}\xi\\ \dot{\xi}\\ r\end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix}0 & I_N & 0\\ A_{21} & A_{22}(|V_{rel}|) & A_{23}\\ 0 & 0 & 0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\xi\\ \dot{\xi}\\ r\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}0\\ B_2(|V_{rel}|)\\ 1\end{bmatrix}}_{B}\underbrace{\dot{r}}_{u} + \underbrace{\begin{bmatrix}0\\ w_2\\ 0\end{bmatrix}}_{w}}$$

Experimental Set Up (5m)



Riser Pipe Unit Model (5m)

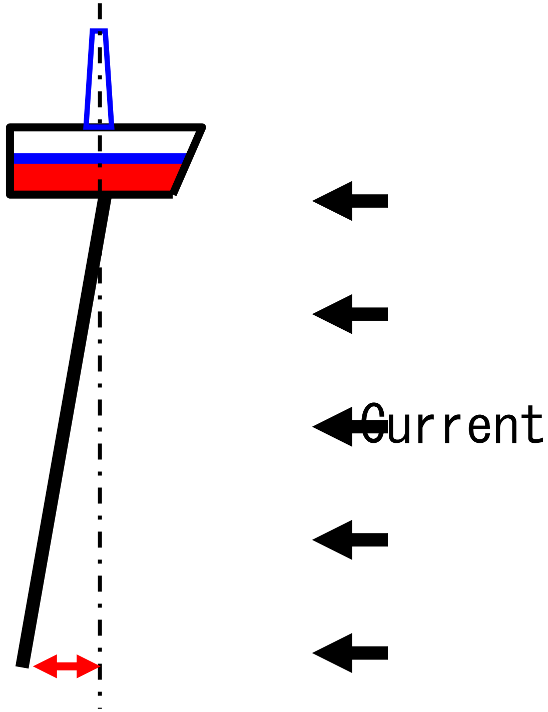
CHIKYU: 210 m length
Vessel Model: 3.8 m length
Scaling factor : 1/55



$1/\sqrt{55}$ period

Dynamic Similarity

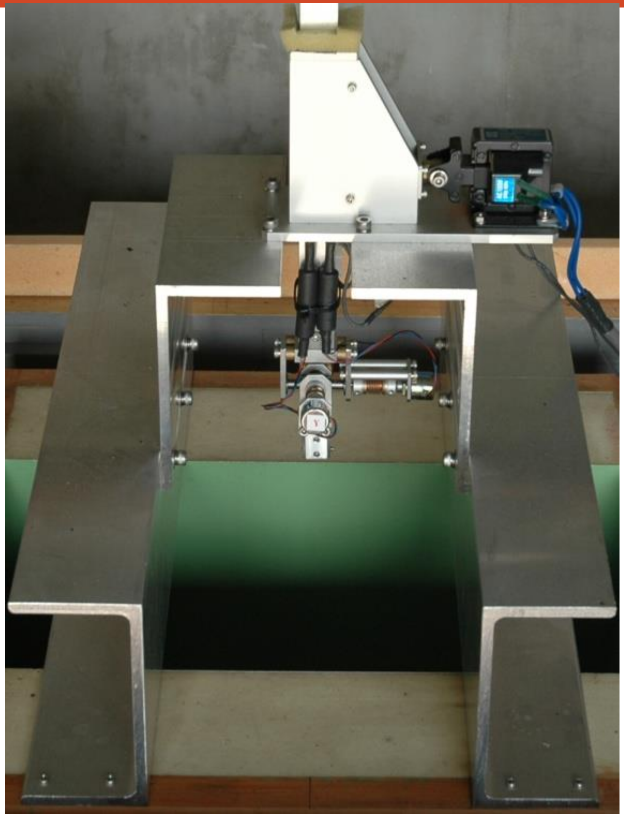
Riser: 2500 m length
Riser Model: 4.8 m length
Scaling factor : 1/500



1/55 deviation

Geometric Similarity

Experimental Parts



Exp#6 (Overview)